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THEORIA MOTUS LUNAE

EXHIBENS

OMNES EIUS INAEQUALITATES

IN

ADDITAMENTO

HOC IDEM ARGUMENTUM ALITER TRACTATUR

SIMULQUE OSTENDITUR

QUEMADMODUM MOTUS LUNAE CUM OMNIBUS

INAEQUALITATIBUS

INNUMERIS ALIIS MODIS

REPRAESENTARI

ATQUE AD CALCULUM REUOCARI POSSIT

AUCTORE

L. EULERO

BIBLIOTECA NAZIONALE VITTORIO EMANUELE III · NAPOLI

IMPENSIS

ACADEMIAE IMPERIALIS SCIENTIARUM

PETROPOLITANAE

ANNO 1753.

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Academiam Scientiarum Petropolitanam triennio abhinc omnes, qui ingenii viribus confiſi, ad examinandam Neutonianam de motu Lunae Theoriam, animum applicare vellent, inuitaſſe, atque ei, qui in hac parte tenuiſſet primas, praemii loco propoſuiſſe nummos aureos centum; poſtmodum Celeberrimum Clairautum huius certaminis exſtitiſſe victorem, publico Academicorum, qui Petropoli agunt, iudicio diuulgatum.

Celeberrimus Eulerus, Academiae Petropolitanae membrum honorarium, officii ſui eſſe exiſtimauit, ferre vna cum ceteris de illa Clairauti diſſertatione iudicium. Tranſmiſit ergo huc ad nos una cum ſua ſententia ampliſſimam de eodem argumento diſſertationem; quae quo celerius innoteſceret, viſum eſt Academiae Praeſidi, minoris Ruſſiae Hetmano Illuſtriſſimo Cyrillo Gregoridae, Comiti Raſumouio, eam tradere Academicis in ſolenni conuentu examinandam, ea fini, vt ſi ſuffragio Academicorum comprobata, dignaque iudicata foret, quae orbis eruditi proponeretur theatro, ea praelo quam maturrime ſubiiceretur; quandoquidem ille mos iam inde a principio obtinuit, vt, quae in publico coetu praeleguntur diſſertationes, eae vel ante ſolennem actum, vel paruulo intermiſſo ſpatio typis diuulgentur poſtea.

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Ceterum diſſertatio illa conſtat magnam partem calculis, veris quidem illis et omnibus numeris abſolutis, ſed propter nimiam ſui molem atque difficultatem auditu moleſtiſſimis: quae ſi recitarentur publice, periculum erat, ne auditorum animi auſcultandis iis deficerent, neue Academia Summorum patientia Virorum abvti, caeteroſque enicare odio videretur. Praeuidit hoc incommodum, prouiditque Illuſtriſſimi Praeſidis ſapientia. Mandavit Aſtronomiae Profeſſori V. C. Nicetae Popouio, cuius id temporis dicendi erat prouincia, vt prolixam illam Euleri diſſertationem, omiſſis calculis, redigeret in compendium, et quae inde excepiſſet, auditorum cauſa recitaret publice. Quo quidem facto et auribus hominum conſuluit, et tamen rerum capitum participes eos facere aequabili temperamento inſtituit: at-

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que diſſertationem, ne quid forte naeuorum obreperet, ipſo auctore coram typis excudi aequum cenſuit.

Iam qui illo tempore interfuerant conuentui Academicorum ſolenni auditores, aequis nimis auribusque auſcultarunt excerpta recitantem Euleriana Popouium: eſt ergo quod ſperare liceat, et iis, quorum intereſt, vt qui hoc genere ſtudiorum maxime delectantur, factum iri ſatis ipſa diſſertatione. Datum Petropoli Nov. 1752.

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INTRODUCTIO.

Cum nullum ſit dubium, quin ſummi Newtoni Theoria, qua motum Planetarum feliciſſimo cum ſucceſſu certis legibus adſtrinxit, plurimum ad motum Lunae accuratius determinandum contulerit, maximi ſane in Aſtronomia momenti eſt quaeſtio: vtrum haec Newtoni Theoria omnino ſit ſufficiens omnibus motus inaequalitatibus, quae in Luna obſeruantur, exactiſſime explicandis nec ne? Quanquam autem a Newtoni aſſeclis plerumque affirmari ſolet, nullam in motu Lunae obſeruari inaequalitatem, cuius ratio in iſta Theoria non contineatur; tamen tantum abeſt, vt hic conſenſus a quoquam perſpicue ſit monſtratus, vt potius applicatio huius Theoriae ad Lunam tantis implicetur calculi difficultatibus, quibus penitus euoluendis vires ingenii humani vix ſufficere videntur. Plurimae quidem adhuc prodierunt Tabulae Lunares, quae ex Theoria Newtoniana deductae perhibentur, ſed praeterquam quod ſaepius ultra 5′ ab obſeruationibus diſcrepent, earum conuenientia cum Theoria ipſa neutiquam eſt euicta; quin potius pleraeque Tabulae inaequalitatum non tam Theoriae quam obſeruationibus ſunt ſuperſtructae. Huiusmodi ergo tabularum ſiue conſenſus ſiue diſſenſus cum obſeruationibus neque ad Theoriam Newtonianam pleniſſime confirmandam, neque ad eam infringendam alle-

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gari poteſt: nam quatenus iſtae tabulae obſeruationibus ſatisfaciunt, hoc non ſoli Theoriae eſt tribuendum; quatenus autem cum obſeruationibus minus conueniunt, hoc ne Theoriae quidem imputari poterit, propterea quod iſtae Tabulae non ſoli Theoriae innituntur.

Quaeſtio itaque, cujus mentionem feci, recte enodari nequit, niſi ante eiusmodi Tabulae exhibeantur, quae ex ſola Theoria, nullis obſeruationibus in ſubſidium vocatis, ſint formatae; tum enim demum ex huiusmodi Tabularum collatione cum ingenti obſeruationum ſummo ſtudio inſtitutarum copia diiudicare licebit, vtrum Theoria omnibus obſeruationibus reſpondeat, an vero correctione quapiam indigeat. Non difficile quidem eſt ex principiis mechanicis motum Lunae aequationibus analyticis complecti; quoniam autem hae aequationes plures variabiles inter ſe permixtas continent, atque adeo differentialibus ſecundi ordinis implicantur, earum reſolutio maximis difficultatibus eſt obnoxia; et quoniam alio modo niſi per approximationem ſuſcipi non poteſt, vtcunque inſtituatur, ſemper non leue dubium remanet, vtrum partes, quae in calculo ſunt neglectae et praetermiſſae, nihil, quod in motu Lunae eſſet notabile, efficere potuerint. Hoc modo explicatio motuum Lunae tota ad ſolam Analyſin transfertur, ac difficultates, quibus premitur, inde oriuntur, quod Analyſis nondum ſatis eſt exculta.

Cum igitur Theoria Newtoniana hoc principio latiſſime patente innitatur, quod omnia corpora coeleſtia ſe mutuo attrahant in ratione reciproca duplicata diſtantiarum,

rum,

rum, ſi motum Lunae ſecundum hanc Theoriam definire velimus, vires erunt ſpectandae omnes, quibus Luna ſollicitatur. Atque inter has vires primaria eſt ea, qua ad terram vrgetur, quae ſi ſola adeſſet, terraque quieſceret, Luna in ellipſi perfecta ſecundum regulas Keplerianas motum ſuum circa terram abſolueret. At cum Luna praeterea aeque ac terra ipſa etiam ad ſolem trahatur, hac vi motus ille regularis non mediocriter perturbabitur: atque haec vis a ſole profecta omnium difficultatum, quae in determinatione motus Lunae offenduntur, cauſa eſt exiſtimanda. Reliquae enim vires, quibus forte Luna ſecundum Theoriam Newtoni ad reliquos planetas vrgeri deberet, tam ſunt exiguae, vt effectus inde oriundus merito pro nihilo haberi queat.

Solas ergo vires ſolis ac terrae in computum duci oportet, ſi motum Lunae ſecundum Theoriam definire velimus, atque cum ex his viribus formulae analyticae fuerint erutae, quae motum Lunae complectantur, omne ſtudium in his formulis ita euoluendis erit impendendum, vt inde ad quoduis tempus propoſitum locus Lunae aſſignari, ac more apud Aſtronomos ſolito ſecundum longitudinem et latitudinem definiri queat. Hinc porro Tabulae Aſtronomicae pro motu Lunae erunt condendae, quibus omnes inaequalitates tam in longitudine quam in latitudine exhibeantur, ex quibus ſi pro cujusuis obſeruationis momento locus Lunae computetur, conſenſus vel diſſenſus calculi ab obſeruationibus Theoriam vel confirmabit, vel ejus defectum declarabit. Neque tamen Theoria ſola hujusmodi Tabulis conſtruendis ſufficit, ſed quaedam ele-

 menta

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menta extrinſecus ab obſeruationibus aſſumi oportet, quae ſunt 1°. Excentricitas orbitae Lunaris, quae ſalua theoria vel major vel minor eſſe potuiſſet; pendet enim a motu Lunae primitus impreſſo, quem Theoria non determinat, ſed tanquam cognitum aſſumit. 2°. Locus Lunae medius pro quapiam Epocha propoſita, qui pariter ex obſeruationibus eſt concludendus. 3°. Locus Apogei orbitae Lunaris pro Epocha quadam data. 4°. Tempus periodicum Lunae ſecundum motum medium, quod pendet a diſtantia Lunae media a Terra, ideoque ex ſola Theoria definiri nequit. 5°. Locus nodorum Lunae pro Epocha quadam data: et 6to denique inclinatio media orbitae Lunaris ad planum Eclipticae.

His autem ſex elementis per obſeruationes definitis reliqua omnia, quibus ad locum Lunae pro quouis tempore aſſignandum opus eſt, ex ſola Theoria ſunt petenda; quae primo ad quoduis tempus locum Apogei eiusque ideo motum verum praebere debet, vt inde ex loco Lunae medio eius anomalia media colligi queat. Deinde Theoria quoque omnes correctiones ſeu Proſtaphaereſes, quae loco Lunae medio vel additae vel ſublatae eius locum verum exhibeant, ſuppeditare debet; atque iſtae correctiones, quae motus inaequalitates appellari ſolent, partim ab Anomalia media Lunae, partim ab eius Phaſi ſeu elongatione a ſole, partimque ab Anomalia ſolis media pendent, ex quo triplici fonte numerus inaequalitatum in immenſum augetur. Porro etiam Theoria motum nodi eiusque omnes inaequalitates indicare tenetur, ac denique etiam pro quouis tempore orbitae Lunaris veram inclinationem

da

ad eclipticam, vt inde eius latitudinem veram eruere liceat.

Cum autem, vt iam innui, nemo adhuc omnes inaequalitates, quae in motu Lunae reperiuntur, ex Theoria elicuerit, vt ex iis iudicium ferri poſſit, quantum haec Theoria cum obſeruationibus conueniat; etſi nullum eſt dubium quin diſcrimen, ſi quod deprehenderetur, admodum paruum ſit futurum: iam pridem haec quaeſtio ex ſolo motu Apogei dirimi eſt coepta, dum aliis motus Apogei ex obſeruationibus cognitus magnopere a Theoria diſcrepare eſt viſus, alii autem etiam hoc loco pulcerrimum conſenſum Theoriae et veritatis iactauerunt. Mirum autem eſt, ipſum Newtonum nihil circa motum Apogei ex Theoria ſtatuiſſe, ſed eum ex ſolis obſeruationibus in calculum tranſtuliſſe, cum tamen motum nodorum ſumma ſagacitate ex Theoria elicuiſſet, atque veritati conſentaneum oſtendiſſet. Cur igitur motum Apogei plane ſilentio praeterierit, nulla alia ratio ſubeſſe videtur, niſi quod animaduerterit hunc motum, prouti ex Theoria prodiret, obſeruationibus parum fore conformem. Ex iis enim, quae Newtonus in ſuo immortali opere de motu abſidum in genere tradidit, non admodum difficile videtur motum apogei Lunae definire: verum hic praeter expectationem euenit, vt motus apogei annuus vix 20° ſuperans reperiatur, cum tamen ex obſeruationibus conſtet, Apogeum Lunae interuallo unius anni vltra 40° promoueri.

Siue autem iſta motus Apogei quantitas 20° legitime ſit ex Theoria deriuata, ſiue minus; conſideratio Apogei tutiſſimum praebet remedium quaeſtionem de ſufficientia

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Theoriae Newtonianae decidendi. Quamuis enim ex Theoria inaequalitas quaepiam in ipſo motu Lunae aliquot minutis ſecundis vel etiam primis maior minorue prodiret, quam experientia monſtraret, tamen tantilla differentia merito vel leui cuipiam errori in obſeruationibus, vel vitio in approximatione commiſſo tribueretur; quandoquidem aliunde certum eſt Theoriam Newtonianam non admodum a veritate recedere. At longe aliter eſt comparata ratio motus Apogei: quodſi enim vires Lunam ſollicitantes tantillum a Theoria Newtoniana diſcrepent, vt ex iis in ipſo motu Lunae vix perceptibile diſcrimen naſceretur, tamen inde in motu Apogei annuo differentia plurium graduum oriri poterit. Quae tanta differentia cum nulli errori vel obſeruationum vel ipſius calculi, ſiquidem omni cura inſtituatur, tribui queat, inueſtigatio motus apogei certiſſimum ſuppeditat criterium iudicandi, vtrum quaepiam Theoria veritati ſit conſentanea nec ne?

Quodſi ergo calculo rite adminiſtrato Theoria Newtoni reperiatur tantum Apogei Lunaris motum exhibere, quantus per obſeruationes deprehenditur, ſcilicet ultra 40° quotannis; fortius certe argumentum, quo veritas hujus Theoriae indubie demonſtretur, deſiderari nequit. Sin autem contra eueniat, vt progreſſio Apogei annua ex Theoria rite deriuata notabiliter a 40° deficiat, hinc certo erit concludendum Theoriam Newtonianam correctione quapiam indigere, neque vires, quibus Luna reuera ſollicitatur, exactiſſime huic Theoriae eſſe conformes.

Verum haec ipſa quaeſtio, vtrum Theoria Newtoniana ad verum apogei Lunae motum perducat nec ne? eſt profun-

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fundiſſimae indaginis, atque ſummam in calculo circumſpectionem ac ſollertiam requirit. Quanquam enim ex principiis generalibus, vnde vulgo motus abſidum definiri ſolet, ſatis luculenter ſemiſſis tantum pro motu Apogei Lunae elicitur; tamen quoniam in calculo plures termini, qui in determinationem motus Lunae ingrediuntur, ob paruitatem ſunt reiecti, merito dubitatio ſuboritur, num hi ipſi termini, ſi eorum ratio eſſet habita, non iſtum defectum compenſare valuerint? Quin etiam non defuere Geometrae, qui conſenſum huius Theoriae cum vero Apogei motu demonſtrare ſunt conati: verum plerumque non difficile erat paralogiſmum in ipſorum ratiociniis deprehendere. Maximam autem hoc loco attentionem meretur iudicium profundiſſimi Geometrae Clairaut, qui cum primum validiſſimis argumentis ſtatuiſſet, Newtoni Theoriam non vltra dimidium veri apogei Lunaris motus ſuppeditare, ſubito in contrariam abiit ſententiam ſtatuens hanc Theoriam eleganiſſime cum veritate conſpirare; neque certe tantae perſpicaciae Vir a priſtina ſententia, quam omni ſtudio propugnauerat, receſſiſſe eſt putandus, niſi firmiſſimis argumentis eo eſſet adactus.

Cum autem omnes rationes, quae Ipſum ad hanc retractationem impulerint, nondum publice expoſuerit, liceat mihi quidem, qui ſemper contrariae ſententiae fui addictus, tantiſper arduam hanc quaeſtionem tanquam nondum deciſam ſpectare, donec per propriam inueſtigationem inuenero, quid de ea ſit ſtatuendum. Poſtquam enim iam a longo temporis interuallo plurimum ſtudii in indagatione motus Lunae conſumſiſſem, ac variis methodis inſiſtens

ſemper

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ſemper concluſionem Theoriae Newtonianae minus fauentem eſſem adeptus; quam tamen pro rite demonſtrata venditare non eram auſus, propterea quod approximatione eſſem vſus, ac ſemper ſuſpicio quaedam ratione terminorum praetermiſſorum remaneret: nuper in aliam incidi viam hanc inueſtigationem ſuſcipiendi, quae mihi multo certior videtur, ita vt per eam nulla dubitatione interiecta ad veritatem penetrare confidam. Ne autem ſi forte Theoriam Newtonianam minus ſufficientem inuenero, calculum ſecundum aliam Theoriam de nouo inſtituere cogar, ſtatim meam inueſtigationem in latiori ſenſu exordiar, viresque quibus Luna ad terram ſollicitatur, non exacte ſed proxime tantum quadratis diſtantiarum reciproce proportionales aſſumam: deinceps ſcilicet innoteſcet, vtrum haec aberratio a regula Newtoniana locum habeat nec ne? Calculum autem ita adornabo, vt quicquid euenerit, non ſolum verum apogei motum aſſequar, ſed etiam omnes Lunae inaequalitates inde elicere valeam, quas deinceps Aſtronomorum more tabulis complecti licebit.

Primum ergo problema in latiſſimo ſignificatu concipiam, vt corporis a viribus quibuscunque ſollicitati motum ſim inueſtigaturus: deinde vires, quibus Luna actu vrgeri cenſenda eſt, in calculum introducam, et aequationes Lunae motum determinantes exhibebo. Has porro aequationes variis modis in alias formas transmutabo, donec eas eo perduxero, vbi ad finem propoſitum maxime accommodatae videbuntur: quo cum peruenero, tandem tam motum Apogei, quam cunctas Lunae inaequalitates motus ex calculo deriuare ſtudebo.

CAPUT

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CAPUT I.

DE MOTU CORPORIS A VIRIBUS QUIBUSCUNQUE SOLLICITATI.

§. 1.

Quoniam corpus a viribus quibuscunque ſollicitari ponimus, fieri poteſt, vt eius motus non in eodem plano abſoluatur. Hinc ad ejus motum per calculum ita repraeſentandum, vt ad quoduis tempus verus locus, in quo corpus verſabitur, aſſignari queat, conueniet corporis motum ad planum quoddam fixum pro lubitu aſſumtum referri. Exhibeat igitur Tabula hoc planum, atque corpus iam verſetur extra hoc planum in puncto L, vnde ad planum demittatur perpendiculum L M; eritque punctum M locus corporis ad planum relatus. Quod ſi ergo ad quoduis tempus propoſitum hunc corporis locum relatum M, ſimulque eius a plano diſtantiam L M indicare valeamus, verus corporis locus L ad hoc tempus innoteſcet. Fig. 1.

§. 2. Ad locum autem puncti M commodius determinandum, aſſumamus in plano rectam quandam fixam CQ pro axe, ita vt ducta ex M ad hanc rectam perpendiculari M P, locus puncti M more apud Geometras recepto per coordinatas orthogonales definiatur. Aſſumto ergo porro in axe puncto quodam fixo C, vnde abſciſſae C P computentur, erit P M applicata puncto M reſpondens, & ipſum punctum L determinabitur per tres coordinatas inter ſe normales CP, PM &

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M L. Cum igitur praeſenti temporis momento corpus in L verſari ponatur, vocentur iſtae tres coordinatae eo ſpectantes:

$$CP = p; \quad PM = q \quad \text{et} \quad ML = r$$

elapſo autem temporis elemento, quod per dt indicemus, coordinatae ternae tum locum corporis indicantes erunt:

$$p + dp; \quad q + dq; \quad \text{et} \quad r + dr.$$

§. 3. Quaecunque nunc fuerint vires, quibus corpus ſollicitatur, eae ſemper per cognitam virium reſolutionem reduci poterunt ad ternas vires ſecundum directiones ternarum coordinatarum vrgentes. Ponamus ergo vim $= P$, qua corpus in L ſecundum directionem ipſi P C parallelam trahitur: eam porro vim $= Q$, qua corpus ſecundum directionem ipſi M P parallelam trahitur: eamque denique vim $= R$, qua corpus ſecundum directionem L M ſollicitatur. Has ſcilicet vires ita directas concipio, vt ſi corpus earum actioni libere obediret, valores coordinatarum p, q, r inde diminuerentur. His poſitis, ex principiis Mechanicae conſtat, ſi elementum temporis dt pro conſtanti aſſumatur, motum corporis his tribus formulis differentiodifferentialibus determinari

$$\text{I. } ddp = -\tfrac{1}{2}Pdt^2; \quad \text{II. } ddq = -\tfrac{1}{2}Qdt^2; \quad \text{III. } ddr = -\tfrac{1}{2}Rdt^2.$$

§. 4. Verum hae coordinatae ad vſum aſtronomicum, ad quem hic potiſſimum reſpicimus, non ſatis ſunt accommodatae. Nam ſi ſpectatorem in C conſtitutum aſſumimus, locus L, vbi corpus cernetur, commodiſſime

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me per quantitatem rectae C M, et angulum Q C M vna cum angulo M C L repraeſentatur: atque ſi tabula planum eclipticae referat, rectaque C Q ad principium arietis ſit directa, angulus Q C M in Aſtronomia vocari ſolet ſideris longitudo, angulus M C L vero eius latitudo, et recta C M eius diſtantia curtata. Vocemus ergo porro:

I. Diſtantiam curtatam ſeu rectam C M $= x$

II. Longitudinem ſeu angulum Q C M $= \phi$

III. Latitudinem ſeu angulum M C L $= \psi$

ac poſito conſtanter ſinu toto $= 1$, erunt valores coordinatarum ante adhibitarum:

$CP = p = x \cos\phi$; $PM = q = x \sin\phi$ & $ML = r = x \tang\psi$

atque diſtantia corporis vera a puncto C erit C L $=$ $x \sec.\psi = \frac{x}{\cos\psi}$.

§. 5. Sumtis nunc differentialibus more conſueto obtinebimus:

$$dp = dx \cos\phi - x\,d\phi \sin\phi\,;\ dq = dx \sin\phi + x\,d\phi \cos\phi$$

$$\text{et } dr = dx \tang\psi + \frac{x\,d\psi}{\cos\psi^2}$$

atque hinc denuo differentialibus ſumendis reperietur,

$$ddp = ddx \cos\phi - 2dx\,d\phi \sin\phi - x\,dd\phi \sin\phi - x\,d\phi^2 \cos\phi$$

$$ddq = ddx \sin\phi + 2dx\,d\phi \cos\phi + x\,dd\phi \cos\phi - x\,d\phi^2 \sin\phi$$

$$ddr = ddx \tang\psi + \frac{2dx\,d\psi}{\cos\psi^2} + \frac{x\,dd\psi}{\cos\psi^2} + \frac{2x\,d\psi^2 \sin\psi}{\cos\psi^3}$$

 Binae

Binae priores formulae rite combinatae ſuppeditabunt ſequentes multo concinniores

$$ddp \cos\phi + ddq \sin\phi = ddx - x\,d\phi^2$$

$$ddq \cos\phi - ddp \sin\phi = 2\,dx\,d\phi + x\,dd\phi$$

ſicque habebitur:

$$ddx - x\,d\phi^2 = -\tfrac{1}{2}\,dt^2\ (P\cos\phi + Q\sin\phi)$$

$$2dx\,d\phi + x\,dd\phi = -\tfrac{1}{2}\,dt^2\ (Q\cos\phi - P\sin\phi)$$

Tertiam vero aequationem deinceps magis tractabilem efficiemus.

§. 6 Manifeſtum autem eſt formulam $P\cos\phi + Q\sin\phi$ praebere vim ex viribus P et Q compoſitam, qua corpus in L ſecundum directionem rectae MC vrgetur, formulam vero alteram $Q\cos\phi - P\sin\phi$ exprimere vim ex eadem reſolutione ſecundum directionem MQ ad MC normalem directam. Cum igitur hae duae vires aſſumtis binis P et Q aequiualeant, ponamus eſſe

I. Vim corpus L ſecundum MC trahentem $= V$

II. Vim corpus L ſecundum MQ trahentem $= T$

manente tertia vi corpus ad planum normaliter ſecundum LM vrgente $= R$. Atque ſequentes habebimus aequationes:

I. $2\,dx\,d\phi + x\,dd\phi = -\tfrac{1}{2}\,T\,dt^2$

II. $ddx - x\,d\phi^2 = -\tfrac{1}{2}\,V\,dt^2$

III. $ddx \operatorname{tang}\psi + \dfrac{2dx\,d\psi}{\cos\psi^2} + \dfrac{x\,dd\psi}{\cos\psi^2} + \dfrac{2x\,d\psi^2\sin\psi}{\cos\psi^3} = -\tfrac{1}{2}\,R\,dt^2$

§. 7. Quo autem effectum tertiae vis R commodius ad calculum reuocemus, more apud Aſtronomos recepto contemplemur planum, in quo corpus durante elemento

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elemento temporis dt mouetur, et quod ſimul per punctum C tranſeat. Hoc igitur planum cum plano aſſumto interſectionem alicubi formabit, quae ſit recta C ☊, ac linea nodorum appellari ſolet; ſicque erit ☊ C L planum orbitae, in qua corpus L praeſenti inſtanti mouetur, et angulus, quo hoc planum ☊ C L ad planum fixum Q C M inclinatur, vocatur inclinatio orbitæ ad eclipticam pro tempore praeſenti. Cum igitur ex his duabus rebus latitudo ſideris definiri ſoleat, ponamus.

Longitudinem nodi aſcendentis ſeu angulum Q C ☊ $= \pi$ ac inclinationem orbitae ☊ C L ad eclipticam $= \rho$ atque loco latitudinis ψ has duas quantitates π et ρ definire oportebit.

§. 8. Tertia ergo aequatio in duas diſpertietur, ad quas inueniendas ex M et L ad lineam nodorum C ☊ ducantur normales M N et L N, eritque angulus L N M menſura inclinationis orbitae ad eclipticam; ideoque L N M $= \rho$. Tum vero ob angulum ☊ C M $= \Phi - \pi$ et C M $= x$ erit:

$$CN = x \operatorname{cof}(\Phi - \pi) \text{ et } MN = x \operatorname{fin}(\Phi - \pi)$$

hinc elicietur M L $= x \operatorname{tang} \rho \operatorname{fin} (\Phi - \pi)$, vnde prodit tang M C L $= \operatorname{tang} \psi = \operatorname{tang} \rho \operatorname{fin} (\Phi - \pi)$, quae formula inſeruit latitudini ψ ex cognita inclinatione ρ et loco nodi eiusue longitudine π inueniendae, ſi quidem iam cognita fuerit longitudo ſideris Φ. Quoniam autem ſidus elemento temporis dt in eodem plano manet, in differentiatione formulae $\operatorname{tang} \psi = \operatorname{tang} \rho \operatorname{fin} (\Phi - \pi)$,

 quanti-

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quantitates π et ϱ tanquam conſtantes ſpectari poterunt, eritque idcirco.

$$\frac{d\,\psi}{\operatorname{coſ}\psi^2} = d\,\Phi \text{ tang } \varrho \operatorname{coſ}(\Phi - \pi)$$

§. 9. Interim tamen nihil impedit, quominus in eadem differentiatione quantitates π et ϱ tanquam variabiles tractemus, quales reuera eſſe poſſunt ſucceſſu temporis; vnde orietur haec aequatio:

$$\frac{d\,\psi}{\operatorname{coſ}\psi^2} = \frac{d\,\varrho}{\operatorname{coſ}\varrho^2} \operatorname{ſin}(\Phi - \pi) + (d\Phi - d\pi) \text{ tang}\,\varrho \operatorname{coſ}(\Phi - \pi)$$

hicque valor ipſius $\frac{d\,\psi}{\operatorname{coſ}\psi^2}$ collatus cum praecedente praebebit hanc aequalitatem:

$$\frac{d\,\varrho}{\operatorname{coſ}\varrho^2} \operatorname{ſin}(\Phi - \pi) = d\pi \text{ tang } \varrho \operatorname{coſ}(\Phi - \pi)$$

vnde obtinemus $\frac{d\,\varrho}{\operatorname{ſin}\varrho \operatorname{coſ}\varrho} = \frac{d\pi \operatorname{coſ}(\Phi - \pi)}{\operatorname{ſin}(\Phi - \pi)} = \frac{d\,\pi}{\text{tang}(\Phi - \pi)}$.
Cum iam ſit $\frac{d\,\varrho}{\operatorname{ſin}\varrho \operatorname{coſ}\varrho} = \frac{d.\text{ tang}\,\varrho}{\text{tang}\,\varrho} = d.\,l\,\text{tang}\,\varrho$, erit

$$d.\,l\,\text{tang}\,\varrho = \frac{d\,\pi}{\text{tang }(\Phi - \pi)}$$

ex quo, ſi longitudo nodi iam fuerit reperta, ſine labore inclinatio ad eclipticam ϱ inveſtigari poterit.

§. 10. Differentiemus formulam primo inuentam

$$\frac{d\,\psi}{\operatorname{coſ}\psi^2} = d\,\Phi \text{ tang } \varrho \operatorname{coſ}(\Phi - \pi)$$

iterum, et cum ſit $d.$ tang $\varrho = \frac{d\pi \text{ tang}\,\varrho \operatorname{coſ}(\Phi - \pi)}{\operatorname{ſin}(\Phi - \pi)}$

erit:

erit:

$$\frac{dd\psi}{\cos\psi^2}+\frac{2d\psi^2\sin\psi}{\cos\psi^3}=dd\Phi\ \mathrm{tang}\,\varrho\ \cos(\Phi-\pi)$$
$$+\frac{d\Phi d\pi\,\mathrm{tang}\,\varrho\cos(\Phi-\pi)^2}{\sin(\Phi-\pi)}-d\Phi(d\Phi-d\pi)\mathrm{tang}\,\varrho\sin(\Phi-\pi)$$

ſeu
$$\frac{dd\psi}{\cos\psi^2}+\frac{2d\psi^2\sin\psi}{\cos\psi^3}=dd\Phi\ \mathrm{tang}\,\varrho\ \cos(\Phi-\pi)$$
$$+\frac{d\Phi d\pi\,\mathrm{tang}\,\varrho}{\sin(\Phi-\pi)}-d\Phi^2\,\mathrm{tang}\,\varrho\sin(\Phi-\pi)$$

qui valores pro ψ in tertia aequatione ſuperiori ſubſtituti ſuppeditabunt:

$$ddx\,\mathrm{tang}\,\varrho\sin(\Phi-\pi)+2dxd\Phi\,\mathrm{tang}\,\varrho\cos(\Phi-\pi)+xdd\Phi\,\mathrm{tang}\,\varrho\cos(\Phi-\pi)$$
$$+\frac{xd\Phi d\pi\,\mathrm{tang}\,\varrho}{\sin(\Phi-\pi)}-xd\Phi^2\,\mathrm{tang}\,\varrho\sin(\Phi-\pi)=-\tfrac{1}{2}Rdt^2$$

quae transmutatur in hanc:

$$(ddx-xd\Phi^2)\mathrm{tang}\,\varrho\sin(\Phi-\pi)+(2dxd\Phi+xdd\Phi)\,\mathrm{tang}\,\varrho\cos(\Phi-\pi)$$
$$+\frac{xd\Phi d\pi\,\mathrm{tang}\,\varrho}{\sin(\Phi-\pi)}=-\tfrac{1}{2}Rdt^2$$

§. 11. Commode, hic euenit vt in iſta formula illae ipſae expreſſiones differentio-differentiales $ddx-xd\Phi^2$ et $2dxd\Phi+xdd\Phi$ occurrant, quae ex actione duarum reliquarum virium ſunt enatae: vnde ſi formularum harum valores aequiualentes $-\tfrac{1}{2}Vdt^2$ et $-\tfrac{1}{2}Tdt^2$ ſubſtituamus, impetrabimus

$$-\tfrac{1}{2}Vdt^2\,\mathrm{tang}\,\varrho\sin(\Phi-\pi)-\tfrac{1}{2}Tdt^2\,\mathrm{tang}\,\varrho\cos(\Phi-\pi)+\frac{xd\Phi d\pi\,\mathrm{tang}\,\varrho}{\sin(\Phi-\pi)}=-\tfrac{1}{2}Rdt^2$$

qua differentiale $d\pi$, quo promotio elementaris lineae nodorum indicatur, ita determinabitur, vt ſit

$$d\pi=\tfrac{1}{2}dt^2.\frac{\sin(\Phi-\pi)}{xd\Phi}\left(V\sin(\Phi-\pi)+T\cos(\Phi-\pi)-\frac{R}{\mathrm{tang}\,\varrho}\right)$$

Deinde

Deinde cum ſit $d.\,l\,\text{tang}\,\varrho = \frac{d\pi}{\text{tang}\,(\phi - \pi)}$, erit

$$d.l\,\text{tang}\,\varrho = \tfrac{1}{2}dt^2.\,\frac{\text{coſ}(\phi-\pi)}{x\,d\phi}(V\,\text{ſin}(\phi-\pi)+T\,\text{coſ}(\phi-\pi)-\frac{R}{\text{tang}\,\varrho})$$

Duas ergo has aequationes loco ſuperioris tertiae, ex qua latitudo ψ inueniri debebat, in calculum introduci conueniet; inuentis enim π et ϱ erit tang ψ = tang ϱ ſin $(\phi - \pi)$.

§. 12. Hinc patet lineam nodorum nunquam eſſe mobilem, quin ſimul inclinatio ϱ variationi ſit obnoxia. Eadem enim vis $V\,\text{ſin}\,(\phi-\pi) + T\,\text{coſ}\,(\phi - \pi) - \frac{R}{\text{tang}\,\varrho}$, quae lineae nodorum motum imprimit eius longitudinem π immutando, ſimul in inclinatione ϱ variationem generat. Nulli autem plane immutationi tam linea nodorum, quam inclinatio erunt ſubjectae, ſi vis illa euaneſcat, quod euenit ſi media directio omnium virium corpus L ſollicitantium in ipſum planum ☊ CL, in quo corpus ſemel moueri coepit, perpetuo incidat; hicque eſt caſus, quo corpus continuo in eodem plano moueri pergit. Generatim ergo corporis a tribus viribus V, T, R ſollicitati motus quatuor ſequentibus aequationibus determinatur.

I. $2\,dx\,d\phi + x\,dd\phi = -\tfrac{1}{2}\,T\,dt^2$

II. $d\,dx - x\,d\phi^2 = -\tfrac{1}{2}\,V\,dt^2$

III. $d\pi = \tfrac{1}{2}dt^2.\,\frac{\text{ſin}\,(\phi-\pi)}{x\,d\phi}(V\,\text{ſin}(\phi-\pi) + T\,\text{coſ}(\phi-\pi) - \frac{R}{\text{tang}\,\varrho})$

IV. $d.\,l\,\text{tang}\,\varrho = \frac{d\pi}{\text{tang}\,(\phi - \pi)}$

quas ergo quouis caſu oblato reſolui oportet.

CAPUT

CAPUT II.

INUESTIGATIO VIRIUM LUNAM SOLLICITANTIUM.

§. 13.

Cum Lunae motus, qualis ex centro terrae ſpectaretur, definiri debeat, ſit C terrae centrum, ad quod etiam praecipua vis, qua Luna vrgetur, directa concipitur; atque tabula exhibeat planum eclipticae, in quo nunc quidem Sol exiſtat in S, Luna vero ſupra hoc planum verſetur in L latitudinem habens borealem, vnde ad planum eclipticae perpendiculum demittatur LM. Hinc ductis rectis CL, CM, CS, itemque CQ initium arietis verſus, vnde longitudines computari ſolent, fiant ſequentes denominationes. Fig. 1.

1. Longitudo Solis ſeu angulus ACS $= \theta$
2. Longitudo Lunae ſeu angulus ACM $= \phi$
3. Latitudo Lunae ſeu angulus MCL $= \psi$
4. Diſtantia Solis a Terra CS $= y$
5. Diſtantia Lunae a Terra curtata CM $= x$

§. 14. Sit iam AM proiectio orbitae lunaris in planum eclipticae; ac planum, in quo Luna nunc mouetur, per centrum terrae ductum, planum eclipticae interſecet ſecundum rectam C☊, quae lineam nodorum pro tempore praeſenti exhibebit: ac terminus ☊ quidem nodum aſcendentem referet, ſiquidem lunam ſecundum regionem AM promoueri ponamus. Quod ſi ergo porro vocemus

6. Lon-

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6. Longitudinem nodi aſc: A C ☊ $= \pi$

7. Incl. orbitae Lunae ad eclipticam $= \varrho$

hinc latitudo Lunae geocentrica ita definietur, vt ſit $\tang \psi = \tang. \sin (\phi - \pi)$. Vnde incrementum latitudinis $d\psi$ commode aſſignabitur, cum ſit, vt ſupra vidimus $d. \tang \psi = \frac{d\psi}{\cos \psi^2} = d\phi \tang \varrho \cos (\phi - \pi)$: ac praeterea ex motu nodi cognito variatio inclinationis ita definietur, vt ſit $d. \tang \varrho = \frac{d\pi \tang \varrho}{\tang (\phi - \pi)}$, ſeu $d\varrho = \frac{d\pi \sin \varrho \cos \varrho}{\tang (\phi - \pi)}$.

§. 15. Cum nunc primum Luna ad centrum terrae C ſecundum directionem LC attrahatur, ſit haec vis $= M$. Deinde ſit vis, qua Luna ad ſolem S vrgetur ſecundum LS $= N$; atque his duabus viribus Luna proprie vrgeri cenſenda eſt. Praeterea vero cum Terra ipſa, ad quam motum Lunae referimus, in motu verſetur, ut eam tanquam quieſcentem conſiderare queamus, non ſolum motum Terrae, ſed etiam vires, quibus Terra vrgetur, toti mundo ſecundum plagas oppoſitas imprimi concipiamus. Sit igitur vis qua Terra ad Solem vrgetur $= S$, & vis qua ad Lunam trahitur $= L$, his viribus contrario modo in lunam translatis, Luna ſequentibus viribus ſollicitata habebitur

1. Secundum directionem LC vi $= M + L$
2. Secundum directionem LS vi $= N$
3. Secundum directionem MR ipſi SC parallelam vi $= S$.

§. 16. Hunc

§. 16. Nunc primo has vires ad ternas directiones ſupra aſſumtas MC, MQ et LM reducamus; ac primo quidem vis $M + L$ dabit

pro directione MC vim $= (M + L) \cos \psi$

pro directione LM vim $= (M + L) \sin \psi$

Secunda vis N vero dabit

pro directione LM vim $= \frac{LM}{LS} . N$

pro directione MS vim $= \frac{MS}{LS} . N$

at haec vlterius reſoluta ducta Mr ipſi CS parallela dabit:

pro directione MC vim $= \frac{MC}{LS} . N$

pro directione Mr vim $= \frac{CS}{LS} . N$

Haec poſtrema a vi tertia S ſubtracta relinquet vim $S - \frac{CS}{LS} . N$, qua Luna ſecundum directionem MR ſollicitatur, quae ob angulum $CMR = SCM = \phi - \theta$ dabit

pro directione MC vim $= (S - \frac{CS}{LS} . N) \cos (\phi - \theta)$

pro directione Mq vim $= (S - \frac{CS}{LS} . N) \sin (\phi - \theta)$

vbi directio Mq contraria eſt directione MQ.

§. 17. His iam viribus cum ternis initio aſſumtis V, T et R comparandis, inueniemus pro his viribus ſequentes valores:

1. pro directione M C vim V $=$

$$(M+L)\,\mathrm{coſ}\,\psi+\frac{MC}{LS}.\,N+(S-\frac{CS}{LS}.\,N)\,\mathrm{coſ}(\varphi-\theta)$$

2. pro directione MQ vim $T=-(S-\frac{CS}{LS}.\,N)\ \mathrm{ſin}\ (\varphi-\theta)$

3. pro directione LM vim $R=(M+L)\,\mathrm{ſin}\,\psi+\frac{LM}{LS}\ N.$

Cum nunc ſit $CM=x$, $CS=y$, et angulus $SCM=\varphi-\theta$; erit $MS=\sqrt{(xx-2xy\,\mathrm{coſ}(\varphi-\theta)+yy)}$, et ob $LM=x\ \mathrm{tang}\ \psi$ erit $LS=\sqrt{(yy-2xy\,\mathrm{coſ}(\varphi-\theta)+xx\ \mathrm{ſec.}\,\psi^2)}$, quae diſtantia Solis a Luna LS breuitatis gratia ponatur $=u$, vt ſit $u=\sqrt{(yy-2xy\,\mathrm{coſ}(\varphi-\theta)+xx\ \mathrm{ſec.}\,\psi^2)}$. His ergo valoribus introductis erunt vires noſtrae:

1. $V=(M+L)\,\mathrm{coſ}\,\psi+\frac{Nx}{u}+S\,\mathrm{coſ}(\varphi-\theta)-\frac{Ny}{u}\,\mathrm{coſ}(\varphi-\theta)$

2. $T=-S\ \mathrm{ſin}\ (\varphi-\theta)+\frac{Ny}{u}\ \mathrm{ſin}\ (\varphi-\theta)$

3. $R=-(M+L)\,\mathrm{ſin}\,\psi+\frac{Nx\ \mathrm{tang}\,\psi}{u}$

§. 18. Quia nunc eſt $\mathrm{tang}\,\psi=\mathrm{tang}\,\rho\ \mathrm{ſin}\ (\varphi-\pi)$ et $\mathrm{ſin}\,\psi=\mathrm{tang}\,\psi\ \mathrm{coſ}\,\psi$ erit:

$$\frac{R}{\mathrm{tang}\,\rho}=(M+L)\,\mathrm{coſ}\,\psi\ \mathrm{ſin}\ (\varphi-\pi)+\frac{Nx\,\mathrm{ſin}\,(\varphi-\pi)}{u}$$

tum vero habebitur

$$V\mathrm{ſin}(\varphi-\pi)+T\mathrm{coſ}(\varphi-\pi)=(M+L)\,\mathrm{coſ}\psi\,\mathrm{ſin}\,(\varphi-\pi)+\frac{Nx\mathrm{ſin}(\varphi-\pi)}{u}$$
$$+S\,\mathrm{coſ}(\varphi-\theta)\,\mathrm{ſin}\,(\varphi-\pi)-\frac{Ny\,\mathrm{coſ}\,(\varphi-\theta)\,\mathrm{ſin}(\varphi-\pi)}{u}$$
$$-S\,\mathrm{ſin}\,(\varphi-\theta)\,\mathrm{coſ}(\varphi-\pi)+\frac{Ny\,\mathrm{ſin}\,(\varphi-\theta)\,\mathrm{coſ}(\varphi-\pi)}{u}$$

quae ob $\mathrm{coſ}(\varphi-\theta)\mathrm{ſin}(\varphi-\pi)-\mathrm{ſin}(\varphi-\theta)\mathrm{coſ}(\varphi-\pi)=\mathrm{ſin}(\varphi-\pi)$, dat

V ſin

$$V \sin(\varphi-\pi) + T\cos(\varphi-\pi) - \frac{R}{\tang \varrho} = S\sin(\theta-\pi) - \frac{Ny}{u}\sin(\theta-\pi)$$

ex quo aequationes motum Lunae continentes erunt:

$$\text{I. } 2dxd\varphi + xdd\varphi = -\tfrac{1}{2}dt^2\left(\frac{Ny}{u} - S\right)\sin(\varphi-\theta)$$

$$\text{II. } ddx - xd\varphi^2 = -\tfrac{1}{2}dt^2\left((M+L)\cos\psi + \frac{Nx}{u} - \left(\frac{Ny}{u} - S\right)\cos(\varphi-\theta)\right)$$

$$\text{III. } d\pi = -\tfrac{1}{2}dt^2 . \frac{\sin(\varphi-\pi)\sin(\theta-\pi)}{x\,d\varphi}\left(\frac{Ny}{u} - S\right)$$

$$\text{IV. } d.\,l\tang\varrho = \frac{dx}{\tang(\varphi-\pi)}$$

vbi $\theta-\pi$ exprimit angulum ☊ CS ſeu diſtantiam Solis a nodo aſcendente.

§. 19. Jam ſecundum Theoriam Newtoni, ſi maſſam Terrae ponamus $= ♁$ ac Lunae $= ☾$, ob diſtantiam $CL = \frac{x}{\cos\psi}$, foret vis $M = \frac{♁\cos\psi^2}{xx}$ et vis $L = \frac{☾\cos\psi^2}{xx}$, ſicque vis tota $M + L = (♁ + ☾) . \frac{\cos\psi^2}{xx}$. Quo autem, ſi forte haec Theoria inſufficiens deprehendatur, rem generalius complectamur, ponamus hanc vim:

$$M + L = (♁ + ☾)\cos\psi^2\left(\frac{1}{xx} - \frac{1}{bb}\right)$$

vbi terminus $\frac{1}{bb}$ defectum huius vis a Theoria Newtoniana exhibeat; qui cum ſit minimus, pro conſtanti haberi poterit ſaltem pro exigua variabilitate, quam diſtantia x ſubit. Vim autem Solis exacte Theoriae Newtonianae conformem aſſumere poterimus; quoniam etiamſi inde recederet, differentia non ſolum foret

quam minima, ſed quia pro Luna aeque diſcreparet ac pro Terra, in noſtris formulis nullius plane eſſet momenti.

§. 20. Poſita ergo Solis maſſa $= \odot$, erit vis, qua Terram ad ſe attrahit $S = \frac{\odot}{yy}$, vis autem qua Lunam ad ſe trahit $N = \frac{\odot}{uu}$. His ergo valoribus virium in calculum inductis, motus lunae ex quatuor ſequentibus aequationibus determinari debet:

$$\text{I. } 2dxd\varphi + xdd\varphi = -\tfrac{1}{2}dt^2\left(\frac{\odot y}{u^3} - \frac{\odot}{yy}\right)\sin(\varphi-\theta)$$

$$\text{II. } ddx - xd\varphi^2 = -\tfrac{1}{2}dt^2\,(♁ + ☾)\cos\psi^3\left(\frac{1}{xx} - \frac{1}{bb}\right)$$
$$-\tfrac{1}{2}dt^2\left(\frac{\odot x}{u^3} - \frac{\odot y}{u^3}\cos(\varphi-\theta) + \frac{\odot}{yy}\cos(\varphi-\theta)\right)$$

$$\text{III. } d\pi = -\tfrac{1}{2}dt^2.\,\frac{\sin(\varphi-\pi)\sin(\theta-\pi)}{xd\varphi}\left(\frac{\odot y}{u^3} - \frac{\odot}{yy}\right)$$

$$\text{IV. } d.\,l\,\tang\varrho = \frac{d\pi}{\tang(\varphi-\pi)}$$

Hic iam primum curandum eſt, vt elementum temporis, quod eſt quantitas heterogenea, ex calculo eliminemus; id quod commodiſſime fiet, ſi motum medium ſolis vtpote tempori proportionalem, loco temporis in calculum introducemus.

§. 21. Cum igitur etiam motus Solis in his aequationibus ſit ratio habenda, eum prius inueſtigemus: et quoniam pro terra quieſcente Sol a ſola vi $\frac{\odot}{yy}$ ad terram

ram ſollicitari concipiendus eſt, ſi formulas pro luna inuentas ad ſolem accommodemus, obtinebimus:

$$2\,dy\,d\theta + y\,dd\theta = 0$$

$$ddy - y\,d\theta^2 = -\tfrac{1}{2}\,dt^2 . \frac{\odot}{yy}$$

ſi iam diſtantiam Solis a terra mediam ponamus $= b$ ejusque anomaliam mediam $= q$; caſu quo excentricitas orbitae ſolaris eſſet nulla, foret ſemper $y = b$ & $d\theta = dq$: vnde altera aequatio dabit $- b\,dq^2 = - \tfrac{1}{2}\,dt^2 . \frac{\odot}{bb}$. Quare loco elementi temporis dt elementum anomaliae mediae ſolis ita in calculum introduci debet, vt vbſque loco $\tfrac{1}{2}\,dt^2$ ſcribatur $\frac{b^3\,dq^2}{\odot}$, id quod tam in his formulis pro Sole, quam in ſuperioribus pro Luna fieri poterit.

§. 22. Cum iam b denotet diſtantiam ſolis a terra mediam, ſit eius vera diſtantia $y = b\,\omega$, et anomalia eius vera $= s$, erit $d\theta = ds$, quandoquidem a motu apogei ſolis animum abſtrahimus. Hinc itaque erit

$$2\,d\omega\,ds + \omega\,dds = 0$$

$$dd\omega - \omega\,ds^2 = - \frac{dq^2}{\omega\omega},$$

quarum prior integrata dat $\omega\omega\,ds = C\,dq$ ob dq conſtans, ideoque $\omega\,ds^2 = \frac{CC\,dq^2}{\omega^3}$; qui valor in altera aequatione ſubſtitutus praebet,

$$dd\omega = \frac{CC\,dq^2}{\omega^3} - \frac{dq^2}{\omega\omega}.$$

quae

quae per $2\,d\omega$ multiplicata et integrata dat:

$$\frac{d\omega^2}{dq^2} = D - \frac{CC}{\omega\omega} + \frac{2}{\omega}$$

vnde fit $dq = \frac{\omega\, d\omega}{V(D\omega\omega + 2\omega - CC)}$

ac proinde $ds = \frac{C\, d\omega}{\omega V(D\omega\omega + 2\omega - CC)}$

§. 23. Quanquam autem hinc valores finiti haud difficulter deduci poſſent, tamen alia vtar methodo, quae in motu Lunae maiorem praeſtabit vtilitatem. Inuento autem $\omega\omega\, ds = C\, dq$, alteram aequationem ita transformo, vt elementi conſtantis dq ratio non amplius habeatur:

$$dq.\, d.\, \frac{d\omega}{dq} - \omega\, ds^2 = - \frac{dq^2}{\omega\omega}$$

Sit nunc $\omega = \frac{1}{u}$, vt habeat $ds = C\, uu\, dq$, et $d\omega = - \frac{du}{uu}$, et ob $dq = \frac{ds}{C\, uu}$ erit $\frac{d\omega}{dq} = - \frac{C du}{ds}$; hinc ſumto iam elemento ds conſtante, erit

$$\frac{-d's}{C\, uu} \cdot \frac{C ddu}{ds} - \frac{ds^2}{u} = - \frac{ds^2}{CC\, uu} \text{ ſeu}$$

$$ddu + u ds^2 = \frac{ds^2}{CC}$$

vnde ſtatim elicitur $u = \frac{1 - e \operatorname{coſ} s}{C\, C}$, vbi e excentricitatem orbitae ſolaris indicabit.

§. 24. Hinc porro habebitur $\omega = \frac{C\, C}{1 - e \operatorname{coſ} s}$, et $y = \frac{C C b}{1 - e \operatorname{coſ} s}$, anomalia vera s ab apogeo computata; vnde diſtantia apogei

apogei a terra poſito $s = o$ erit $= \frac{CCb}{1-e}$, et diſtantia perigei poſito $s = 180°$ prodit $= \frac{CCb}{1+e}$; ſicque diſtantia media fiet $= \frac{CCb}{1-ee}$, quae cum per hypotheſin aequalis eſſe debeat ipſi b, ſtatui oportet $CC = 1-ee$: hincque erit

$$y = \frac{b(1-ee)}{1-e\,\mathrm{cof}\,s} \quad \text{et} \quad \omega = \frac{1-ee}{1-e\,\mathrm{cof}\,s}$$

Porro autem aequatio $\omega\omega\,ds = C\,dq = dq\,\sqrt{(1-ee)}$ abibit in hanc:

$$dq = \frac{(1-ee)^{\frac{3}{2}}\,ds}{(1-e\,\mathrm{cof}\,s)^2} \text{ et } q = \int \frac{(1-ee)^{\frac{3}{2}}\,ds}{(1-e\,\mathrm{cof}\,s)^2}$$

ex qua, vti ſatis conſtat, vel data anomalia vera s inueniri poteſt anomalia media q, vel viciſſim. His itaque formulis motum Solis continentibus in determinatione motus Lunae vtamur.

§. 25. Primo ergo loco $\frac{2}{⊙}\,dt^2$ vbique ſcribamus $\frac{b^3\,dq^2}{⊙}$ et $b\,\omega$ loco y, quo facto noſtrae aequationes fient

$$\text{I. } 2\,dx\,d\Phi + x\,dd\Phi = -b^3\,dq^2\left(\frac{b\omega}{u^3} - \frac{1}{bb\omega\omega}\right)\mathrm{fin}\,(\Phi-\theta)$$

$$\text{II. } ddx - x\,d\Phi^2 = -\frac{(♁+☾)\,b^3 dq^2}{⊙}\,\mathrm{cof}\,\psi^3\left(\frac{1}{xx} - \frac{1}{bb}\right)$$

$$-b^3 dq^2\left(\frac{x}{u^3} - \frac{b\omega\,\mathrm{cof}(\Phi-\theta)}{u^3} + \frac{\mathrm{cof}(\Phi-\theta)}{bb\omega\omega}\right)$$

$$\text{III. } d\pi = -b^3\,dq^2.\,\frac{\mathrm{fin}(\Phi-\pi)\,\mathrm{fin}\,(\theta-\pi)}{x\,d\Phi}\left(\frac{b\omega}{u^3} - \frac{1}{bb\omega\omega}\right)$$

Ponatur porro $u = bv$, adque in calculum quoque intro-

ducatur diſtantia media lunae a terra, quae ſit $=a$, poſitoque $x=az$, prodibit:

$$\text{I. } 2dzd\phi + zdd\phi = -\frac{bdq^2}{a}\left(\frac{\omega}{v^3} - \frac{1}{\omega\omega}\right)\text{ſin}\,(\phi-\theta)$$

$$\text{II. } ddz - zd\phi^2 = -\frac{(\text{♁}+\text{☾})\,b^3}{\odot\, a^3}\, dq^2 \,\text{coſ}\,\psi^3 \left(\frac{1}{zz} - \frac{aa}{bb}\right)$$

$$- \frac{zdq^2}{v^3} + \frac{b\omega dq^2\,\text{coſ}\,(\phi-\theta)}{av^3} - \frac{bdq^2\,\text{coſ}\,(\phi-\theta)}{a\omega\omega}$$

$$\text{III. } d\pi = -\frac{bdq^2}{azd\phi}\,\text{ſin}\,(\phi-\pi)\,\text{ſin}\,(\theta-\pi)\left(\frac{\omega}{v^3} - \frac{1}{\omega\omega}\right)$$

§. 26. Ponamus nunc ad abbreuiandum:

$$\frac{(\text{♁}+\text{☾})\,b^3}{\odot\, a^3} = m;\quad \frac{(\text{♁}+\text{☾})\,b^3}{\odot\, abb} = \mu,\ \text{ſeu}\ \mu = \frac{maa}{bb}$$

quarum litterarum valores m et μ per obſeruationes definiri debent; tum vero ſit $\frac{a}{b} = \nu$, quae eſt quantitas valde parua a parallaxi ſolis pendens. Hisque valoribus introductis, aequationes noſtrae ſequentes induent formas:

$$\text{I. } 2dzd\Phi + zdd\Phi = -\frac{1}{\nu}dq^2\left(\frac{\omega}{v^3} - \frac{1}{\omega\omega}\right)(\text{ſin}\,\Phi-\theta)$$

$$\text{II. } ddz - zd\Phi^2 = -\frac{m\,dq^2\,\text{coſ}\,\psi^3}{zz} + \mu dq^2\,\text{coſ}\,\psi^3$$

$$- \frac{zdq^2}{v^3} + \frac{1}{\nu}\,dq^2\left(\frac{\omega}{v^3} - \frac{1}{\omega\omega}\right)\text{coſ}\,(\Phi-\theta)$$

$$\text{III. } d\pi = -\frac{dq^2}{\nu zd\Phi}\,\text{ſin}\,(\phi-\pi)\,\text{ſin}\,(\theta-\pi)\left(\frac{\omega}{v^3} - \frac{1}{\omega\omega}\right)$$

$$\text{IV. } d.\,l\,\text{tang}\,\rho = \frac{d\pi}{\text{tang}\,(\Phi-\pi)}$$

§. 27.

§. 27. Cum iam poſuerimus:

$$x = az;\quad y = b\omega;\quad v = \frac{u}{b} \quad \text{et} \quad \nu = \frac{a}{b}$$

erit $u = \sqrt{(bb\omega\omega - 2ab\omega z \operatorname{coſ}(\varphi - \theta) + aazz \operatorname{ſec.} \psi^2)}$
atque $v = \sqrt{(\omega\omega - 2\nu\omega z \operatorname{coſ}(\varphi - \theta) + \nu\nu zz \operatorname{ſec.} \psi^2)}$
vbi notandum eſt quantitates ω, q et s ex motu ſolis ita inter ſe pendere, vt ſit

$$\omega = \frac{1-ee}{1-e\operatorname{coſ}s} \quad \text{et} \quad dq = \frac{(1-ee)^{\frac{3}{2}}\,ds}{(1-e\operatorname{coſ}s)^2} = \frac{\omega\omega\,ds}{\sqrt{(1-ee)}}$$

ita vt huic ad datum quoduis tempus tam valor ipſius ω quam anomaliae verae s definiri poſſit. Modum autem has formulas ad calculum reuocandi hic non trado, quia eum alias fuſius iam expoſui: hoc ſolum hic notari conueniet, excentricitatis orbitae ſolaris valorem ex obſeruationibus colligi $e = 0{,}01680$.

§. 28. Nunc antequam vlterius progredi queamus, valorem irrationalem ipſius v tolli conueniet, quod facile per ſeriem praeſtabitur maxime conuergentem, ob ν fractionem valde paruam; ſumta enim parallaxi ſolis $= 12''$, quia parallaxis lunae media eſt $= 3380''$, erit $\frac{a}{b} = \nu = \frac{12}{3380} = \frac{1}{282}$. Hinc ſufficit ſeriei illius conuergentis, quam reperiemus, aliquot tantum terminos ab initio asſumſiſſe; quia reliqui ob paruitatem continuo magis creſcentem tuto omitti poterunt. Cum autem angulus $\varphi - \theta$, qui diſtantiam ſolis a luna ſecundum longitudinem denotat, in hac reſolutione frequentiſſime occurret, breuitatis gratia ponamus $\varphi - \theta = \eta$
ita vt pro v ſequentem habeamus valorem irrationalem

$$v = \sqrt{(\omega\omega - 2\nu\omega z \operatorname{coſ}\eta + \nu\nu zz \operatorname{ſec.} \psi^2)}.$$

§. 29. Quoniam ergo in noſtris formulis occurrit $\frac{1}{v^3}$ ob $\frac{1}{v^3} = (\omega\omega - 2v\omega z\,\text{coſ}\,\eta + vvzz\,\text{ſec}\,\psi^2)^{-\frac{3}{2}}$, nanciſcemur

$$\frac{1}{v^3} = \frac{1}{\omega^3} + \frac{3vz\,\text{coſ}\,\eta}{\omega^4} - \frac{3vvzz\,\text{ſec.}\,\psi^2}{2\omega^5} + \frac{15vvzz}{2\omega^5}\,\text{coſ}\,\eta^2;$$

vbi terminos altiores ipſius v poteſtates inuoluentes ſine haeſitatione reiicere poſſumus; in ipſis aequationibus autem tantum in prima ipſius v poteſtate ſubſiſtemus. Habebimus ergo:

$$\frac{1}{v}\left(\frac{\omega}{v^3} - \frac{1}{\omega\omega}\right) = \frac{3z\,\text{coſ}\,\eta}{\omega^3} + \frac{3vzz}{2\omega^4}(5\,\text{coſ}\,\eta^2 - \text{ſec.}\,\psi^2) \text{ ſeu}$$

$$\frac{1}{v}\left(\frac{\omega}{v^3} - \frac{1}{\omega\omega}\right) = \frac{3z\,\text{coſ}\,\eta}{\omega^3} + \frac{3vzz}{4\omega^4}(5 + 5\,\text{coſ}\,2\eta - 2\,\text{ſec.}\,\psi^2)$$

hincque porro:

$$\frac{1}{v}\left(\frac{\omega}{v^3} - \frac{1}{\omega\omega}\right)\text{ſin}(\varphi - \theta) = \frac{3z\,\text{ſin}\,2\eta}{2\omega^3} + \frac{3vzz}{8\omega^4}(5\,\text{ſin}\,\eta + 5\,\text{ſin}\,3\eta - 4\,\text{ſin}\,\eta\,\text{ſec.}\,\psi^2)$$

$$\frac{1}{v}\left(\frac{\omega}{v^3} - \frac{1}{\omega\omega}\right)\text{coſ}(\varphi - \theta) = \frac{3z}{2\omega^3}(1 + \text{coſ}\,2\eta) + \frac{3vzz}{8\omega^4}(15\,\text{coſ}\,\eta + 5\,\text{coſ}\,3\eta - 4\,\text{coſ}\,\eta\,\text{ſec.}\,\psi^2)$$

$$\text{atque } \frac{z}{v^3} = \frac{z}{\omega^3} + \frac{3vzz}{\omega^4}\,\text{coſ}\,\eta$$

§. 30. Subſtituamus hos valores in noſtris aequationibus atque obtinebimus:

$$\text{I. } 2dz\,d\varphi + zdd\varphi = -dq^2\left(\frac{3z\,\text{ſin}\,2\eta}{2\omega^3} + \frac{3vzz}{8\omega^4}(5\,\text{ſin}\,\eta + 5\,\text{ſin}\,3\eta - 4\,\text{ſin}\,\eta\,\text{ſec.}\,\psi^2)\right)$$

$$\text{II. } ddz - zd\varphi^2 = -\frac{m\,dq^2\,\text{coſ}\,\psi^3}{zz} + \mu dq^2\,\text{coſ}\,\psi^3 + \frac{zdq^2}{2\omega^3} + \frac{3zdq^2}{2\omega^3}\,\text{coſ}\,2\eta$$

$$+ \frac{3vzz\,dq^2}{8\omega^4}(7\,\text{coſ}\,\eta + 5\,\text{coſ}\,3\eta - 4\,\text{coſ}\,\eta\,\text{ſec.}\,\psi^2)$$

III. $d\pi$

$$\text{III. } d\pi = -\frac{dq^2}{2d\Phi}\,\text{ſin}(\Phi-\pi)\,\text{ſin}(\theta-\pi)\left(\frac{3z\,\text{coſ}\,\eta}{\omega^3} + \frac{3vzz}{4\omega^4}(5+5\,\text{coſ}\,2\eta - 2\,\text{ſec.}\,\psi^2)\right)$$

$$\text{IV. } d.\,l\,\text{tang}\,\varrho = \frac{d\pi}{\text{tang}\,(\Phi-\pi)}.$$

Hic iam obſervare licet, cum angulus ψ nunquam fere 5° ſuperet, eiusque ſecans nonniſi in terminis iam per v multiplicatis, ac propterea reſpectu reliquorum valde parvis occurrat, ſine vllius erroris ſenſibilis metu in his terminis poni poſſe ſec. $\psi = 1$.

§. 31. Deinde vt etiam ex maioribus terminis coſ ψ eliminemus; conſideremus formulam tang ψ = tang ϱ ſin $(\Phi-\pi)$, eritque ſec. $\psi = \frac{1}{\text{coſ}\,\psi} = \sqrt{(1+\text{tang}\,\varrho^2\,\text{ſin}(\Phi-\pi)^2)}$

Hinc ergo habebimus:

$$\text{coſ}\,\psi^3 = (1+\text{tang}\,\varrho^2.\,\text{ſin}\,(\Phi-\pi)^2)^{-\frac{3}{2}}$$

et cum tang ϱ^2 nunquam fere fractionem $\frac{1}{144}$ ſuperet erit ſatis exacte:

$$\text{coſ}\,\psi^3 = 1 - \tfrac{3}{2}\,\text{tang}\,\varrho^2.\ \text{ſin}\,(\Phi-\pi)^2 \text{ — vel etiam}$$

$$\text{coſ}\,\psi^3 = 1 - \tfrac{3}{4}\,\text{tang}\,\varrho^2 + \tfrac{3}{4}\,\text{tang}\,\varrho^2\,\text{coſ}\,2\,(\Phi-\pi)$$

qui valor pro coſ ψ^3 in termino maiore $\frac{m\,dq^2\,\text{coſ}\,\psi^3}{zz}$ ſubſtitui poteſt: in altero autem termino $\mu\,dq^2$ coſ ψ^3 quia per ſe eſt valde paruus, atque adeo ſecundum Theoriam Newtoni euaneſceret, nihil impedit, quo minus loco coſ ψ^3 ſcribamus vnitatem.

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§. 32. Hoc ergo modo ſi aequationes noſtras a conſideratione latitudinis Lunae ψ liberemus ad ſequentes peruenìemus aequationes:

$$\text{I. } 2dzd\Phi + zdd\Phi = -dq^2\left(\frac{3z\,\text{ſin}\,2\eta}{2\omega^3} + \frac{3vzz}{8\omega^4}(\text{ſin}\,\eta + 5\,\text{ſin}\,3\eta)\right)$$

$$\text{II. } ddz - zd\Phi^2 = -\frac{mdq^2}{zz}\left(1 - \tfrac{3}{4}\text{tang}\,\varrho^2 + \tfrac{3}{4}\text{tang}\,\varrho^2\,\text{coſ}\,2(\Phi-\pi)\right) + \mu dq^2$$

$$+ \frac{zdq^2}{2\omega^3} + \frac{3zdq^2}{2u^3}\,\text{coſ}\,2\eta + \frac{3vzzdq^2}{8\omega^4}(3\,\text{coſ}\,\eta + 5\,\text{coſ}\,3\eta)$$

$$\text{III. } d\pi = -\frac{dq^2}{zd\Phi}\,\text{ſin}(\Phi-\pi)\,\text{ſin}(\theta-\pi)\left(\frac{3z\,\text{coſ}\,\eta}{\omega^3} + \frac{3vzz}{4\omega^4}(3 + 5\,\text{coſ}\,2\eta)\right)$$

$$\text{IV. } d.\,l\,\text{tang}\,\varrho = \frac{d\pi}{\text{tang}(\Phi-\pi)}.$$

Nunc igitur in hoc erit incumbendum, vt ex his quatuor aequationibus omnia motus phaenomena, quae in Luna ſecundum Theoriam adeſſe debent, ſollicite eruantur, atque tum cum obſeruationibus conferantur.

CAPUT III.

INTRODUCTIO ANOMALIAE VERAE LUNAE IN PRAECEDENTES AEQUATIONES.

§. 33.

Quoniam noſtra quaeſtio circa Lunam verſatur, loco anomaliae mediae ſolis, quam pro tempore in calculum introduximus, magis conueniet motu Lunae medio vti, qui itidem tempori eſt proportionalis. Verum ex ſequentibus patebit calculum commodiorem reddi, ſi loco motus medii adhibeamus anomaliam Lunae mediam, cuius incrementa itidem tempori ſunt proportionalia. Sit itaque ad datum tempus anomalia media Lunae $= p$; et cum eius incrementum dp ad incrementum anomaliae mediae ſolis eodem tempuſculo acceptum datum ac per obſeruationes cognitam teneat rationem, ponamus $dp = ndq$. Tabulae autem Aſtronomicae pro interuallo 365 dierum praebent:

Motum anomaliae mediae Solis $11^s, 29^\circ, 44', 39'' = 1295079''$

Motum anomaliae mediae Lunae

$$13^{\text{Rev.}}\ 2^s, 28^\circ, 43'\ 13'' = 17167393''$$

vnde fit $n = \frac{17167393}{1295079} = 13,25586$

§. 34. Poſito ergo $\frac{dp}{n}$ loco dq, aequationes noſtrae erunt

$$\text{I. } 2dzd\Phi + zdd\Phi = -\frac{dp^2}{nn}\left(\frac{3z\sin 2\eta}{2w^3} + \frac{3vzz}{8w^4}(\sin\eta + 5\sin 3\eta)\right)$$

II. ddz

$$\text{II. } ddz - zd\Phi^2 = -\frac{mdp^2}{nnzz}\left(1 - \tfrac{3}{4}\mathrm{tang}\,\varrho^2 + \tfrac{3}{4}\mathrm{tang}\,\varrho^2 \operatorname{cos} 2(\Phi-\pi)\right) + \frac{\mu dp^2}{nn}$$
$$+ \frac{zdp^2}{2n^2u^3} + \frac{3zdp^2}{2n^2\omega^3}\operatorname{cos} 2\eta + \frac{3\nu zzdp^2}{8nn\omega^4}(3\operatorname{cos}\eta + 5\operatorname{cos} 3\eta)$$

$$\text{III. } d\pi = \frac{-dp^2}{nnzd\Phi}\operatorname{sin}(\Phi-\pi)\operatorname{sin}(\theta-\pi)\left(\frac{3z\operatorname{cos}\eta}{\omega^3} + \frac{3\nu zz}{4\omega^4}(3+5\operatorname{cos} 2\eta)\right)$$

$$\text{IV. } d.\,l\,\mathrm{tang}\,\varrho = \frac{d\pi}{\mathrm{tang}(\Phi-\pi)}$$

atque hic elementum dp aſſumtum eſt conſtans: ſimul autem patet terminos, qui per nn ſunt diuiſi, prae ceteris ſatis eſſe paruos, cum ſit $nn = 175,71795$. Quae circumſtantia ſequentes approximationes non mediocriter adiuuabit.

§. 35. Nunc antequam vlterius progrediamur, aequationem primam per z multiplicemus, atque integratione in priori parte inſtituta obtinebimus

$$zzd\Phi = Cdp - \frac{dp}{nn}\int dp\left(\frac{3z^2\operatorname{sin} 2\eta}{2\cdot\omega^3} + \frac{3\nu z^3}{8\omega^4}(\operatorname{sin}\eta + 5\operatorname{sin} 3\eta)\right)$$

ponamus breuitatis gratia hoc membrum integrale

$$\int dp\left(\frac{3zz}{2\omega^4}\operatorname{sin} 2\eta + \frac{3\nu z^3}{8\omega^4}(\operatorname{sin}\eta + 5\operatorname{sin} 3\eta)\right) = S$$

quod integrale, ne introductio conſtantis incertitudinem pariat, ita capi aſſumo, vt nullum terminum mere conſtantem contineat, quippe qui iam in C eſſet comprehenſus. Hoc ergo circa determinationem integrationis probe obſeruato, erit $zzd\Phi = dp\left(C - \frac{S}{nn}\right)$: vbi terminus S aequabilem arearum deſcriptionem, quam Regula Kepleri in planetis primariis infert, perturbat; eſt enim

enim $\frac{1}{2} zz\, d\Phi$ elementum areae deſcriptae, quod ſi ipſi C dp eſſet aequale, tempori exacte eſſet proportionale.

§. 36. Cum igitur ſit $d\Phi = \frac{dp}{zz}\left(C - \frac{S}{nn}\right)$, erit $z\, d\Phi^2 = \frac{dp^2}{z^3}\left(CC - \frac{2}{nn}CS + \frac{1}{n^4}SS\right)$, quo valore ſubſtituto reliquae noſtrae aequationes ſequentes induent formas:

$$\text{II. } ddz = \frac{dp^2}{z^3}\left(CC - \frac{2}{nn}CS + \frac{1}{n^4}SS\right) - \frac{m\,dp^2}{nnzz}\left(1 - \tfrac{3}{4}\operatorname{tang}\rho^2 + \tfrac{3}{4}\operatorname{tang}\rho^2 \operatorname{cof} 2(\Phi - \pi)\right) + \frac{\mu\, dp^2}{nn} + \frac{z\,dp^2}{2nn\omega^3} + \frac{3z\,dp^2}{2nnu^3}\operatorname{cof} 2\eta + \frac{3\nu zz\,dp^2}{8nnu^4}(3\operatorname{cof}\eta + 5\operatorname{cof} 3\eta)$$

$$\text{III. } d\pi = -\frac{z\,dp}{Cnn - S}\operatorname{fin}(\Phi - \pi)\operatorname{fin}(\theta - \pi)\left(\frac{3z\operatorname{cof}\eta}{u^3} + \frac{3\nu zz}{4\omega^4}(3 + 5\operatorname{cof} 2\eta)\right)$$

et quarta manet $d.\, l\operatorname{tang}\rho = \frac{d\pi}{\operatorname{tang}(\Phi - \pi)}$ vt ante.

Eo igitur pertigimus, vt inueſtigari oporteat quantitates z, π et ρ, quibus inuentis obtinebitur Φ ex formula primum eruta. Cum autem ſit $d\eta = d\Phi - d\theta$, ob $d\theta = ds = \frac{dq\sqrt{(1-ee)}}{uu} = \frac{dp\sqrt{(1-ee)}}{nu\omega}$, erit $d\eta = \frac{dp}{zz}\left(C - \frac{S}{nn}\right) - \frac{dp\sqrt{(1-ee)}}{n\omega\omega}$. Tum vero eſt vti vidimus $u = \frac{1-ee}{1-e\operatorname{cof} s}$ vnde et huius differentiale ad dp reduci poterit.

§. 37. Si hunc calculum proſequi vellemus, tota inueſtigatio tandem eo rediret, vt definiretur quantum

 longi-

longitudo Lunae vera ab eius longitudine media, quae ex anomalia media p haberetur, diſcreparet: hoc autem diſcrimen nonnunquam vltra 8 gradus exſurgere poſſet, ideoque correctiones admodum notabiles requireret. Vt igitur nobis quam minimae correctiones inueſtigandae relinquantur, expediet differentiam inter locum Lunae verum, et locum corporis quod ſecundum regulas Kepleri in ellipſi circa Terram reuolueretur, ita tamen mobili, vt eius motus abſidum cum motu apogei Lunae per obſeruationes cognito conueniret. Seu quod eodem redit, quaeramus primo ex anomalia Lunae media p ſecundum regulas Kepleri anomaliam eius veram quae ſit $= r$, vnde ſi longitudo apogei fuerit $= w$, quantita, $w + r$ nunquam multum vltra gradum a longitudine Lunae vera differet: vnde diſcrimen multo facilius inueniri poterit, ſi quidem debita orbitae lunaris excentricitas in calculum inducatur. Hinc loco anomaliae Lunae mediae p eius anomaliam veram, quae ſcilicet mediae pro excentricitate rite aſſumta conueniat, in aequationes noſtras inferamus.

§. 38. Tabulae quidem aſtronomicae excentricitatem orbitae lunaris plerumque variabilem ſtatuunt; ſed cum hic non de vera huius orbitae excentricitate quaeſtio ſit, quam de excentricitate illius orbitae ellipticae mobilis, in qua corpus motum proxime motum Lunae referat; huius excentricitas media erit ſtatuenda inter maximam ac minimam, quae vulgo orbitae lunari tribuuntur: vnde iſta excentricitas media colligitur $= 0,05445$. Ne autem huic concluſioni nimium fidamus genera-

generatim hanc excentricitatem ponamus $= k$; atque anomalia vera per mediam ita determinabitur, vt ſit $dp = \frac{(1-kk)^{\frac{3}{2}} dr}{(1-k\cos r)^2}$, vel ſit breuitatis gratia $\frac{1-kk}{1-k\cos r} = t$, vt ſit $dp = \frac{tt\,dr}{\sqrt{(1-kk)}}$. Porro autem reliqua differentialia ita ad elementum dr reuocabuntur, vt ſit:

$$ds = \frac{tt\,dr\sqrt{(1-ee)}}{nu\omega\sqrt{(1-kk)}} = d\theta, \text{ et } d\eta = \frac{tt\,dr}{zz\sqrt{(1-kk)}}\left(C - \frac{S}{nn}\right) - \frac{tt\,dr\sqrt{(1-ee)}}{nu\omega\sqrt{(1-kk)}}.$$

§. 39. Si motus Lunae cum motu huius corporis, quod imaginamur, perfecte conueniret, tum vbique foret $z = \frac{1-kk}{1-k\cos r}$ ſeu $z = t$: quoniam autem hi duo motus inter ſe non conueniunt, non erit $z = t$. Ponamus ergo eſſe:

$$z = tu = \frac{(1-kk)u}{1-k\cos r} \text{ ſeu } x = \frac{(1-kk)au}{1-k\cos r}$$

vbi primum obſeruo, quantitatem u valde parum ab vnitate recedere. Erit autem quantitas variabilis, quae alium terminum conſtantem praeter vnitatem non inuoluet: nam ſi alium terminum conſtantem contineret, is in a poſſet comprehendi, idque indicio eſſet diſtantiam mediam a non recte eſſe aſſumtam. Habebit ergo u huiusmodi formam $1 + Z$, vbi Z ex terminis nonniſi variabilibus conſtabit. Praeterea autem animaduerto, hanc quantitatem Z nullum terminum huius formae $\alpha \cos r$ complecti debere; quoniam hoc indicio eſſet excentricitatem k non recte eſſe aſſumtam, ſed eam vel maiorem vel minorem accipi oportuiſſe.

 §. 40. His

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§. 40. His igitur notatis, quod quantitas u primo terminum conſtantem $= 1$ contineat, tum vero nullum terminum formae $\alpha \operatorname{cos} r$ inuoluat, ſtatuamus $z = tu$ ſeu $z = \frac{(1-kk)u}{1-k\operatorname{cos} r}$ poſito breuitatis gratia $t = \frac{1-kk}{1-k\operatorname{cos} r}$. Atque cum ſupra elementum dp conſtans poſuiſſemus, hac conditione exuenda erit $ddz = dp\, d.\frac{dz}{dp}$, et $\frac{ddz}{dp^2} = \frac{1}{dp}\, d.\frac{dz}{dp}$ Diviſa ergo ſecunda aequatione per dp^2, erit:

$$\text{II.}\quad \frac{1}{dp}\, d.\frac{dz}{dp} = \frac{CC}{t^3u^3} - \frac{2CS}{nnt^3u^3} + \frac{SS}{n^4t^3u^3}$$

$$-\frac{m}{nnttuu}\left(1-\tfrac{3}{4}\operatorname{tang}\rho^2+\tfrac{3}{4}\operatorname{tang}\rho^2\operatorname{cos}2(\Phi-\pi)\right)+\frac{\mu}{nn}+\frac{tu}{2nn\omega^3}$$

$$+\frac{3tu\operatorname{cos}2\eta}{2nn\omega^3}+\frac{3vttuu}{8nn\omega^4}(3\operatorname{cos}\eta+5\operatorname{cos}3\eta)$$

$$\text{III.}\quad d\pi = \frac{-tu\,dp}{Cnn-S}\operatorname{sin}(\Phi-\pi)\operatorname{sin}(\theta-\pi)\left(\frac{3tu\operatorname{cos}\eta}{\omega^3}+\frac{3vttuu}{4\omega^4}(3\mp 5\operatorname{cos}2\eta)\right)$$

vbi nunc nullum differentiale aſſumtum eſt conſtans, ſed iam pro lubitu quoduis differentiale conſtans aſſumi poterit.

§. 41. Poſito autem $z = tu$ et $dp = \frac{ttdr}{\sqrt{(1-kk)}}$ exiſtente $t = \frac{1-kk}{1-k\operatorname{cos} r}$ erit primo:

$$S = \int \frac{ttdr}{\sqrt{(1-kk)}}\left(\frac{3ttuu}{2\omega^3}\operatorname{sin}2\eta+\frac{3vt^3u^3}{8\omega^4}(\operatorname{sin}\eta+5\operatorname{sin}3\eta)\right) \text{ ſeu}$$

$$S = \int \frac{dr}{\sqrt{(1-kk)}}\left(\frac{3t^4uu}{2\omega^3}\operatorname{sin}2\eta+\frac{3vt^5u^3}{8\omega^4}(\operatorname{sin}\eta+5\operatorname{sin}3\eta)\right)$$

Hinc

Hinc fiet $d\Phi = \frac{dr}{uu\sqrt{(1-kk)}}\left(C - \frac{S}{nn}\right)$ atque

$$d\eta = \frac{dr}{uu\sqrt{(1-kk)}}\left(C - \frac{S}{nn}\right) - \frac{ttdr\sqrt{(1-ee)}}{n\omega\omega\sqrt{(1-kk)}}$$

Porro autem ob $dz = tdu + udt$, erit $\frac{dz}{dp} = \frac{tdu+udt}{ttdr}\sqrt{(1-kk)}$; at eſt $dt = -\frac{(1-kk)kdr\,\text{ſin}\,r}{(1-k\,\text{coſ}\,r)^2} = -\frac{ktt dr\,\text{ſin}\,r}{1-kk}$; ſicque fiet $\frac{dz}{dp} = \frac{du\sqrt{(1-kk)}}{tdr} - \frac{ku\,\text{ſin}\,r}{\sqrt{(1-kk)}}$; ac poſito elemento dr conſtante erit $d.\frac{dz}{dp} = \frac{ddu\sqrt{(1-kk)}}{tdr} - \frac{dudt\sqrt{(1-kk)}}{ttdr} - \frac{kdu\,\text{ſin}\,r}{\sqrt{(1-kk)}} - \frac{kudr\,\text{coſ}\,r}{\sqrt{(1-kk)}}$ hincque ob $\frac{dt}{tt} = -\frac{kdr\,\text{ſin}\,r}{1-kk}$ habebitur:

$$d.\frac{dz}{dp} = \frac{ddu\sqrt{(1-kk)}}{tdr} - \frac{kudr\,\text{coſ}\,r}{\sqrt{(1-kk)}}.$$

§. 42. Hinc iam porro obtinemus pro ſecunda aequatione

$$\frac{1}{dp}\,d.\frac{dz}{dp} = \frac{(1-kk)\,ddu}{t^3dr^2} - \frac{ku\,\text{coſ}\,r}{tt}$$

qui valor ſubſtitutus in aequatione per $\frac{t^3}{1-kk}$ multiplicata orietur haec aequatio:

$$\text{II.}\quad \frac{ddu}{dr^2} - \frac{ktu\,\text{coſ}\,r}{1-kk} = \frac{CC}{(1-kk)u^3} - \frac{2CS}{(1-kk)nnu^3} + \frac{SS}{n^4(1-kk)u^3}$$

$$-\frac{mt}{nn(1-kk)uu}\left(1-\tfrac{3}{4}\text{tang}\,\rho^2 + \tfrac{3}{4}\text{tang}\,\rho^2\,\text{coſ}\,2(\Phi-\pi)\right) + \frac{\mu t^3}{nn(1-kk)} + \frac{t^4u}{2nn(1-kk)\omega^3}$$

$$+\frac{3t^4u\,\text{coſ}\,2\eta}{2nn(1-kk)\omega^3} + \frac{3\nu t^5uu}{8nn\omega^4(1-kk)}(3\,\text{coſ}\,\eta + 5\,\text{coſ}\,3\eta)$$

$$\text{III.}\quad d\pi = -\frac{uudr\,\text{ſin}(\Phi-\pi)\,\text{ſin}(\theta-\pi)}{(Cnn-S)\sqrt{(1-kk)}}\left(\frac{3t^4\,\text{coſ}\,\eta}{\omega^3} + \frac{3\nu t^5u}{4\omega^4}(3+5\,\text{coſ}\,3\eta)\right)$$

 Quartam

Quartam aequationem $d.\, l \tang \varrho = \frac{d\pi}{\tang(\Phi - \pi)}$, cum nullam mutationem ſubeat, ſuperfluum foret continuo repetere.

§. 43. Conueniet autem quantitates conſtantes C et m, quarum valores nondum nouimus, ſaltem vero proxime indagare, quo facilius deinceps ipſam aequationum reſolutionem dirigere queamus. Perſpicuum autem eſt, ſi omnes quantitates a ſitu ſolis pendentes ex calculo deleantur, tum vtique fieri debere $u = 1$. Cum igitur primum S ab angulo η pendeat, terminos tam S quam η inuoluentes omittamus, ac pro ω quidem ſcribamus 1; quia tantum determinationem ad verum accedentem requirimus, quem in finem quoque inclinationem orbitae negligamus. Hinc aequatio ſecunda dabit:

$$-\frac{kt\cos r}{1-kk} = \frac{CC}{1-kk} - \frac{mt}{nn(1-kk)} + \frac{\mu t^3}{n^2(1-kk)} + \frac{t^4}{2nn(1-kk)} \text{ ſiue}$$

$$CC = \frac{mt}{nn} - \frac{\mu t^3}{nn} - kt\cos r - \frac{t^4}{2nn}$$

Cum autem ſit $t = \frac{1-kk}{1-k\cos r} = 1 + k\cos r$ proxime, ob k valde paruum habebitur.

$$CC = \frac{m}{nn} - \frac{\mu}{nn} - \frac{1}{2nn}$$
$$+ \frac{m}{nn}k\cos r - \frac{3\mu k}{nn}\cos r - k\cos r - \frac{2k}{nn}\cos r$$

vnde perſpicuum eſſe oportere.

$$\frac{m}{nn} = 1 + \frac{2+3\mu}{nn} \text{ et } CC = 1 + \frac{3+4\mu}{2nn}$$

§. 44. His

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§. 44. His igitur conſtantium $\frac{m}{nn}$ et CC valoribus proximis inuentis ponamus eſſe reuera:

$$\frac{m}{nn} = 1 + \frac{2+3\mu+\gamma}{nn} \text{ et } CC = 1 + \frac{3+4\mu+\delta}{2nn} = \lambda\lambda$$

ſcribamus enim λ pro C, quia litteris maiuſculis A, B, C, D etc. deinceps in operationibus ſequentibus vtemur: ſicque fiet

$$S = \frac{1}{\sqrt{(1-kk)}} \int dr \left(\frac{3t^4 uu}{2\omega^3} \sin 2\eta + \frac{3\nu t^5 u^3}{8\omega^4} (\sin\eta + 5\sin 3\eta)\right)$$

$$d\Phi = \frac{dr}{uu\sqrt{(1-kk)}}\left(\lambda - \frac{S}{nn}\right)$$

$$d\eta = \frac{dr}{nu\sqrt{(1-kk)}}\left(\lambda - \frac{S}{nn}\right) - \frac{ttdr\sqrt{(1-ee)}}{n\omega\omega\sqrt{(1-kk)}}$$

II. $$\frac{(1-kk)\,ddu}{dr^2} = ktu\cos r + \frac{\lambda\lambda}{u^3} - \frac{2\lambda S}{nnu^3} + \frac{SS}{n^4u^3} + \frac{\mu t^3}{nn} + \frac{t^4 u}{2nn\omega^3}$$
$$- \frac{mt}{nnuu}\left(1 - \tfrac{3}{4}\tan\rho^2 + \tfrac{3}{4}\tan\rho^2 \cos 2(\Phi-\pi)\right)$$
$$+ \frac{3t^4 u\cos 2\eta}{2nn\omega^3} + \frac{3\nu t^5 uu}{8nn\omega^4}(3\cos\eta + 5\cos 3\eta).$$

III. $$d\pi = -\frac{uudr\sin(\Phi\,\pi)\sin(\theta\,\pi)}{(\lambda nn - S)\sqrt{(1-kk)}}\left(\frac{3t^4}{\omega^3}\cos\eta + \frac{3\nu t^5 u}{4\omega^4}(3+5\cos 2\eta)\right)$$

§. 45. Ponatur $\lambda = \kappa\sqrt{(1-kk)}$, vt ſit $\kappa\kappa = 1 + \frac{3+4\mu+\delta}{2nn}$ defectum enim in termino indefinito δ complecti licet, exiſtente $m = nn + 2 + 3\mu + \gamma$; tum vero ponatur $S = (1-kk)^{\frac{1}{2}} \int R dr$, vt ſit $R = \frac{dS}{dr\sqrt{(1-kk)}}$; ac ſi pro t et ω valores reſtituamus, qui erant,

$t = \frac{1-kk}{1-k\cos r}$ et $\omega = \frac{1-ee}{1-e\cos s}$ habebimus:

R =

$$R = \frac{3\,(1-kk)^3\,(1-e\cos s)^3}{2\,(1-ee)^3\,(1-k\cos r)^4}\,nn\,\sin 2\eta$$
$$+\frac{3v(1-kk)^4\,(1-e\cos s)^4}{8\,(1-ee)^4\,(1-k\cos r)^5}\,u^3\,(\sin\eta+5\sin 3\eta)$$

$$d\Phi = \frac{dr}{uu}\left(\varkappa-\frac{1}{nn}\int R\,dr\right);\ d\theta = ds = \frac{(1-kk)^{\frac{3}{2}}(1-e\cos s)^2}{n(1-ee)^{\frac{3}{2}}(1-k\cos r)^2}\,dr$$

$$\frac{d\eta}{dr} = \frac{\varkappa}{uu}-\frac{\int R\,dr}{nnuu}-\frac{(1-kk)^{\frac{3}{2}}(1-e\cos s)^2}{n(1-ee)^{\frac{3}{2}}(1-k\cos r)^2}$$

§. 46. Aequatio autem ſecunda facta hac ſubſtitutione, ſi per $1-kk$ diuidatur, abibit in ſequentem:

$$\text{II. } \frac{ddu}{dr^2} = \frac{ku\cos r}{1-k\cos r}+\frac{\varkappa\varkappa}{u^3}-\frac{2\varkappa\int R\,dr}{nnu^3}+\frac{(\int R\,dr)^2}{n^4u^3}+\frac{\mu(1-kk)^2}{nn(1-k\cos r)^3}$$
$$-\frac{m}{nn(1-k\cos r)\,uu}\left(1-\tfrac{3}{4}\operatorname{tang}\rho^2+\tfrac{3}{4}\operatorname{tang}\rho^2\cos 2(\Phi-\pi)\right)$$
$$+\frac{(1-kk)^3(1-e\cos s)^3}{2nn\,(1-ee)^3(1-k\cos r)^4}\,n\,(1+3\cos 2\eta)$$
$$+\frac{3v(1-kk)^4(1-e\cos s)^4}{8nn(1-ee)^4(1-k\cos r)^5}\,u^2\,(3\cos\eta+5\cos 3\eta)$$

$$\text{III. } d\pi = -\frac{dr\sin(\varphi-\pi)\sin(\theta-\pi)}{(\varkappa nn - R\int dr)}\left(\frac{3(1-kk^3(1-e\cos s)^3}{(1-ee)^3(1-k\cos r)^4}\,uu\cos\eta\right.$$
$$\left.+\frac{3v(1-kk)^4(1-e\cos s)^4}{4(1-ee)^4(1-k\cos r)^5}\,n^3\,(3+5\cos 2\eta)\right)$$

Ac ſi ϵ denotet inclinationem mediam orbitae lunaris, quantitas $1-\frac{3}{4}\operatorname{tang}\rho^2+\frac{3}{4}\operatorname{tang}\rho^2\cos 2(\varphi-\pi)$ in has duas partes diſceſpi poterit:

$(1-\frac{3}{4}\operatorname{tang}\epsilon^2)+\frac{3}{4}(\operatorname{tang}\epsilon^2-\operatorname{tang}\rho^2+\operatorname{tang}\rho^2\cos 2(\varphi-\pi))$ quarum illa eſt conſtans, haec vero proprie a nodo et inclinatione pendet.

§. 47.

§. 47. Euoluamus autem producta illa ex μ et ω orta, et quoniam excentricitates k et e ſunt valde parvae, ſufficit ad eos vſque terminos tantum progredi, qui coefficientes habeant kk, ek et ee, eosque qui per altiores poteſtates ſint multiplicati omittere. Hinc erit:

$$\frac{1}{1-k\,\mathrm{coſ}\,r} = 1 + \tfrac{1}{2}kk + k\,\mathrm{coſ}\,r + \tfrac{1}{2}k^2\,\mathrm{coſ}\,2r$$

$$\frac{k\,\mathrm{coſ}\,r}{1-k\,\mathrm{coſ}\,r} = \tfrac{1}{2}kk + k\,\mathrm{coſ}\,r + \tfrac{1}{2}k^2\,\mathrm{coſ}\,2r$$

$$\frac{(1-kk)^2}{(1-k\,\mathrm{coſ}\,r)^3} = 1 + 3k\,\mathrm{coſ}\,r,$$ quia hic terminus per μ multiplicatur.

$$\frac{(1-kk)^{\frac{3}{2}}}{(1-k\,\mathrm{coſ}\,r)^2} = 1 + 2k\,\mathrm{coſ}\,r + \tfrac{3}{2}kk\,\mathrm{coſ}\,2r$$

$$\frac{(1-kk)^3}{(1-k\,\mathrm{coſ}\,r)^4} = 1 + 2kk + 4k\,\mathrm{coſ}\,r + 5kk\,\mathrm{coſ}\,2r$$

$$\frac{(1-kk)^4}{(1-k\,\mathrm{coſ}\,r)^5} = 1 + 5k\,\mathrm{coſ}\,r,$$ quia hic terminus iam per ν eſt multiplicatus.

§. 48. Porro vero pro terminis ex ω enatis eſt:

$$\frac{(1-e\,\mathrm{coſ}\,s)^2}{(1-ee)^{\frac{3}{2}}} = 1 + 2ee - 2e\,\mathrm{coſ}\,s + \tfrac{1}{2}ee\,\mathrm{coſ}\,2s$$

$$\frac{(1-e\,\mathrm{coſ}\,s)^3}{(1-ee)^3} = 1 + \tfrac{9}{2}ee - 3e\,\mathrm{coſ}\,s + \tfrac{3}{2}ee\,\mathrm{coſ}\,2s$$

$$\frac{(1-e\,\mathrm{coſ}\,s)^4}{(1-ee)^4} = 1 - 4e\,\mathrm{coſ}\,s,$$ quia hic factor tantum in minimis terminis occurrit.

Hinc ergo colligimus:

$$\frac{(1-kk)^{\frac{3}{2}}(1-e\,\mathrm{coſ}\,s)^2}{(1-ee)^{\frac{3}{2}}(1-k\,\mathrm{coſ}\,r)^2} = 1 + 2ee + 2k\,\mathrm{coſ}\,r + \tfrac{3}{2}kk\,\mathrm{coſ}\,2r - 2e\,\mathrm{coſ}\,s - 2ek\,\mathrm{coſ}(r+s) - 2ek\,\mathrm{coſ}(r-s) + \tfrac{1}{2}ee\,\mathrm{coſ}\,2s$$

$$\frac{(1-kk)^3(1-e\cos s)^3}{(1-ee)^3(1-k\cos r)^4} = 1 + 2kk + \tfrac{9}{2}ee + 4k\cos r + 5kk\cos 2r$$
$$-3e\cos s - 6ek\cos(r-s) - 6ek\cos(r+s) + \tfrac{3}{2}ee\cos 2s$$

$$\frac{(1-kk)^4(1-e\cos s)^4}{(1-ee)^4(1-k\cos r)^5} = 1 + 5k\cos r - 4e\cos s,$$

atque hinc fiet: $d\Phi = \frac{dr}{uu}\left(\varkappa - \frac{1}{nn}\int R\,dr\right)$ atque

$$\frac{ds}{dr} = \frac{1+2ee}{n} + \frac{2k}{n}\cos r - \frac{2e}{n}\cos s - \frac{2ek}{n}(\cos r - s)$$
$$+ \frac{3kk}{2n}\cos 2r + \frac{ee}{2n}\cos 2s - \frac{2ek}{n}\cos(r+s)$$

$$\frac{d\eta}{dr} = \frac{\varkappa}{uu} - \frac{\int R\,dr}{nnuu} - \frac{1-2ee}{n} - \frac{2k}{n}\cos + r\frac{2e}{n}\cos s + \frac{2ek}{n}\cos(r-s)$$
$$- \frac{3kk}{2n}\cos 2r - \frac{ee}{2n}\cos 2s + \frac{2ek}{n}\cos(r+s)$$

§. 49. Introductis nunc his valoribus euolutis in formulas noſtras, iisque, qui per ſinum coſinumue alterius anguli ſunt multiplicati, pariter ſecundum ſimplices angulos explicatis, obtinebimus primum valorem ipſius R, qui erit:

$$R = \tfrac{3}{2}u^2 \begin{cases} (1+2kk+\tfrac{9}{2}ee)\sin 2\eta + 2k\sin(2\eta - r) + 2k\sin(2\eta + r) \\ + \tfrac{5}{2}kk\sin(2\eta - 2r) + \tfrac{5}{2}kk\sin(2\eta + 2r) \\ - \tfrac{3}{2}e\sin(2\eta - s) - \tfrac{3}{2}e\sin(2\eta + s) \\ + \tfrac{3}{4}ee\sin(2\eta - 2s) + \tfrac{3}{4}ee\sin(2\eta + 2s) \\ - 3ek\sin(2\eta - r + s) - 3ek\sin(2\eta + r - s) \\ - 3ek\sin(2\eta - r - s) - 3ek\sin(2\eta + r + s) \end{cases}$$

$$+ \tfrac{3}{8}\nu u^3 \begin{cases} \sin\eta + \tfrac{5}{2}k\sin(\eta - r) - 2e\sin(\eta - s) \\ 5\sin 3\eta + \tfrac{5}{2}k\sin(\eta + r) - 2e\sin(\eta + s) \\ + \tfrac{25}{2}k\sin(3\eta - r) - 10e\cos(3\eta - s) \\ + \tfrac{25}{2}k\sin(3\eta + r) - 10e\cos(3\eta + s) \end{cases}$$

§. 50.

§. 50. Aequatio autem ſecunda principalis ſequentem induet formam:

$$\text{II.}\quad \frac{ddu}{dr^2} = \frac{nn}{u^3} - \frac{2n\int R\,dr}{nnu^3} + \frac{(\int R\,dr)^2}{n^4u^3}$$

$$+ \frac{3m\,\mathrm{tang}\,\rho^2}{4nnuu}(1-\cos 2(\Phi-\pi))(1+k\cos r)$$

$$- \frac{m}{nnuu}(1+\tfrac{1}{2}kk+k\cos r+\tfrac{1}{2}k^2\cos 2r)+\frac{\mu}{nn}(1+3k\cos r)$$

$$+ u(\tfrac{1}{2}kk+k\cos r+\tfrac{1}{2}kk\cos 2r)$$

$$+ \frac{u}{2nn}\left\{\begin{array}{l} 1+2kk+\tfrac{9}{2}ee+4k\cos r-3e\cos s-6ek\cos(r-s) \\ \quad +5kk\cos 2r+\tfrac{3}{2}ee\cos 2s-6ek\cos(r+s) \end{array}\right.$$

$$+ \frac{3u}{2nn}\left\{\begin{array}{l} (1+2kk+\tfrac{9}{2}ee)\cos 2\eta+2k\cos(2\eta-r)+2k\cos(2\eta+r) \\ +\tfrac{5}{2}kk\cos(2\eta-2r)+\tfrac{5}{2}kk\cos(2\eta+2r) \\ -\tfrac{3}{2}e\cos 2(2\eta-s)-\tfrac{3}{2}e\cos(2\eta+s) \\ +\tfrac{3}{4}ee\cos(2\eta-2s)+\tfrac{3}{4}ee\cos(2\eta+2s) \\ -3ek\cos(2\eta-r+s)-3ek\cos(2\eta+r-s) \\ -3ek\cos(2\eta-r-s)-3ek\cos(2\eta+r+s) \end{array}\right.$$

$$+ \frac{3\nu uu}{8nn}\left\{\begin{array}{l} 3\cos\eta+5\cos 3\eta+\tfrac{15}{2}k\cos(\eta-r)+\tfrac{15}{2}k\cos(\eta+r) \\ -6e\cos(\eta-s)-6e\cos(\eta+s)+\tfrac{25}{2}k\cos(\eta-r)+\tfrac{25}{2}k\cos(\eta+r) \\ -10e\cos(3\eta-s)-10e\cos(3\eta+s) \end{array}\right.$$

vbi terminos, qui adhuc vlteriori euolutione indigent, primo loco poſui, et cum terminus tang ρ^2 implicans iam ſit valde paruus, in eius multiplicatore ſecundam ipſius k poteſtatem omiſi: ſin autem alicuius momenti videantur, loco $1+k\cos r$ ſcribi poterit $1+\frac{1}{2}kk+k\cos r+\frac{1}{2}kk\cos 2r$.

§. 51. Pro longitudine vero nodi inuenienda aequatio ſequens prodibit reſoluenda:

$$d\pi = -\frac{3uudr\,\mathrm{ſin}(\Phi-\pi)\,\mathrm{ſin}(\theta-\pi)}{unn-\int R\,dr}\,\mathrm{coſ}\,\eta\,(1+2kk+\tfrac{9}{2}ee+4k\,\mathrm{coſ}\,r$$
$$+5kk\,\mathrm{coſ}\,2r-3e\,\mathrm{coſ}\,s+\tfrac{3}{2}ee\,\mathrm{coſ}\,2s)$$
$$-\frac{3vu^3dr\,\mathrm{ſin}(\Phi-\pi)\,\mathrm{ſin}(\theta-\pi)}{4(unn-\int R\,dr)}\,(3+5\,\mathrm{coſ}\,2\eta)\,(1+5k\,\mathrm{coſ}\,r)$$

At eſt $\mathrm{ſin}\,(\Phi-\pi)\,\mathrm{ſin}\,(\theta-\pi) = \frac{1}{2}\,\mathrm{coſ}\,\eta - \frac{1}{2}\,(\Phi+\theta-2\pi)$; vnde
$\mathrm{ſin}(\Phi-\pi)\,\mathrm{ſin}(\theta-\pi)\,\mathrm{cſ}\,\eta = \frac{1}{4}+\frac{1}{4}\,\mathrm{cſ}\,2\eta-\frac{1}{4}\,\mathrm{cſ}\,2(\Phi-\pi)-\frac{1}{4}\,\mathrm{cſ}\,2(\theta-\pi)$
et $\mathrm{ſin}(\Phi-\pi)\,\mathrm{ſin}(\theta-\pi)\,\mathrm{coſ}\,2\eta = \frac{1}{4}\,\mathrm{coſ}\,\eta+\frac{1}{4}\,\mathrm{coſ}\,3\eta$
$-\frac{1}{4}\,\mathrm{coſ}\,(3\Phi-\theta-2\pi)-\frac{1}{4}\,\mathrm{coſ}\,(3\theta-\Phi-2\pi)$

Tum vero ob $\int R\,dr$ valde paruum prae unn, erit ſatis exacte

$$\frac{1}{unn-\int R\,dr} = \frac{1}{unn}+\frac{\int R\,dr}{uun^4}+\frac{(\int R\,dr)^2}{u^3n^6}$$

vbi quidem poſtremus terminus tuto omitti poteſt.

§. 52. Praeterea vero ponatur $u = 1+\frac{v}{nn}$, vt ſit $ddu = \frac{ddv}{nn}$, et reiectis terminis per n^4 diuiſis, qui iam per exiguam quantitatem ſunt multiplicati, erit

$$\frac{d\Phi}{dr} = u-\frac{2uv}{nn}+\frac{3uv^2}{n^4}-\frac{\int R\,dr}{nn}+\frac{2v\int R\,dr}{n^4}$$

$$\frac{d\eta}{dr} = u-\frac{1-2ee}{n}-\frac{2k}{n}\,\mathrm{coſ}\,r+\frac{2e}{n}\,\mathrm{coſ}\,s-\frac{3kk}{2n}\,\mathrm{coſ}\,2r-\frac{ee}{2n}\,\mathrm{coſ}\,2s$$
$$+\frac{2ek}{n}\,\mathrm{coſ}\,(r-s)+\frac{2ek}{n}\,\mathrm{coſ}\,(r+s)$$
$$-\frac{2uv-\int R\,dr}{nn}+\frac{3uv^2+2v\int R\,dr}{n^4}\qquad\text{atque}$$

$$R = \tfrac{3}{2}\,(1+2kk\mp\tfrac{9}{2}ee)\,\mathrm{ſin}\,2\eta+3k\,\mathrm{ſin}(2\eta-r)-\tfrac{9}{4}e\,\mathrm{ſin}(2\eta-s)$$
$$+3k\,\mathrm{ſin}(2\eta+r)-\tfrac{9}{4}e\,\mathrm{ſin}(2\eta+s)$$
$$+\frac{3v}{nn}\,\mathrm{ſin}\,2\eta\quad+\tfrac{15}{4}kk\,\mathrm{ſin}(2\eta-2r)+\tfrac{9}{8}ee\,\mathrm{ſin}(2\eta-2s)$$
$$+\tfrac{15}{4}kk\,\mathrm{ſin}(2\eta+2r)+\tfrac{9}{8}ee\,\mathrm{ſin}(2\eta+2s)$$
$$+\frac{3vv}{2n^4}\,\mathrm{ſin}\,2\eta\quad-\tfrac{9}{2}ek\,\mathrm{ſin}(2\eta-r+s)-\tfrac{9}{2}ek\,\mathrm{ſin}(2\eta+r-s)$$
$$-\tfrac{9}{2}ek\,\mathrm{ſin}(2\eta-r-s)-\tfrac{9}{2}ek\,\mathrm{ſin}(2\eta+r+s)$$
$$+$$

$$+\frac{6kv}{nn}\sin(2\eta-r)+\frac{6kv}{nn}\sin(2\eta+r)$$
$$-\frac{9ev}{2nn}\sin(2\eta-s)-\frac{9ev}{2nn}\sin(2\eta+s)$$
$$+\tfrac{3}{8}v\sin\eta+\tfrac{15}{16}vk\sin(\eta-r)-\tfrac{3}{4}ve\sin(\eta-s)$$
$$+\tfrac{75}{16}vk\sin(3\eta-r)-\tfrac{15}{4}ve\sin(3\eta-s)$$
$$+\tfrac{15}{8}v\sin 3\eta+\tfrac{15}{16}vk\sin(\eta+r)-\tfrac{3}{4}ve\sin(\eta+s)$$
$$+\tfrac{75}{16}vk\sin(3\eta+s)-\tfrac{15}{4}ve\sin(3\eta+s)$$

§. 53. Ipſa vero aequatio ſecunda per hanc ſubſtitutionem, poſtquam per nn fuerit multiplicata, in formam ſequentem abibit.

$$\text{II. } \frac{ddv}{dr^2}=\varkappa\varkappa nn-3\varkappa\varkappa v+\frac{6\varkappa\varkappa vv}{nn}-2\varkappa\int R\,dr$$
$$+\frac{6\varkappa v}{nn}\int R\,dr+\frac{1}{nn}\left(\int R\,dr\right)^2$$
$$+\frac{3m\,\mathrm{tang}\,\rho^2}{4}(1-\cos 2(\varphi-\pi))(1+\tfrac{1}{2}kk+k\cos r+\tfrac{1}{2}kk\cos 2r)$$
$$-\frac{3mv\,\mathrm{tang}\,\rho^2}{2nn}(1-\cos 2(\varphi-\pi))\quad(1+k\cos r)$$
$$-\frac{3mvv}{n^4}(1+k\cos r)-m(1+\tfrac{1}{2}kk+k\cos r+\tfrac{1}{2}kk\cos 2r)$$
$$+\frac{2mv}{nn}(1+\tfrac{1}{2}kk+k\cos r+\tfrac{1}{2}kk\cos 2r)+nn(\tfrac{1}{2}kk+k\cos r+\tfrac{1}{2}kk\cos 2r)$$
$$+v(\tfrac{1}{2}kk+k\cos r+\tfrac{1}{2}kk\cos 2r)+\mu(1+3k\cos r)$$
$$+\tfrac{1}{2}+kk+\tfrac{9}{4}ee+2k\cos r-\tfrac{3}{2}e\cos s-3ek\cos(r-s)$$
$$+\tfrac{5}{2}kk\cos r+\tfrac{3}{4}ee\cos 2s-3ek\cos(r+s)$$
$$+\frac{v}{2nn}(1+4k\cos r-3e\cos s)$$
$$+\tfrac{3}{2}(1+2kk+\tfrac{9}{2}ee)\cos 2\eta+3k\cos(2\eta-r)-\tfrac{9}{4}e\cos(2\eta-s)$$
$$+3\cos(2\eta+r)-\tfrac{9}{4}e\cos(2\eta+s)$$

$$+\tfrac{15}{4}kk\,\mathrm{cof}(2\eta-2r)+\tfrac{9}{8}ee\,\mathrm{cof}(2\eta-2s)$$
$$+\tfrac{15}{4}kk\,\mathrm{cof}(2\eta+2r)+\tfrac{9}{8}ee\,\mathrm{cof}(2\eta+2s)$$
$$-\tfrac{9}{2}ek\,\mathrm{cof}(2\eta-r+s)-\tfrac{9}{2}ek\,\mathrm{cof}(2\eta+r-s)$$
$$-\tfrac{9}{2}ek\,\mathrm{cof}(2\eta-r-s)-\tfrac{9}{2}ek\,\mathrm{cof}(2\eta+r+s)$$

$$+\frac{3v}{2nn}\begin{cases}\mathrm{cof}\,2\eta+2k\,\mathrm{cof}(2\eta-r)+2k\,\mathrm{cof}(2\eta+r)\\ -\tfrac{3}{2}e\,\mathrm{cof}(2\eta-s)-\tfrac{3}{2}e\,\mathrm{cof}(2\eta+s)\end{cases}$$

$$+\tfrac{3}{8}v\begin{cases}+3\,\mathrm{cof}\,\eta+\tfrac{15}{2}k\,\mathrm{cof}(\eta-r)-6e\,\mathrm{cof}(\eta-s)\\ +5\,\mathrm{cof}\,3\eta+\tfrac{15}{2}k\,\mathrm{cof}(\eta+r)-6e\,\mathrm{cof}(\eta+s)\\ +\tfrac{25}{2}k\,\mathrm{cof}(3\eta-r)-10e\,\mathrm{cof}(3\eta-s)\\ +\tfrac{25}{2}k\,\mathrm{cof}(3\eta+r)-10e\,\mathrm{cof}(3\eta+s)\end{cases}$$

$$+\frac{3vv}{4nn}(3\,\mathrm{cof}\,\eta+5\,\mathrm{cof}\,3\eta)$$

§. 54. Cum autem ſit $m=nn+2+3\mu+\gamma$ et $\varkappa\varkappa=1+\frac{3+4\mu+\delta}{2nn}$, ſi hi valores ſubſtituantur, plures termini ſe mutuo deſtruent, aequatioque prodibit ſequenti forma concinniori contenta: vbi quidem in terminis per ſe minimis loco m ſcribi licebit nn, et 1 loco $\varkappa\varkappa$ vel $\varkappa$.

II. Aequatio.

$$\frac{ddv}{dr^2}=\tfrac{1}{2}\delta-\gamma+\tfrac{9}{4}ee-\gamma k\,\mathrm{cof}\,r+\tfrac{3}{2}kk\,\mathrm{cof}\,2r$$
$$-2\left(1+\frac{3+4\mu+\delta}{4nn}\right)\int R\,dr+\frac{1}{nn}\left(\int R\,dr\right)^2$$
$$-v\left(1-\tfrac{3}{2}kk-3k\,\mathrm{cof}\,r-\tfrac{3}{2}kk\,\mathrm{cof}\,2r\right)$$
$$+\frac{vv}{nn}\big(3-3k\,\mathrm{cof}\,r)-\tfrac{3}{2}e\,\mathrm{cof}\,s+\tfrac{3}{4}ee\,\mathrm{cof}\,2s-3ek\,\mathrm{cof}(r-s)$$
$$-3ek\,\mathrm{cof}(r+s)+\tfrac{3}{2}(1+2kk+\tfrac{9}{2}ee)\,\mathrm{cof}\,2\eta$$

+

$$+ 3k\cos(2\eta-r) + \tfrac{15}{4}kk\cos(2\eta-2r) - \tfrac{9}{4}e\cos(2\eta-s)$$
$$+ 3k\cos(2\eta+r) + \tfrac{15}{4}kk\cos(2\eta+2r) - \tfrac{9}{4}e\cos(2\eta-s)$$
$$+ \tfrac{9}{8}ee\cos(2\eta-2s) - \tfrac{9}{2}ek\cos(2\eta-r+s) - \tfrac{9}{2}ek\cos(2\eta-r-s)$$
$$+ \tfrac{9}{8}ee\cos(2\eta+2s) - \tfrac{9}{2}ek\cos(2\eta+r-s) - \tfrac{9}{2}ek\cos(2\eta+r+s)$$

$$+ \frac{v}{nn}\begin{cases} 2\gamma - \tfrac{3}{2}\delta + 6k\cos r + 2(3\mu+\gamma)k\cos r + 6\int R\,dr \\ - \tfrac{3}{2}e\cos s + \tfrac{3}{2}\cos 2\eta + 3k\cos(2\eta-r) + 3k\cos(2\eta+r) \\ - \tfrac{9}{4}e\cos(2\eta-s) - \tfrac{9}{4}e\cos(2\eta+s) \end{cases}$$

$$+ \tfrac{3}{8}v\begin{cases} 3\cos\eta + \tfrac{15}{2}k\cos(\eta-r) - 6e\cos(\eta-s) \\ + 5\cos 3\eta + \tfrac{15}{2}k\cos(\eta+r) - 6e\cos(\eta+s) \\ + \tfrac{25}{2}k\cos(3\eta-r) - 10e\cos(3\eta-s) \\ + \tfrac{25}{2}k\cos(3\eta+r) - 10e\cos(3\eta+s) \end{cases}$$

$$+ \frac{3vv}{4nn}(3\cos\eta + 5\cos 3\eta)$$

$$+ \tfrac{3}{4}(nn+2+3\mu+\gamma)\frac{(1-2v)}{nn}\,\mathrm{tang}\,\rho^2(1-\cos 2(\Phi-\pi))$$
$$(1 + \tfrac{3}{2}kk + k\cos r + \tfrac{3}{2}kk\cos 2r)$$

§. 55. Pro loco nodi autem inveniendo prodibit ſequens aequatio.

$$\frac{d\pi}{dr} = \frac{-3}{unn}\left(1+\frac{2uv+\int R\,dr}{unn}\right)(1 + 2kk + \tfrac{9}{4}ee)$$
$$\begin{cases} \tfrac{1}{4} + \tfrac{1}{4}\cos 2\eta - \tfrac{1}{4}\cos 2(\Phi-\pi) - \tfrac{1}{4}\cos 2(\theta-\pi) \\ + k\cos r + \tfrac{1}{2}k\cos(2\eta-r) - \tfrac{1}{2}k\cos(2\Phi-2\pi-r) \\ - \tfrac{3}{4}e\cos s + \tfrac{1}{2}k\cos(2\eta+r) - \tfrac{1}{2}k\cos(2\Phi-2\pi+r) \\ \quad - \tfrac{3}{8}e\cos(2\eta-s) - \tfrac{1}{2}k\cos(2\theta-2\pi-r) \\ \quad - \tfrac{3}{8}e\cos(2\eta+r) - \tfrac{1}{2}k\cos(2\theta-2\pi+r) \end{cases}$$
$$- \frac{3v}{4unn}\begin{cases} \tfrac{11}{4}\cos\eta + \tfrac{5}{4}\cos 3\eta - \tfrac{5}{4}\cos(3\Phi+\theta-2\pi) \\ - \tfrac{5}{4}\cos(3\Phi-\theta-2\pi) - \tfrac{5}{4}\cos(3\theta-\Phi-2\pi) \end{cases}$$

At

At pro inclinatione orbitae habebitur:

$$\frac{d.l\,\mathrm{tang}\,\varrho}{dr} = \frac{-3}{unn}\left(1 + \frac{2uv + \int \mathrm{R}dr}{unn}\right)\left(1 + 2kk + \tfrac{9}{2}ee\right)$$

$$\left\{\begin{array}{l} \tfrac{1}{4}\,\mathrm{ſin}\,2(\varphi - \pi) + \tfrac{1}{4}\,\mathrm{ſin}\,2(\theta - \pi) - \tfrac{1}{4}\,\mathrm{ſin}\,2\eta) \\ - \tfrac{1}{2}k\,\mathrm{ſin}\,(2\eta - r) + \tfrac{1}{2}k\,\mathrm{ſin}\,(2\varphi - 2\pi - r) \\ - \tfrac{1}{2}k\,\mathrm{ſin}\,(2\eta + r) + \tfrac{1}{2}k\,\mathrm{ſin}\,(2\varphi - 2\pi + r) \\ + \tfrac{3}{8}e\,\mathrm{ſin}\,(2\eta - s) + \tfrac{1}{2}k\,\mathrm{ſin}\,(2\theta - 2\pi - r) \\ + \tfrac{3}{8}e\,\mathrm{ſin}\,(2\eta + s) + \tfrac{1}{2}k\,\mathrm{ſin}\,(2\theta - 2\pi + r) \end{array}\right.$$

$$- \frac{3\nu}{4unn}\left(-\tfrac{1}{4}\,\mathrm{ſin}\,\eta - \tfrac{5}{4}\,\mathrm{ſin}\,3\eta + \tfrac{3}{2}\,\mathrm{ſin}\,(\varphi + \theta - 2\pi)\right)$$

$$+ \tfrac{5}{4}\,\mathrm{ſin}\,(3\varphi - \theta - 2\pi) + \tfrac{5}{4}\,\mathrm{ſin}\,(3\theta - \varphi - 2\pi))$$

Quomodo igitur his aequationibus ad motum Lunae cognoſcendum vti conueniat, in ſequentibus capitibus videamus.

CAPUT

CAPUT IV.

INUESTIGATIO INAEQUALITATIS LUNAE ABSOLUTAE, QUAE VARIATIO DICITUR.

§. 33.

Ex his aequationibus perſpicitur in determinationem motus Lunae plurimorum angulorum vel ſinus vel coſinus ingredi, qui anguli formantur per variam combinationem ſequentium 4 angulorum:

1. ex diſtantia Solis a Luna, quem angulum poſuimus $= \eta$
2. ex anomalia Lunae vera $= r$
3. ex anomalia Solis vera $= s$
4. ex diſtantia Lunae a nodo aſcendente $= \Phi - \pi$.

Ne igitur a tanta angulorum multitudine obruamur, a caſibus ſimplicioribus ordiamur: ac primo quidem in eas tantum motus inaequalitates inquiramus, quae a ſolo angulo η pendeant, neque idcirco excentricitatem vel Solis vel Lunae implicent, neque ab orbitae lunaris inclinatione ad eclipticam afficiantur.

§. 57. Has igitur inaequalitates, quae a ſolo ſitu Solis reſpectu Lunae naſcuntur, atque ab Aſtronomis ſub nomine variationis comprehendi ſolent, ex praecedentibus aequationibus eliciemus, ſi tam excentricitatem Lunae k quam ſolis e pro nihilo habeamus, atque inclinationem orbitae lunaris ad eclipticam euaneſcentem ſtatuamus, ita vt ſit $k = o$, $e = o$ et tang $\rho = o$. Sic enim obtinebimus eas inaequalitates Lunae, quae ab his elementis non pendent, ideoque tantum per angu-

lum η determinantur; quae cum vnica tabula comprehendi queant, haec tabula variationem Lunae indicare dicitur. Interim tamen hic animaduerti oportet, partem quandam exiguam variationis quoque ab excentricitate orbitae Lunae k pendere, quam partem deinceps ſupplebimus, cum huius excentricitatis rationem ſumus habituri.

§. 58. Reiectis ergo terminis k, e, et tang ϱ continentibus, habebimus:

$$\frac{d\Phi}{dr} = \varkappa - \frac{2\varkappa v - \int R\,dr}{nn} + \frac{3\varkappa v^2 + 2v\int R\,dr}{n^4}$$

$$\frac{d\eta}{dr} = \varkappa - \frac{1}{n} - \frac{2\varkappa v - \int R\,dr}{nn} + \frac{3\varkappa v^2 + 2v\int R\,dr}{n^4}$$

$R = \frac{3}{2}\,\text{ſin}\,2\eta + \frac{3v}{nn}\,\text{ſin}\,2\eta + \frac{3}{8}v\,\text{ſin}\,\eta + \frac{15}{8}v\,\text{ſin}\,3\eta$, ac denique

$$\frac{ddv}{dr^2} = \tfrac{1}{2}\delta - \gamma - 2\left(1 + \frac{3+4\mu+\delta}{4nn}\right)\int R\,dr + \frac{1}{nn}\left(\int R\,dr\right)^2 - v + \frac{3vv}{nn}$$
$$+ \tfrac{3}{2}\,\text{coſ}\,2\eta + \frac{v}{nn}\left(2\gamma - \tfrac{3}{2}\delta\right) + \frac{3v\,\text{coſ}\,2\eta}{2nn} + \frac{6v}{nn}\int R\,dr$$
$$+ \tfrac{9}{8}\,\text{coſ}\,\eta + \tfrac{15}{8}v\,\text{coſ}\,3\eta$$

Hic autem notandum eſt eſſe $\varkappa = \sqrt{\left(1 + \frac{3+4\mu+\delta}{2nn}\right)}$; quoniam vero valores litterarum μ et δ demum cum per conſenſum obſeruationum, tum per indolem calculi definire inſtituimus, hic ex obſeruationibus petamus valores ipſius $\varkappa$; cum enim ſit $\varkappa : 1 = d\Phi : dr$, hoc eſt vt motus Lunae medius ad motum anomaliae, erit $\varkappa = 1,0085272$. Fieri quidem poteſt, vt hic valor aliquantulum a vero differat, ſed errorem ſi quis lateat infra detegemus, facillimeque emendabimus.

§. 59.

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§. 59. Cum igitur iam ſupra inuenerimus eſſe $n = 13,25586$ ac proinde $nn = 175,71795$ erit $\frac{1}{n} = 0,075438$, ideoque $\varkappa - \frac{1}{n} = 0,933089$ Hic autem numerus, qui iam quaſi medium valorem rationis $\frac{d\eta}{dr}$ exprimit, in omnibus operationibus, quae ſequuntur, frequentiſſime occurret, hincque breuitatis gratia ponamus

$$\varkappa - \frac{1}{n} = \alpha, \text{ vt ſit } \alpha + \frac{1}{n} = V\left(+\frac{3+4\mu+\delta}{2nn}\right)$$

eritque ergo $\alpha = 0,933089$, qui valor quam minime a vero diſcrepat, vti mox patebit Quod autem verus ipſius α valor aliquantulum diuerſus eſſe poſſit, inde primo patet, quod minutias, quae ex terminis $\frac{3\varkappa v^2 + 2v\int R dr}{n^4}$ quantitati conſtanti accreſcere potuiſſent, hic negleximus; tum vero fieri poteſt, vt ratio media differentialium $d\eta$ ad dr alia ſit atque quantitatum finitarum η et r.

§. 60. Si has formulas attente contemplemur, mox deprehendemus valorem integralis $\int R dr$ conſtare ex coſinibus angulorum 2η, η, 3η, et 4η. Quanquam enim altiora quoque multipla huius anguli ingredientur, tamen facile patet, coefficientes eorum continuo fieri minores, ita vt in quadruplo tuto ſubſiſtere poſſimus: ſimilis autem erit ratio valoris ipſius v. Hinc ponamus:

$$\int R\,dr = \mathfrak{A}\cos 2\eta + \mathfrak{B}\cos 4\eta + \mathfrak{a}v\cos\eta + \mathfrak{b}v\cos 3\eta$$

$$v = A\cos 2\eta + B\cos 4\eta + av\cos\eta + bv\cos 3\eta$$

atque hos valores fictitios in formulis noſtris ſubſtituamus:

mus, vt inde valores iſtorum coefficientium aſſumtorum determinare poſſimus: quippe qui modus aptiſſimus videtur ad cognitionem integralium perueniendi. Quia autem eſt circiter $\nu = \frac{1}{282}$, patet terminos per ν multiplicatos prae reliquis tam eſſe exiguos, vt eos qui multo fuerint minores, ſine haeſitatione praetermittere poſſimus.

§. 61. Per hos ergo valores aſſumtos conſequemur:

$$\frac{d\Phi}{dr} = \varkappa - \frac{(2\varkappa A + \mathfrak{A})}{nn} \operatorname{cof} 2\eta - \frac{(2\varkappa B + \mathfrak{B})}{nn} \operatorname{cof} 4\eta$$
$$+ \frac{A(3\varkappa A + 2\mathfrak{A})}{2n^4} \operatorname{cof} 2\eta + \frac{A(3\varkappa A + 2\mathfrak{A})}{2n^4} \operatorname{cof} 4\eta$$
$$- \frac{(2\varkappa a + \mathfrak{a})}{nn} \nu \operatorname{cof} \eta - \frac{(2\varkappa b + \mathfrak{b})}{nn} \nu \operatorname{cof} 3\eta$$

atque ob $\varkappa - \frac{1}{n} = \alpha$ erit minimis terminis omiſſis, quia hi in operatione multo magis diminuerentur:

$$\frac{d\eta}{dr} = \alpha - \frac{(2\varkappa A + \mathfrak{A})}{nn} \operatorname{cof} 2\eta - \frac{(2\varkappa B + \mathfrak{B})}{nn} \operatorname{cof} 4\eta$$
$$- \frac{(2\varkappa a + \mathfrak{a})}{nn} \nu \operatorname{cof} \eta - \frac{(2\varkappa b + \mathfrak{b})}{nn} \nu \operatorname{cof} 3\eta$$

His poſitis erit:

$$\frac{d.\operatorname{cſ} 2\eta}{dr} = -\operatorname{ſn} 2\eta . \frac{2d\eta}{dr} = -2\alpha \operatorname{ſn} 2\eta - \frac{(2\varkappa B + \mathfrak{B})}{nn} \operatorname{ſn} 2\eta + \frac{(2\varkappa A + \mathfrak{A})}{nn} \operatorname{ſin} 4\eta$$
$$+ \frac{(2\varkappa a + \mathfrak{a})}{nn} \nu \operatorname{ſin} \eta + \frac{(2\varkappa a + \mathfrak{a})}{nn} \nu \operatorname{ſin} 3\eta$$
$$- \frac{(2\varkappa b + \mathfrak{b})}{nn} \nu \operatorname{ſin} \eta$$

d.

$$\frac{d.\cos 4\eta}{dr} = -\sin 4\eta.\ \frac{4\,d\eta}{dr} = -4\alpha\sin 4\eta + \frac{2(2\kappa A + \mathfrak{A})}{nn}\sin 2\eta$$

$$\frac{d.\ \cos\eta}{dr} = -\sin\eta.\ \frac{d\eta}{dr} = -\alpha\sin\eta$$

$$\frac{d.\cos 3\eta}{dr} = -\sin 3\eta.\ \frac{3d\eta}{dr} = -3\alpha\sin 3\eta$$

§. 62. Quod ſi iam ſecundum has formulas quantitas integralis $\int R\,dr$ differentietur, obtinebitur:

$$R = \left(-2\alpha\mathfrak{A} - \frac{\mathfrak{A}(2\kappa B + \mathfrak{B})}{nn} + \frac{2\mathfrak{B}(2\kappa A + \mathfrak{A})}{nn}\right)\sin 2\eta$$
$$+ \left(\frac{\mathfrak{A}(2\kappa A + \mathfrak{A})}{nn} - 4\alpha\mathfrak{B}\right)\sin 4\eta$$
$$+ \left(\frac{\mathfrak{A}(2\kappa a + \mathfrak{a})}{nn} + \frac{\mathfrak{A}(2\kappa b + \mathfrak{b})}{nn} - \alpha\mathfrak{a}\right)\nu\sin\eta$$
$$+ \left(\frac{\mathfrak{A}(2\kappa a + \mathfrak{a})}{nn} - 3\alpha\mathfrak{b}\right)\nu\sin 3\eta$$

Cum iam ſit per hypotheſin

$$R = \tfrac{3}{2}\sin 2\eta \quad + \frac{3A}{2nn}\sin 4\eta \quad + \tfrac{3}{8}\nu\sin\eta \quad + \tfrac{15}{8}\nu\sin 3\eta$$
$$-\frac{3B}{2nn} \qquad\qquad\qquad + \frac{3a}{2nn}\nu \qquad + \frac{3a}{2nn}\nu$$
$$-\frac{3b}{2nn}\nu$$

prodibit terminis homogeneis comparandis:

$$2\alpha\mathfrak{A} = -\tfrac{3}{2} - \frac{\mathfrak{A}(2\kappa B + \mathfrak{B}) + 2\mathfrak{B}(2\kappa A + \mathfrak{A})}{nn} + \frac{3B}{2nn}$$

$$4\alpha\mathfrak{B} = -\frac{3A}{2nn} + \frac{\mathfrak{A}(2\kappa A + \mathfrak{A})}{nn}$$

$$\alpha\mathfrak{a} = -\tfrac{3}{8} - \frac{3(a-b)}{2nn} + \frac{\mathfrak{A}(2\kappa a + \mathfrak{a}) - \mathfrak{A}(2\kappa b + \mathfrak{b})}{nn}$$

$$3\alpha\mathfrak{b} = -\tfrac{15}{8} - \frac{3a}{2nn} + \frac{\mathfrak{A}(2\kappa a + \mathfrak{a})}{nn}$$

 §. 63.

§. 63. Aequatio autem noſtra differentio-differentialis, ſi pro $\int R\, dr$ et v valores aſſumti ſubſtituantur, ſequentem induet formam:

$$\frac{ddv}{dr^2} = (\tfrac{1}{2}\delta - \gamma) - 2\varkappa\mathfrak{A}\cos 2\eta - 2\varkappa\mathfrak{B}\cos 4\eta - 2\varkappa\mathfrak{a}v\cos\eta - 2\varkappa\mathfrak{b}v\cos 3\eta$$

$$\begin{array}{lllll}
+\frac{\mathfrak{A}\mathfrak{A}}{2nn} & & +\frac{\mathfrak{A}\mathfrak{A}}{2nn} & & \\
 & -A & -B & -av & -bv \\
+\frac{3AA}{2nn} & & +\frac{3AA}{2nn} & & \\
 & +\frac{1}{4} & & & \\
 & +\frac{(4\gamma-3\delta)}{2nn}A & +\frac{(4\gamma-3\delta)}{2nn}B & +\frac{3b}{4nn}v & \\
+\frac{3A}{4nn} & +\frac{3B}{4nn} & +\frac{3A}{4nn} & +\frac{3a}{4nn}v & +\frac{3a}{4nn}v \\
+\frac{3A\mathfrak{A}}{nn} & & +\frac{3A\mathfrak{A}}{nn} & +\frac{5}{8}v & +\frac{15}{8}v
\end{array}$$

vbi quidem perſpicuum eſt, quinam termini reſpectu reliquorum tam ſint parui, vt ſine errore deleri queant.

§. 64. Quaeramus ergo primum differentiale $\frac{dv}{dr}$ ac reperietur:

$$\frac{dv}{dr} = \begin{array}{l}
\left(-2A\alpha - \frac{A(2\varkappa B + \mathfrak{B})}{nn} + \frac{2B(2\varkappa A + \mathfrak{A})}{nn}\right)\sin 2\eta \\
\left(-4\alpha B + \frac{A(2\varkappa A \mp \mathfrak{A})}{nn}\right)\sin 4\eta \\
\left(-\alpha a + \frac{A(2\varkappa a + \mathfrak{a})}{nn} - \frac{A(2\varkappa b + \mathfrak{b})}{nn}\right)v\sin\eta \\
\left(-3\alpha b + \frac{A(2\varkappa a + a)}{nn}\right)v\sin 3\eta
\end{array}$$

pona-

ponatur autem breuitatis ergo:

$$\frac{dv}{dr} = -\mathrm{A}'\,\text{ſin}\,2\eta - \mathrm{B}'\,\text{ſin}\,4\eta - a'\nu\,\text{ſin}\,\eta - b'\nu\,\text{ſin}\,3\eta$$

vt ſit:

$$\mathrm{A}' = 2\alpha\mathrm{A} + \frac{\mathrm{A}(2\kappa\mathrm{B}+\mathfrak{B})}{nn} - \frac{2\mathrm{B}(2\kappa\mathrm{A}+\mathfrak{A})}{nn}$$

$$\mathrm{B}' = 4\alpha\mathrm{B} - \frac{\mathrm{A}(2\kappa\mathrm{A}+\mathfrak{A})}{nn}$$

$$a' = \alpha a - \frac{\mathrm{A}(2\kappa a+\mathfrak{a})}{nn} + \frac{\mathrm{A}(\kappa b+\mathfrak{b})}{nn}$$

$$b' = 3\alpha b - \frac{\mathrm{A}(2\kappa a+\mathfrak{a})}{nn}$$

§. 65. Hinc cum ſit:

$$\frac{d.\text{ſn}\,2\eta}{dr} = \text{cſ}\,2\eta.\frac{2d\eta}{dr} = 2\alpha\,\text{cſ}\,2\eta - \frac{(2\kappa\mathrm{B}+\mathfrak{B})}{nn}\text{cſ}\,2\eta - \frac{(2\kappa\mathrm{A}+\mathfrak{A})}{nn}\text{cſ}\,4\eta - \frac{(2\kappa\mathrm{A}+\mathfrak{A})}{nn} - \frac{(2\kappa a+\mathfrak{a})}{nn}\nu\,\text{cſ}\,\eta - \frac{(2\kappa b+\mathfrak{b})}{nn}\nu\,\text{cſ}\,\eta - \frac{(2\kappa a+\mathfrak{a})}{nn}\nu\,\text{coſ}\,3\eta$$

$$\frac{d.\text{ſin}\,4\eta}{dr} = \text{coſ}\,4\eta.\frac{4d\eta}{dr} = 4\alpha\,\text{cſ}\,4\eta - \frac{2(2\kappa\mathrm{A}+\mathfrak{A})}{nn}\text{cſ}\,2\eta - \frac{2(2\kappa\mathrm{B}+\mathfrak{B})}{nn}$$

$$\frac{d.\text{ſin}\,\eta}{dr} = \text{coſ}\,\eta.\frac{d\eta}{dr} = \alpha\,\text{coſ}\,\eta;\ \text{et}\ \frac{d.\text{ſin}\,3\eta}{dr} = \text{coſ}\,3\eta.\frac{3d\eta}{dr} = 3\alpha\,\text{coſ}\,3\eta$$

prodibit

$$\frac{ddv}{dr^2} = \begin{array}{l} +\dfrac{\mathrm{A}'(2\kappa\mathrm{A}+\mathfrak{A})}{nn} + \dfrac{2\mathrm{B}'(2\kappa\mathrm{B}+\mathfrak{B})}{nn} \\ \left(-2\alpha\mathrm{A}' + \dfrac{\mathrm{A}'(2\kappa\mathrm{B}+\mathfrak{B})}{nn} + \dfrac{2\mathrm{B}'(2\kappa\mathrm{A}+\mathfrak{A})}{nn}\right)\text{coſ}\,2\eta \\ \left(-4\alpha\mathrm{B}' + \dfrac{\mathrm{A}'(2\kappa\mathrm{A}+\mathfrak{A})}{nn}\right)\text{coſ}\,4\eta \\ \left(-\alpha a' + \dfrac{\mathrm{A}'(2\kappa a+\mathfrak{a})}{nn}\right)\nu\,\text{coſ}\,\eta \\ \left(-3\alpha b' + \dfrac{\mathrm{A}'(2\kappa a+\mathfrak{a})}{nn}\right)\nu\,\text{coſ}\,3\eta \end{array}$$

ſeu

ſeu ſubſtitutis ſuperioribus valoribus:

$$\frac{ddv}{dr^2} = \begin{cases} +\frac{2A\alpha(2\varkappa A+\mathfrak{A})}{nn} + \frac{8\alpha B(2\varkappa B+\mathfrak{B})}{nn} \\ \left(-4\alpha\alpha A + \frac{12\alpha B(2\varkappa A+\mathfrak{A})}{nn}\right)\cos 2\eta \\ \left(-16\alpha\alpha B + \frac{8\alpha A(2\varkappa A+\mathfrak{A})}{nn}\right)\cos 4\eta \\ \left(-\alpha\alpha a + \frac{3\alpha A(2\varkappa a+\mathfrak{a})}{nn} - \frac{\alpha A(2\varkappa b+\mathfrak{b})}{nn}\right) v\cos\eta \\ \left(-9\alpha\alpha b + \frac{5\alpha A(2\varkappa a+\mathfrak{a})}{nn}\right) v\cos 3\eta \end{cases}$$

§ 66. Hi iam termini ſingulatim illis, qui §. 63. ſunt exhibiti, aequales ſtatuantur, atque ſequentes prodibunt determinationes,

$$\tfrac{1}{2}\delta - \gamma + \frac{3AA+6A\mathfrak{A}+\mathfrak{A}\mathfrak{A}}{2nn} + \frac{3A}{4nn} = \frac{2\alpha A(2\varkappa A+\mathfrak{A})}{nn} + \frac{8\alpha B(2\varkappa B+\mathfrak{B})}{nn}$$

$$-A + \tfrac{3}{2} - 2\varkappa\mathfrak{A} + \frac{(4\gamma-3\delta)}{2nn}A + \frac{3B}{4nn} = -4\alpha\alpha A + \frac{12\alpha B(2\varkappa A+\mathfrak{A})}{nn}$$

$$-B - 2\varkappa\mathfrak{B} + \frac{3AA+6A\mathfrak{A}+\mathfrak{A}\mathfrak{A}}{2nn} + \frac{3A}{4nn} + \frac{(4\gamma-3\delta)}{2nn}B = -16\alpha\alpha B + \frac{8\alpha A(2\varkappa A+\mathfrak{A})}{nn}$$

$$-a + \tfrac{9}{8} - 2\varkappa\mathfrak{a} + \frac{3a+3b}{4nn} = -\alpha\alpha a + \frac{3\alpha A(2\varkappa a+\mathfrak{a})}{nn} - \frac{\alpha A(2\varkappa b+\mathfrak{b})}{nn}$$

$$-b + \tfrac{15}{8} - 2\varkappa\mathfrak{b} + \frac{3a}{4nn} = -9\alpha\alpha b + \frac{5\alpha A(2\varkappa a+\mathfrak{a})}{nn}$$

vnde primum quaeri debent valores vero proximi, qui ſunt:

$$\mathfrak{A} = -\frac{3}{4\alpha};\quad \mathfrak{a} = -\frac{3}{8\alpha};\quad \mathfrak{b} = -\frac{5}{8\alpha};$$

$$A = -\frac{\frac{3}{2}+2\varkappa\mathfrak{A}}{4\alpha\alpha-1};\quad a = \frac{\frac{9}{8}-2\varkappa\mathfrak{a}}{1-\alpha\alpha};\quad b = -\frac{\frac{15}{8}+2\varkappa\mathfrak{b}}{9\alpha\alpha-1}$$

§. 67.

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§. 67. Calculus ergo ſequenti modo inſtituatur:

$\alpha = 0,933089$; $l\alpha = 9,969923$

$\varkappa = 1,008527$; $l\varkappa = 0,003687$

$l2\varkappa = 0,304717$

Iam eſt $l8\alpha = 0,873013$

ſubtr. a $\begin{cases} l3 = 0,477121 \\ l5 = 0,698970 \end{cases}$

$\mathfrak{a} = -0,402$; erit $l-\mathfrak{a} = 9,604108$

$\mathfrak{b} = -0,635$; $l-\mathfrak{b} = 9,825957$

$\mathfrak{A} = -0,804$; $l-\mathfrak{A} = 9,905138$

atque hinc conficietur:

$$A = -\frac{3,121}{4\alpha\alpha-1};\quad a = +\frac{1,936}{1-\alpha\alpha};\quad b = -\frac{3,156}{9\alpha\alpha-1}$$

quarum ergo litterarum valores proximi ſunt

$$A = -1,2583;\quad a = +14,968;\quad b = -0,4613$$

§. 68. Quaeramus hinc primum valores litterarum $\mathfrak{B}$ et B.

$\mathfrak{A} = -0,804$; $l-\mathfrak{A} = 9,905138$

$2\varkappa A + \mathfrak{A} = -3,341$; vnde colligitur

$4\alpha\mathfrak{B} = +\frac{4,573}{nn}$ $\quad l4,573 = 0,660201$

hinc erit $\quad lnn = 2,244816$

$8,415385$

$l4\alpha = 0,571983$

$\mathfrak{B} = +0,00697$ $\quad l\mathfrak{B} = 7,843402$

Deinde eſt

$$(16\alpha\alpha-1)B = 2\varkappa\mathfrak{B} - \frac{3A}{4nn} - \frac{3AA-6A\mathfrak{A}-\mathfrak{A}\mathfrak{A}}{2nn} + \frac{8\alpha A(2\varkappa A+\mathfrak{A})}{nn}$$

ſeu $B = +\frac{0,16819}{16\alpha\alpha-1}$; vnde reperitur

$$B = +0,012792 \quad \text{et} \quad lB = 8,106947$$

§. 69. His iam valoribus proxime veris inuentis quaerantur exacti, ac primo quidem

$$2\alpha\mathfrak{A} = -\tfrac{3}{2} + \frac{\tfrac{3}{2}B - \mathfrak{A}(2\varkappa B + \mathfrak{B}) + 2\mathfrak{B}(2\varkappa A + \mathfrak{A})}{nn}$$

vnde reperitur vt ante:

$$\mathfrak{A} = -0,80378 \quad . \; . \; . \quad l{-}\mathfrak{A} = 9,905138$$

$$\alpha\mathfrak{a} = -\tfrac{3}{8} - \frac{\tfrac{3}{4}(a-b) + \mathfrak{A}(2\varkappa a + \mathfrak{a}) - \mathfrak{A}(2\varkappa b + \mathfrak{b})}{nn} = -0,65361$$

$$\mathfrak{a} = -0,70048 \quad . \; . \; . \quad l{-}\mathfrak{a} = 9,845396$$

$$3\alpha\mathfrak{b} = -\tfrac{15}{8} - \frac{\tfrac{3}{4}a + \mathfrak{A}(2\varkappa a + \mathfrak{a})}{nn} = -2,13900$$

$$\mathfrak{b} = -0,76413 \quad . \; . \; . \quad l{-}\mathfrak{b} = 9,883167$$

$$(4\alpha\alpha - 1)A = -\tfrac{3}{2} + 2\varkappa\mathfrak{A} - \frac{\tfrac{3}{4}B + 12\alpha B(2\varkappa A + \mathfrak{A})}{nn} = -3,12379$$

$$A = -1,25826 \quad . \; . \; . \quad l{-}A = 0,099771$$

$$(1-\alpha\alpha)a = \tfrac{9}{8} - 2\varkappa\mathfrak{a} + \frac{\tfrac{3}{4}(a+b) - 3\alpha A(2\varkappa a + \mathfrak{a}) + \alpha A(2\varkappa b + \mathfrak{b})}{nn} \text{ vel}$$

$$\left(1 - \alpha\alpha - \frac{3}{4nn} + \frac{(6\alpha\varkappa A)}{nn}\right) a = \tfrac{9}{8} - 2\varkappa\mathfrak{a}$$
$$+ \frac{\tfrac{3}{4}b - 3\alpha A\mathfrak{a} + \alpha A(2\varkappa b + \mathfrak{b})}{nn} = -2,53335$$

hinc $a = +30,989$ et $l\,a = 1,491207$

Vnde patet valorem ipſius $\mathfrak{a}$ ante inuentum non ſatis eſſe exactum, exactior ergo prodibit ex hac formula

$$\left(\alpha - \frac{\mathfrak{A}}{nn}\right)\mathfrak{a} = -\tfrac{3}{8} - \frac{\tfrac{3}{4}(a-b) + \mathfrak{A}(2\varkappa a - 2\varkappa b - \mathfrak{b})}{nn} = -0,93709$$

hinc $\mathfrak{a} = -0,99939$ et $l{-}\mathfrak{a} = 9,999735$

vnde etiam exactius valor ipſius a reperitur, ex quo denuo

denuo valor ipſius $\mathfrak{a}$ corrigetur, ſicque tandem ſatis exacte obtinebitur

$\mathfrak{a} = - 1,2537$ . . . $l{-}\mathfrak{a} = 0,098200$
$a = + 44,48$ . . . $l\,a = 1,648165$
$\mathfrak{b} = - 0,95003$ . . . $l{-}\mathfrak{b} = 9,977736$

§. 70. Hinc iam accuratius quaeramus valorem ipſius $\mathfrak{b}$

$$(9\alpha\alpha - 1)\,b = -\tfrac{\gamma}{8} + 2\kappa\mathfrak{b} - \frac{\tfrac{3}{4}a + 5\alpha A(2\kappa a + \mathfrak{a})}{nn}$$

$b = -1,0146$ . . . $l{-}b = 0,006314$

vnde ſi denuo praecedentes valores corrigantur, fiet

$\mathfrak{a} = - 1,2630$ . . . $l{-}\mathfrak{a} = 0,101403$
$\mathfrak{b} = - 0,9500$ . . . $l{-}\mathfrak{b} = 9,977736$
$a = + 44,525$ . . . $l\,a = 1,648604$
$b = - 1,015$ . . . $l{-}b = 0,006400$
$\mathfrak{A} = - 0,80378$ . . . $l{-}\mathfrak{A} = 9,905138$
$\mathfrak{B} = + 0,00697$ . . . $l\mathfrak{B} = 7,843402$
$A = - 1,25826$ . . . $l{-}A = 0,099771$
$B = + 0,01279$ . . . $l\,B = 8,106947$

His autem valoribus inuentis colligitur fore

$\tfrac{1}{2}\delta - \gamma = + 0,01742$

Hic autem valor partem inſuper accipit cum ab excentricitate vtriusque orbitae, tum ab inclinatione oriundam, quam deinceps determinabimus.

§. 71. Ex his ergo valoribus habebimus:

$\int R dr = - 0,80378 \cos 2\eta + 0,00697 \cos 4\eta$
$\qquad 9,905138 \qquad\qquad 7,843402$
$\quad - 1,2630\, v \cos \eta - 0,9500\, v \cos 3\eta$
$\qquad 0,101403 \qquad\qquad 9,977736$

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$$
\begin{array}{llll}
v = & -\,1{,}25826 \cos 2\eta & + & 0{,}01279 \cos 4\eta \\
& \quad 0{,}099771 & & 8{,}106947 \\
& +\,44{,}525\, \nu \cos \eta & - & 1{,}015\, \nu \cos 3\eta \\
& \quad 1{,}648604 & & 0{,}006400
\end{array}
$$

hincque porro

$$
\begin{array}{llll}
\frac{d\Phi}{dr} = \varkappa & +\ 0{,}019015 \cos 2\eta & - & 0{,}0000762 \cos 4\eta \\
& \quad 8{,}279096 & & 5{,}881955 \\
+\,0{,}0001103 & & & \\
& -\ 0{,}50381\, \nu \cos \eta & + & 0{,}017068\, \nu \cos 3\eta \\
& \quad 9{,}702270 & & 8{,}232184
\end{array}
$$

at eſt $\frac{d\eta}{dr} = \frac{d\Phi}{dr} - \frac{1}{n}$, poſuimusque $\varkappa - \frac{1}{n} = \alpha$, exiſtente $\varkappa = V\ (1 + \frac{3+4\mu+\delta}{2nn})$

§. 72. Ponatur breuitatis gratia

$$\frac{d\Phi}{dr} = \mathfrak{O} + \mathfrak{P} \cos 2\eta - \mathfrak{Q} \cos 4\eta - \mathfrak{R}\nu \cos \eta + \mathfrak{S}\nu \cos 3\eta$$

erit $\frac{d\eta}{dr} = \alpha + \mathfrak{P} \cos 2\eta - \mathfrak{Q} \cos 4\eta - \mathfrak{R}\nu \cos \eta + \mathfrak{S}\nu \cos 3\eta$

ſitque ad integrandum:

$$\Phi = \mathfrak{o}\, r + \mathfrak{p} \sin 2\eta - \mathfrak{q} \sin 4\eta - \mathfrak{r}\nu \sin \eta + \mathfrak{s}\nu \sin 3\eta$$

vnde per differentiationem elicitur:

$$
\begin{array}{l}
\frac{d\Phi}{dr} = \mathfrak{o} + 2\alpha\mathfrak{p} \cos 2\eta - 4\alpha\mathfrak{q} \cos 4\eta - \alpha\mathfrak{r}\nu \cos \eta + 3\alpha\mathfrak{s}\nu \cos 3\eta \\
\quad \mathfrak{P}\mathfrak{p} - \mathfrak{Q}\mathfrak{p} \cos 2\eta + \mathfrak{P}\mathfrak{p} \cos 4\eta - \mathfrak{R}\mathfrak{p}\nu \cos \eta - \mathfrak{R}\mathfrak{p}\nu \cos 3\eta \\
+\,2\mathfrak{Q}\mathfrak{q} - 2\mathfrak{P}\mathfrak{q} \cos 2\eta \qquad\qquad\qquad\qquad\qquad + 2\mathfrak{R}\mathfrak{q}\nu \cos 3\eta
\end{array}
$$

hinc ergo fit:

$$\mathfrak{o} = \mathfrak{O} - \mathfrak{P}\mathfrak{p} - 2\mathfrak{Q}\mathfrak{q} = \varkappa + 0{,}0001103 - \mathfrak{P}\mathfrak{p} - 2\mathfrak{Q}\mathfrak{q}$$

$$(2\alpha - \mathfrak{Q})\mathfrak{p} = \mathfrak{P} + 2\mathfrak{P}\mathfrak{q};\quad 4\alpha\mathfrak{q} = \mathfrak{Q} + \mathfrak{P}\mathfrak{p};$$

$$\alpha\mathfrak{r} = \mathfrak{R} - \mathfrak{R}\mathfrak{p};\quad 3\alpha\mathfrak{s} = \mathfrak{S} + \mathfrak{R}(\mathfrak{p} - 2\mathfrak{q})$$

Ergo

Ergo $\mathfrak{p} = 0{,}010191$ . . . $l\mathfrak{p} = 8{,}008208$
$\mathfrak{q} = 0{,}000072$ . . . $l\mathfrak{q} = 5{,}859381$
$\mathfrak{r} = 0{,}53453$ . . . $l\mathfrak{r} = 9{,}727977$
$\mathfrak{s} = 0{,}00790$ . . . $l\mathfrak{s} = 7{,}897466$
$\mathfrak{v} = \varkappa - 0{,}000080$

§. 73. Longitudo igitur lunae ϕ quatenus pendet a ſola diſtantia lunae a ſole erit

$$\phi = (\varkappa - 0{,}000080)r + 0{,}010191 \operatorname{ſin} 2\eta - 0{,}53453\, \nu \operatorname{ſin} \eta$$
$$- 0{,}000072 \operatorname{ſin} 4\eta + 0{,}00790\, \nu \operatorname{ſin} 3\eta$$

Simili modo cum diſtantia lunae a terra poſita ſit $= \frac{a(1-kk)u}{1-k\operatorname{coſ} r}$, ob $u = 1 + \frac{v}{nn}$, quatenus valor ipſius u a ſola phaſi lunae pendet, erit

$$u = 1 - 0{,}00716 \operatorname{coſ} 2\eta + 0{,}00287\, \nu \operatorname{coſ} \eta$$
$$+ 0{,}00007 \operatorname{coſ} 4\eta - 0{,}00009\, \nu \operatorname{coſ} 3\eta$$

Verum tamen hic valor litterae α ac praecipue ipſius a non admodum certus videtur, cum a terminis neglectis licet minimis inſignem mutationem perpeti queat. Hic enim pro α non ſolum $\varkappa - \frac{1}{n}$ ſed $\varkappa - \frac{1}{n} + 0{,}0001103$ accipi debuiſſet; quare cum valorem ipſius α propius cognoſcimus, hanc determinationem repeti conueniet.

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CAPUT V.

INUESTIGATIO INAEQUALITATUM LUNAE AB EIUS EXCENTRICITATE SIMPLICI SOLUM PEDENTIUM.

§. 74.

Quemadmodum in praecedenti capite inaequalitas abſoluta ſeu variatio duabus partibus conſtans eſt inuenta, quarum poſterior a littera *v* ſeu a parallaxi ſolis pendebat, ac maiorem curam requirebat; ita etiam inaequalitates, quas hoc capite ſcrutamur, partes continent ab eadem parallaxi ſolis pendentes; quarum indagatio quoque accuratiorem cognitionem quorundam elementorum exigit. Hancobrem et praecedentis capitis et huius partes, quae litteram *v* inuoluunt deinceps, cum reliquas inaequalitates, a parallaxi ſolis non pendentes determinauerimus, ſeorſim inueſtigabimus, atque titulo inaequalitatum parallacticarum complectemur.

§. 75. In hoc ergo capite ac ſequentibus, donec ad parallaxin ſolis perueniamus, terminos formularum noſtrarum per *v* multiplicatos tantisper remouebimus; et quoniam hoc loco tantum propoſitum eſt in motus lunae inaequalitates a ſola excentricitate orbitae lunaris ortas inquirere, eos terminos qui vel excentricitatem ſolis e vel inclinationem ϱ continent, praetermittemus. Cum autem in formulis noſtris duplicis generis termini relinquantur, quorum alteri per k, alteri per kk ſunt affecti, inaequalitates ab excentricitate lunae k pendentes

in duas partes diſtribui conueniet; quarum altera excentricitatem tantum ſimplicem k implicet, cui hoc caput deſtinatur, altera vero excentricitatis huius quadrato kk afficiatur, de quo in ſequenti capite agemus.

§. 76. Verum tam in huius generis inaequalitates, quam in ſequentes, omnes inaequalitates abſolutae in praecedenti capite erutae praecipue ingrediuntur; ex quo eas quoque in calculum introduci oportebit. Retinendae ergo erunt in calculo litterae $\mathfrak{A}$, $\mathfrak{B}$ et A, B, quarum valores cum iam conſtent, calculus vehementer contrahetur: imprimis autem quia valores litterarum $\mathfrak{B}$ et B per ſe ſunt admodum parui, quatenus illi in valores ſequentium terminorum influunt, effectum pro nihilo habendum praeſtabunt. Inueſtigationem ergo noſtram ita incipiemus, vt pro $\int R\,dr$ et v valores fictos aſſumamus, et quoniam $\int R\,dr$ nullum terminum conſtantem, v vero neque conſtantem neque terminum huius formae $\alpha \cos r$ continere debet, ponamus:

$$\begin{aligned}\int R\,dr = {} & \mathfrak{A}\cos 2\eta + \mathfrak{B}\cos 4\eta + \mathfrak{C}k\cos r \\ & + \mathfrak{D}k\cos(2\eta - r) + \mathfrak{F}k\cos(4\eta - r) \\ & + \mathfrak{E}k\cos(2\eta + r) + \mathfrak{G}k\cos(4\eta + r)\end{aligned}$$

$$\begin{aligned}v = {} & A\cos 2\eta + B\cos 4\eta \\ & + Dk\cos(2\eta + r) + Fk\cos(4\eta - r) \\ & + Ek\cos(2\eta + r) + Gk\cos(4\eta + r)\end{aligned}$$

vbi quidem facile colligere licet, coefficientes $\mathfrak{F}$, $\mathfrak{G}$, F et G fore minimos.

§. 77. Ex his autem valoribus aſſumtis obtinebimus ex (§. 52.) ſequentes expreſſiones.

$$\frac{d\varphi}{dr} = \begin{aligned} &\varkappa + \frac{A(3\varkappa A+2\mathfrak{A})}{2n^4} - \frac{(2\varkappa A+\mathfrak{A})}{nn}\cos 2\eta \\ &\left(-\frac{(2\varkappa B+\mathfrak{B})}{nn} + \frac{A(3\varkappa A+2\mathfrak{A})}{2n^4}\right)\cos 4\eta \\ &\left(-\frac{\mathfrak{C}}{nn} + \frac{3\varkappa AD}{n^4} + \frac{A\mathfrak{D}+\mathfrak{A}D}{n^4}\right)k\cos r \\ &-\frac{(2\varkappa D+\mathfrak{D})}{nn}k\cos(2\eta-r) - \frac{(2\varkappa E+\mathfrak{E})}{nn}k\cos(2\eta+r) \\ &-\frac{(2\varkappa F+\mathfrak{F})}{nn}k\cos(4\eta-r) - \frac{(2\varkappa G+\mathfrak{G})}{nn}k\cos(4\eta+r) \\ &+\frac{(3\varkappa AD+A\mathfrak{D}+\mathfrak{A}D)}{n^4}k\cos(4\eta-r) \end{aligned}$$

Patebit enim ex valoribus qui inuenientur, litteras D et $\mathfrak{D}$ tantum prae reliquis fore notabiles, vnde terminos ex combinatione reliquarum litterarum oriundos tuto omittere licet. Pro valore autem ipſius $\frac{d\eta}{dr}$ etiam hi termini ex combinatione orti omitti poterunt. Poſito ergo $\varkappa + \frac{A(3\varkappa A+2\mathfrak{A})}{2n^4} - \frac{1}{n}$ ſeu $\varkappa + 0{,}0001103 - \frac{1}{n} = \alpha$ erit:

$$\frac{d\eta}{dr} = \alpha - \frac{(2\varkappa A+\mathfrak{A})}{nn}\cos 2\eta - \frac{2k}{n}\cos r$$
$$-\frac{(2\varkappa D+\mathfrak{D})}{nn}k\cos(2\eta-r) - \frac{(2\varkappa E+\mathfrak{E})}{nn}k\cos(2\eta+r)$$

Cum enim haec formula differentiationibus inſtituendis inſeruiat, reliqui termini poſt primum cum aliis angulis combinantur, ſicque tanto minores terminos producunt,

cunt, qui ex calculo ſine errore expungi poterunt: atque ob hanc cauſam in expreſſione valoris $\frac{d\eta}{dr}$, ſtatim terminos prae reliquis admodum paruos praetermittere viſum eſt.

§. 78. Valorem autem ipſius R atque $\frac{ddv}{dr^2}$ accuratiſſime exhiberi oportet, propterea quod his expreſſionibus totus calculus praecipue innititur, dum valor $\frac{d\eta}{dr}$ formulam tantum ſubſidiariam ſuppeditat. Erit ergo

$$R = \tfrac{3}{4}\sin 2\eta + \frac{3A}{2nn}\sin 4\eta + 3k\sin(2\eta - r) + 3k\sin(2\eta + r)$$
$$+ \frac{3A}{nn}k\sin(4\eta - r) + \frac{3A}{nn}k\sin(4\eta + r)$$
$$+ \frac{3D}{2nn}k\sin r \qquad - \frac{3E}{2nn}k\sin r$$
$$+ \frac{3D}{2nn}k\sin(4\eta - r) + \frac{3E}{2nn}k\sin(4\eta + r)$$
$$- \frac{3A}{nn}k\sin r \qquad + \frac{3A}{nn}k\sin r$$

vbi quidem terminos ab k non pendentes omittere poſſumus, quia illorum iam habuimus rationem, ita vt ſit

$$R = \ldots\ldots + \frac{3(D-E)}{2nn}k\sin r + 3k\sin(2\eta - r) + 3k\sin(2\eta + r)$$
$$+ \frac{3D}{2nn}k\sin(4\eta - r) + \frac{3E}{2nn}k\sin(4\eta + r)$$
$$+ \frac{3A}{nn}k\sin(4\eta - r) + \frac{3A}{nn}k\sin(4\eta + r)$$

§. 79. Simili modo terminis a k non pendentibus omittendis habebitur:

$$\frac{ddv}{dr^2} = -\gamma k \,\text{cſ}\, r + 3k \,\text{cſ}\,(2\eta - r) + 3k \,\text{cſ}\,(2\eta + r) - 2\varkappa \mathfrak{F} k \,\text{cſ}\,(4\eta - r) - 2\varkappa \mathfrak{G} k \,\text{cſ}\,(4\eta + r)$$

$$\begin{array}{lllll}
-2\varkappa\mathfrak{C} & -2\varkappa\mathfrak{D} & -2\varkappa\mathfrak{E} & +\frac{\mathfrak{A}\mathfrak{D}}{2nn} & +\frac{\mathfrak{A}\mathfrak{E}}{2nn} \\
+\frac{\mathfrak{A}\mathfrak{D}}{2nn} & +\frac{\mathfrak{A}\mathfrak{C}}{2nn} & +\frac{\mathfrak{A}\mathfrak{C}}{2nn} & & \\
+\frac{\mathfrak{A}\mathfrak{E}}{2nn} & -D & -E & -F & -G \\
+\frac{3AD}{nn} & +\frac{3}{2}A & +\frac{3}{2}A & +\frac{3}{2}B & +\frac{3}{2}B \\
+\frac{3AE}{nn} & +\frac{(2\gamma-\frac{3}{2}\delta)}{nn}D & +\frac{(2\gamma-\frac{3}{2}\delta)}{nn}E & +\frac{3AD}{nn} & +\frac{3AE}{nn} \\
-\frac{3AA}{2nn} & +\frac{(3+3\mu+\gamma)}{nn}A & +\frac{(3+3\mu+\gamma)}{nn}A & -\frac{3AA}{4nn} & -\frac{3AA}{4nn} \\
+\frac{3A\mathfrak{D}}{nn} & & & +\frac{3A\mathfrak{D}}{nn} & +\frac{3A\mathfrak{E}}{nn} \\
+\frac{3A\mathfrak{E}}{nn} & & & +\frac{3\mathfrak{A}D}{nn} & +\frac{3\mathfrak{A}E}{nn} \\
+\frac{3\mathfrak{A}D}{nn} & & & & \\
+\frac{3\mathfrak{A}E}{nn} & & & &
\end{array}$$

Hic ſcilicet plures terminos, qui nullius futuri eſſent momenti, omiſimus, ne calculus nimium implicaretur: notandum autem eſt eſſe $\varkappa = \sqrt{\left(1 + \frac{3+4\mu+\delta}{2nn}\right)} = 1 + \frac{3+4\mu+\delta}{4nn}$ proxime; vnde $\mu = (\varkappa - 1)nn - \frac{3-\delta}{4}$ et $\frac{3+3\mu+\gamma}{4nn}$ $= 3(\varkappa - 1) + \frac{3-3\delta+4\gamma}{4nn}$.

§. 80.

§. 80. Quaeramus nunc quoque ex forma pro $\int R\,dr$ ficta valorem ipſius R, atque excluſis terminis ab k non pendentibus reperiemus:

$$R = \begin{cases} \left(-\frac{\mathfrak{A}(2\varkappa E+\mathfrak{E})}{nn}+\frac{\mathfrak{A}(2\varkappa D+\mathfrak{D})}{nn}-\mathfrak{C}-\frac{\mathfrak{D}(2\varkappa A+\mathfrak{A})}{nn}+\frac{\mathfrak{E}(2\varkappa A+\mathfrak{A})}{nn}\right)k\sin r \\ \left(+\frac{2\mathfrak{A}}{n}-(2\alpha-1)\mathfrak{D}\right)k\sin(2\eta-r) \\ \left(+\frac{2\mathfrak{A}}{n}-(2\alpha+1)\mathfrak{E}\right)k\sin(2\eta+r) \\ \left(+\frac{\mathfrak{A}(2\varkappa D+\mathfrak{D})}{nn}+\frac{4\mathfrak{B}}{n}+\frac{\mathfrak{D}(2\varkappa A+\mathfrak{A})}{nn}-(4\alpha-1)\mathfrak{F}\right)k\sin(4\eta-r) \\ \left(+\frac{\mathfrak{A}(2\varkappa E+\mathfrak{E})}{nn}+\frac{4\mathfrak{B}}{n}+\frac{\mathfrak{E}(2\varkappa A+\mathfrak{A})}{nn}-(4\alpha+1)\mathfrak{G}\right)k\sin(4\eta+r) \end{cases}$$

atque inſtituta comparatione inuenietur:

$$\mathfrak{C} = \frac{\mathfrak{A}(2\varkappa D+\mathfrak{D})-(\mathfrak{D}-\mathfrak{E})(2\varkappa A+\mathfrak{A})-\frac{3}{2}(D-E)-\mathfrak{A}(2\varkappa E+\mathfrak{E})}{nn}$$

$$(2\alpha-1)\,\mathfrak{D} = \frac{2\mathfrak{A}}{n}-3;\quad (2\alpha+1)\,\mathfrak{E} = \frac{2\mathfrak{A}}{n}-3;$$

$$(4\alpha-1)\,\mathfrak{F} = \frac{4\mathfrak{B}}{n}+\frac{\mathfrak{A}(2\varkappa D+\mathfrak{D})+\mathfrak{D}(2\varkappa A+\mathfrak{A})-\frac{3}{2}(2A+D)}{nn}$$

$$(4\alpha+1)\,\mathfrak{G} = \frac{4\mathfrak{B}}{n}+\frac{\mathfrak{A}(\varkappa E+\mathfrak{E})-\frac{3}{2}(2A+E)+\mathfrak{E}(2\varkappa A+\mathfrak{A})}{nn}$$

§. 81. Pro differentiali $\frac{dv}{dr}$ inueniendo, praeter terminos ſupra inventos habebimus:

$$\frac{dv}{dr} = -A'\sin 2\eta - B'\sin 4\eta$$

$$\left(+\frac{A(2\varkappa D+\mathfrak{D})}{nn}-\frac{A(2\varkappa E+\mathfrak{E})}{nn}-\frac{D(2\varkappa A+\mathfrak{A})}{nn}+\frac{E(2\varkappa A+\mathfrak{A})}{nn}\right)k\sin r$$

$$\left(+\frac{2A}{n}-(2\alpha-1)D\right)k\sin(2\eta-r)+\left(\frac{2A}{n}-(2\alpha+1)E\right)k\sin(2\eta+r)$$

$$(+\frac{A(2\kappa D+\mathfrak{D})}{n}+\frac{4B}{n}+\frac{D(2\kappa A+\mathfrak{A})}{nn}-(4\alpha-1)F)\,k\,\text{ſin}(4\eta-r)$$

$$(+\frac{A(2\kappa E+\mathfrak{E})}{nn}+\frac{4B}{n}+\frac{E(2\kappa A+\mathfrak{A})}{nn}-(4\alpha+1)G)\,k\,\text{ſin}(4\eta+r)$$

Ponatur autem breuitatis gratia:

$$\frac{dv}{dr}=-A'\,\text{ſin}\,2\eta-B'\,\text{ſin}\,4\eta-C'k\,\text{ſin}\,r-D'k\,\text{ſin}(2\eta-r)$$
$$-E'k\,\text{ſin}(2\eta+r)-F'k\,\text{ſin}(4\eta-r)-G'k\,\text{ſin}(4\eta+r)$$

vt ſit:

$$A'=2A\alpha+\frac{A(2\kappa B+\mathfrak{B})-2B(2\kappa A+\mathfrak{A})}{nn};\ B'=4\alpha B-\frac{A(2\kappa A+\mathfrak{A})}{nn}$$

$$C'=\frac{-A(2\kappa D+\mathfrak{D})+A(2\kappa E+\mathfrak{E})+(D-E)(2\kappa A+\mathfrak{A})}{nn}$$

$$\text{ſive }C'=-\frac{A(\mathfrak{D}-\mathfrak{E})+\mathfrak{A}(D-E)}{nn}$$

$$D'=(2\alpha-1)D-\frac{2A}{n};\quad E'=(2\alpha+1)E-\frac{2A}{n}$$

$$F'=(4\alpha-1)F-\frac{4B}{n}-\frac{A(2\kappa D+\mathfrak{D})-D(2\kappa A+\mathfrak{A})}{nn}$$

$$G'=(4\alpha+1)G-\frac{4B}{n}-\frac{A(2\kappa E+\mathfrak{E})-E(2\kappa A+\mathfrak{A})}{nn}$$

§. 82. Hinc denuo differentiando obtinebitur terminis tantum per k multiplicatis ſcribendis:

$$\frac{ddv}{dr^2}=(-C'+\frac{A'(2\kappa D+\mathfrak{D})}{nn}+\frac{A'(2\kappa E+\mathfrak{E})}{nn}+\frac{D'(2\kappa A+\mathfrak{A})}{nn}+\frac{E'(2\kappa A+\mathfrak{A})}{nn})\,k\,\text{coſ}\,r$$
$$(+\frac{2A'}{n}-(2\alpha-1)D')\,k\,\text{coſ}(2\eta-r)$$
$$(+\frac{2A'}{n}--(2\alpha+1)E')\,k\,\text{coſ}(2\eta+r)$$

+

$$\left(+\frac{4B'}{n}+\frac{A'(2\varkappa D+\mathfrak{D})}{nn}+\frac{D'(2\varkappa A+\mathfrak{A})}{nn}-(4\alpha-1)F'\right)k\cos(4\eta-r)$$

$$\left(+\frac{4B'}{n}+\frac{A'(2\varkappa E+\mathfrak{E})}{nn}+\frac{E'(2\varkappa A+\mathfrak{A})}{nn}-(4\alpha+1)G'\right)k\cos(4\eta+r)$$

vnde comparatione inſtituta orietur:

$$\gamma=-2\varkappa\mathfrak{C}-\frac{\frac{3}{2}AA+3A(D+E)+2A(\mathfrak{D}+2\mathfrak{E})+2\mathfrak{A}(2D+E)+\frac{1}{2}\mathfrak{A}(\mathfrak{D}+\mathfrak{E})}{nn}$$

$$-\frac{A'(2\varkappa D+\mathfrak{D})-A'(2\varkappa E+\mathfrak{E})-(D'+E')(2\varkappa A+\mathfrak{A})}{nn}$$

$$\left.\begin{matrix}(2\alpha-1)^2D-\dfrac{2(2\alpha-1)}{n}A-\dfrac{2A'}{n}+3-2\varkappa\mathfrak{D}-D\\ +\frac{3}{2}A+\dfrac{\frac{1}{2}\mathfrak{A}\mathfrak{C}+(2\gamma-\frac{3}{2}\delta)D+(3+3\mu\mp\gamma)A}{nn}\end{matrix}\right\}=0$$

$$\left.\begin{matrix}(2\alpha+1)^2E-\dfrac{2(2\alpha+1)}{n}A-\dfrac{2A'}{n}+3-2\varkappa\mathfrak{E}-E\\ +\frac{3}{2}A+\dfrac{\frac{1}{2}\mathfrak{A}\mathfrak{C}+(2\gamma-\frac{3}{2}\delta)E+(3+3\mu+\gamma)A}{nn}\end{matrix}\right\}=0$$

$$\left.\begin{matrix}(4\alpha-1)^2F-\dfrac{4(4\alpha-1)}{n}B-\dfrac{(4\alpha-1)A(2\varkappa D+\mathfrak{D})-(4\alpha-1)D(2\varkappa A+\mathfrak{A})}{nn}\\ -\dfrac{4B'}{n}-\dfrac{A'(2\varkappa D+\mathfrak{D})-D'(2\varkappa A+\mathfrak{A})}{nn}-2\varkappa\mathfrak{F}-F\\ +\frac{3}{2}B+\dfrac{\frac{1}{2}\mathfrak{A}\mathfrak{D}+3AD-\frac{3}{4}AA+3A\mathfrak{D}+3\mathfrak{A}D}{nn}\end{matrix}\right\}=0$$

$$\left.\begin{matrix}(4\alpha+1)^2G-\dfrac{4(4\alpha+1)}{n}B-\dfrac{(4\alpha+1)A(2\varkappa E+\mathfrak{E})-(4\alpha+1)E(2\varkappa A+\mathfrak{A})}{nn}\\ -\dfrac{4B'}{n}-\dfrac{A'(2\varkappa E+\mathfrak{E})-E'(2\varkappa A+\mathfrak{A})}{nn}-2\varkappa\mathfrak{G}-G\\ +\frac{3}{2}B+\dfrac{\frac{1}{2}\mathfrak{A}\mathfrak{E}+3AE-\frac{3}{4}AA+3A\mathfrak{E}+3\mathfrak{A}E}{nn}\end{matrix}\right\}=0$$

§. 83. Incipiamus a coefficientibus $\mathfrak{D}$, $\mathfrak{E}$, et D,E; et quia $\mathfrak{E}$ eſt quantitas admodum exigua, erit:

$$(2\alpha-1)\mathfrak{D} = -3 + \frac{2\mathfrak{A}}{n}; \quad (2\alpha+1)\mathfrak{E} = -3 + \frac{2\mathfrak{A}}{n}$$

$$\left((2\alpha-1)^2 - 1 + \frac{2\gamma-\frac{3}{2}\delta}{nn}\right) D = -3 - \tfrac{3}{2} A + 2\varkappa\mathfrak{D}$$
$$+ 2\frac{(2\alpha-1)}{n} A + \frac{2A'}{n} - \left(3\varkappa - 3 + \frac{3-3\delta+4\gamma}{4nn}\right) A$$

$$\left((2\alpha+1)^2 - 1 + \frac{2\gamma-\frac{3}{2}\delta}{nn}\right) E = -3 - \tfrac{3}{2} A + 2\varkappa\mathfrak{E}$$
$$+ 2\frac{(2\alpha+1)}{n} A + \frac{2A'}{n} - \left(3\varkappa - 3 + \frac{3-3\delta+4\gamma}{4nn}\right) A$$

vnde reperitur:

$\mathfrak{D} = -3,6035 \quad . \quad . \quad . \quad l-\mathfrak{D} = 0,556724$
$\mathfrak{E} = -1,0890 \quad - \quad - \quad - \quad l-\mathfrak{E} = 0,037028$

ac porro

$$\left.\begin{array}{l}\left(-0,24973 + \frac{(2\gamma-\frac{3}{2}\delta)}{nn}\right) D = -1,40048 - 7,4315 \\ \left(+7,21497 + \frac{(2\gamma-\frac{3}{2}\delta)}{nn}\right) E = -1,40048 - 2,7403\end{array}\right\} + \frac{(2\gamma-\frac{3}{2}\delta)}{nn}.0,629$$

§. 84. Quoniam autem valorem ipſius $\frac{2\gamma-\frac{3}{2}\delta}{nn}$ nondum nouimus, hunc terminum, cum certo ſit valde paruus, reiiciamus. Poſtmodum vero cum iſtum terminum cognouerimus, facile erit correctionem inde oriundam, ſi operae pretium videbitur, inuenire.

$D = +35,3662 \quad . \quad . \quad . \quad l\,D = 1,548588$
$E = -\quad 0,5739 \quad . \quad . \quad . \quad l-E = 9,758848$

Porro autem litterae $\mathfrak{F}$ et $\mathfrak{G}$ ita elicientur, vt ſit.

$(4\alpha-1)\mathfrak{F} = 0,000529 - 0,30976 + 0,06852 - 0,28042$
$(4\alpha+1)\mathfrak{G} = 0,000529 + 0,01028 + 0,02071 + 0,02638$

$\mathfrak{F} =$

$\mathfrak{F} = -0,1907$ . . . $l-\mathfrak{F} = 9,280416$

$\mathfrak{G} = +0,0122$ . . . $l\,\mathfrak{G} = 8,087607$

Praeterea autem colligimus fore

$\mathfrak{C} = -0,67465$ . . . $l-\mathfrak{C} = 9,829072$

vnde erit proxime $\frac{\mathfrak{A}\mathfrak{C}}{2nn} = 0,00154$, ex quo accuratius concluditur fore

$D = +35,3724$ . . $l\,D = 1,548664$

$E = -0,5741$ . . $l-E = 9,758988$

§. 85. Reliquae aequationes nobis praebebunt

$$6,4655\,F + 3,67820 = 0$$

$$21,3946\,G - 0,29574 = 0$$

vnde obtinebitur

$F = -0,56890$; $l-F = 9,755033$

$G = +0,01382$. $l\,G = 8,140620$

ac denique $\gamma = 1,40673$.

Supra autem iam inuenimus $\frac{1}{2}\delta - \gamma = 0,01742$, vnde ambas iſtas quantitates γ et δ, quas initio ad veros valores conſtantium litterarum m et n determinandos aſſumſimus, nunc cognitas habemus, erit enim:

$\delta = 2,84830$, et $2\gamma - \frac{3}{2}\delta = -1,45899$

ac propterea particulae illius $\frac{2\gamma - \frac{3}{2}\delta}{nn}$ hactenus neglectae valor erit $\frac{2\gamma - \frac{3}{2}\delta}{nn} = -0,00832$, cuius ope iam litterae D et E accuratius definiri poterunt.

§. 86.

§. 86. Hinc autem potiſſimum valor ipſius D mutationem patitur, fiet enim re vera

$-0,25805\ D = -8,83698$ ſeu

$D = 34,24520 \quad . \quad . \quad . \quad l\, D = 1,534600$

$7,20665\ E = -4,14578$ ſeu

$E = -0,57527 \quad . \quad . \quad . \quad l{-}E = 9,759874$

et quoniam D parte ſua triceſima diminuitur, in eadem fere ratione diminuentur valores litterarum $\mathfrak{C}$ et γ, ita vt exactius ſit:

$\mathfrak{C} = -0,65217 \quad . \quad . \quad . \quad l{-}\mathfrak{C} = 9,814361$

$\gamma = +1,35984 \quad . \quad . \quad . \quad l\,\gamma = 0,133490$

$\delta = +2,75336 \quad . \quad . \quad . \quad l\,\delta = 0,439863$

et $\frac{2\gamma - \frac{3}{2}\delta}{nn} = -0,00804$

Deinceps autem operae erit pretium in hos valores adhuc diligentius inquirere.

§. 87. Cum igitur finxerimus ſequentes valores:

$$\int R\,dr = \mathfrak{A}\,\mathrm{coſ}\,2\eta + \mathfrak{B}\,\mathrm{coſ}\,4\eta + \mathfrak{C}\,k\,\mathrm{coſ}\,r + \mathfrak{D}\,k\,\mathrm{coſ}\,(2\eta - r) + \mathfrak{F}\,k\,\mathrm{coſ}(4\eta - r) + \mathfrak{E}\,k\,\mathrm{coſ}\,(2\eta + r) + \mathfrak{G}\,k\,\mathrm{coſ}\,(4\eta + r)$$

$$v = A\,\mathrm{coſ}\,2\eta + B\,\mathrm{coſ}\,4\eta + D\,k\,\mathrm{coſ}\,(2\eta - r) + F\,k\,\mathrm{coſ}(4\eta - r) + E\,k\,\mathrm{coſ}(2\eta + r) + G\,k\,\mathrm{coſ}(4\eta + r)$$

horum coefficientium valores ſunt.

$\mathfrak{A} = -0,80378$	$A = -1,25826$
$\mathfrak{B} = +0,00697$	$B = -0,01279$
$\mathfrak{C} = -0,65217$	. .
$\mathfrak{D} = -3,60350$	$D = +34,24520$
$\mathfrak{E} = -1,08900$	$E = -0,57527$
$\mathfrak{F} = -0,19070$	$F = -0,56890$
$\mathfrak{G} = +0,01220$	$G = +0,01382$

vnde

vnde pro diſtantia lunae a terra $x = \frac{(1-kk)au}{1-k\cos r}$ fit

$$\begin{aligned} u = 1 &- 0,007161 \cos 2\eta \quad + 0,000073 \cos 4\eta \\ &+ 0,194888\, k\cos(2\eta-r) - 0,003274\, k\cos(2\eta+r) \\ &- 0,003235\, k\cos(4\eta-r) + 0,000078\, k\cos(4\eta+r) \end{aligned}$$

§. 88. His valoribus in §. 77. ſubſtitutis obtinebimus:

$$\begin{aligned} \frac{d\Phi}{dr} = \quad & \varkappa \quad + 0,019015 \cos 2\eta - 0,001255\, k\cos r \\ & + 0,0001103 - 0,000076 \cos 4\eta \\ & - 0,38410\, k\cos(2\eta-r) + 0,01278\, k\cos(2\eta+r) \\ & + 0,002647\, k\cos(4\eta-r) - 0,000229\, k\cos(4\eta+r) \end{aligned}$$

ad cuius integrale inueniendum ponamus:

$$\begin{aligned} \Phi = O r &+ \mathfrak{A}' \sin 2\eta + \mathfrak{B}' \sin 4\eta + \mathfrak{C}' k \sin r \\ &+ \mathfrak{D}' k \cos(2\eta-r) + \mathfrak{E}' k \cos(2\eta+r) \\ &+ \mathfrak{F}' k \cos(4\eta-r) + \mathfrak{G}' k \cos(4\eta+r) \end{aligned}$$

eritque differentiando et terminis iam cognitis omittendis.

$$\begin{aligned} \frac{d\Phi}{dr} = &\left(\mathfrak{C}' - \frac{\mathfrak{A}'(2\varkappa D+\mathfrak{D})}{nn} - \frac{\mathfrak{A}'(2\varkappa E+\mathfrak{E})}{nn} - \frac{\mathfrak{D}'(2\varkappa A+\mathfrak{A})}{nn} - \frac{\mathfrak{E}'(2\varkappa A+\mathfrak{A})}{nn}\right) k\cos r \\ &\left(-\frac{2\mathfrak{A}'}{n} + (2\alpha-1)\mathfrak{D}'\right) k\cos(2\eta-r) \\ &\left(-\frac{2\mathfrak{A}'}{n} + (2\alpha+1)\mathfrak{E}'\right) k\cos(2\eta+r) \\ &\left(-\frac{4\mathfrak{B}'}{n} - \frac{\mathfrak{A}'(2\varkappa D+\mathfrak{D})}{nn} - \frac{\mathfrak{D}'(2\varkappa A+\mathfrak{A})}{nn} + (4\alpha-1)\mathfrak{F}'\right) k\cos(4\eta-r) \\ &\left(-\frac{4\mathfrak{B}'}{n} - \frac{\mathfrak{A}'(2\varkappa E+\mathfrak{E})}{nn} - \frac{\mathfrak{E}'(2\varkappa A+\mathfrak{A})}{nn} + (4\alpha+1)\mathfrak{G}'\right) k\cos(4\eta+r) \end{aligned}$$

Pro terminis autem iam inuentis eſt

$$O = \varkappa - 0,000080; \quad \mathfrak{A}' = 0,010191; \quad \mathfrak{B}' = -0,000072$$
$$l\mathfrak{A}' = 8,008208; \quad l\mathfrak{B}' = -5,859381$$

K §. 89.

§. 89. Comparatione iam inſtituta fiet:

$$(2a-1)\mathfrak{D}' = -0{,}38410 + \frac{2\mathfrak{A}'}{n}\,;\quad (2a+1)\mathfrak{E}' = +0{,}01278 + \frac{2\mathfrak{A}'}{n}$$

$$(4a-1)\mathfrak{F}' = +0{,}002647 + \frac{4\mathfrak{B}'}{n} + \frac{\mathfrak{A}'(2\kappa D+\mathfrak{D})+\mathfrak{D}'(2\kappa A+\mathfrak{A})}{nn}$$

$$(4a+1)\mathfrak{G}' = -0{,}000229 + \frac{4\mathfrak{B}'}{n} + \frac{\mathfrak{A}'(2\kappa E+\mathfrak{E})+\mathfrak{E}'(2\kappa A+\mathfrak{A})}{nn}$$

$$\mathfrak{C}' = -0{,}001255 + \frac{\mathfrak{A}'(2\kappa D+\mathfrak{D})+\mathfrak{D}'(2\kappa A+\mathfrak{A})}{nn} + \frac{\mathfrak{A}'(2\kappa E+\mathfrak{E})+\mathfrak{E}'(2\kappa A+\mathfrak{A})}{nn}$$

vnde colligitur fore

$\mathfrak{C}' = +0{,}01083$	
$\mathfrak{D}' = -0{,}44167$ . . .	$l{-}\mathfrak{D}' = 9{,}645092$
$\mathfrak{E}' = +0{,}00499$ . . .	$l\,\mathfrak{E}' = 7{,}698640$
$\mathfrak{F}' = +0{,}00546$ . . .	$l\,\mathfrak{F}' = 7{,}737733$
$\mathfrak{G}' = -0{,}00010$ . . .	$l{-}\mathfrak{G}' = 6{,}002537$

ita vt ſit

$$\varphi = (\kappa - 0{,}000080r) + 0{,}010191\,\text{ſin}\,2\eta + 0{,}01083\,k\,\text{ſin}\,r - 0{,}000072\,\text{ſin}\,4\eta$$
$$-\,0{,}44167\,k\,\text{ſin}\,(2\eta-r) + 0{,}00499\,k\,\text{ſin}\,(2\eta+r)$$
$$+\,0{,}00546\,k\,\text{ſin}\,(4\eta-r) - 0{,}00010\,k\,\text{ſin}\,(4\eta+r)$$

vnde ex comparatione motus medii ad modum anomaliae erit $\kappa = 1{,}008607$, et $a = 0{,}933279$, qui valores iam propius ad veritatem accedunt, quam hactenus vſurpati.

CAPUT

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CAPUT VI.

INUESTIGATIO INAEQUALITATUM LUNAE A QUADRATO EXCENTRICITATIS IPſIUS ORTARUM.

§. 90.

Peruenimus nunc ad alteram partem inaequalitatum in motu Lunae, quae ab eius excentricitate k pendent, eiusque quadratum inuoluunt, ita vt hic non niſi eos terminos ſimus contemplaturi, qui per quadratum excentricitatis lunae kk ſunt multiplicati. Hic autem tam in valorem ipſius $\int R dr$, quam ipſius v termini formae $kk \cos 2\eta$ et $kk \cos 4\eta$ ingredientur, qui poſtquam fuerint inuenti, terminis huius generis iam ante inuentis adiici debent; praeterea vero vtrinque etiam termini formae $kk \cos 2r$ accedent. Hinc ponamus:

$$\begin{aligned}\int R dr = {} & \mathfrak{A} \cos 2\eta + \mathfrak{a}\, kk \cos 2\eta + \mathfrak{B} \cos 4\eta + \mathfrak{b}\, kk \cos 4\eta \\ & + \mathfrak{C}\, k \cos r + \mathfrak{D}\, k \cos(2\eta - r) + \mathfrak{E}\, k \cos(2\eta + r) \\ & + \mathfrak{F}\, k \cos(4\eta - r) + \mathfrak{G}\, k \cos(4\eta + r) \\ & + \mathfrak{H}\, kk \cos 2r + \mathfrak{J}\, kk \cos(2\eta - 2r) + \mathfrak{K}\, kk \cos(2\eta + 2r) \\ & + \mathfrak{L}\, kk \cos(4\eta - 2r) + \mathfrak{M}\, kk \cos(4\eta + 2r)\end{aligned}$$

$$\begin{aligned}v = {} & A \cos 2\eta + a\, kk \cos 2\eta + B \cos 4\eta + b\, kk \cos 4\eta \\ & + D\, k \cos(2\eta - r) + E\, k \cos(2\eta + r) \\ & + F\, k \cos(4\eta - r) + G\, k \cos(4\eta + r) \\ & + H\, kk \cos 2r + J\, kk \cos(2\eta - 2r) + K\, kk \cos(2\eta + 2r) \\ & + L\, kk \cos(4\eta - 2r) + M\, kk \cos(4\eta + 2r)\end{aligned}$$

§. 91. Nunc ad terminos, quibus ante valorem ipſius $\frac{d\Phi}{dr}$ exprimi inuenimus, inſuper ſequentes per kk multiplicati accedent:

$$\frac{d\Phi}{dr} = \ldots + \frac{D(3\varkappa D + 2\mathfrak{D})}{2n^4}k^2 + \left(-\frac{(2\varkappa a + \mathfrak{a})}{nn} + \frac{\mathfrak{C}D}{n^4}\right)k^2 \cos 2\eta$$

$$\left(-\frac{(2\varkappa b + \mathfrak{b})}{nn} + \frac{D(3\varkappa E + 2\mathfrak{E})}{2n^4} + \frac{E(3\varkappa D + 2\mathfrak{D})}{2n^4}\right) k^2 \cos 4\eta$$

$$\left\{\begin{matrix} -\frac{(2\varkappa H + \mathfrak{H})}{nn} + \frac{D(3\varkappa E + 2\mathfrak{E})}{2n^4} + \frac{E(3\varkappa D + 2\mathfrak{D})}{2n^4} \\ + \frac{A(3\varkappa J + 2\mathfrak{J})}{2n^4} + \frac{J(3\varkappa A + 2\mathfrak{A})}{2n^4} \end{matrix}\right\} k^2 \cos 2r$$

$$\left(-\frac{(2\varkappa J + \mathfrak{J})}{nn} + \frac{A(3\varkappa H + 2\mathfrak{H})}{2n^4} + \frac{H(3\varkappa A + 2\mathfrak{A})}{2n^4} + \frac{\mathfrak{C}D}{n^4}\right) k^2 \cos(2\eta - 2r)$$

$$\left(-\frac{(2\varkappa K + \mathfrak{K})}{nn} + \frac{A(3\varkappa H + 2\mathfrak{H})}{2n^4} + \frac{H(3\varkappa A + 2\mathfrak{A})}{2n^2}\right) k \cos(42\eta + 2r)$$

$$\left(-\frac{(2\varkappa L + \mathfrak{L})}{nn} + \frac{A(3\varkappa J + 2\mathfrak{J})}{2n^4} + \frac{J(3\varkappa A + 2\mathfrak{A})}{2n^4} + \frac{D(3\varkappa D + 2\mathfrak{D})}{2n^4}\right) k^2 \cos(4\eta - 2r)$$

$$-\frac{(2\varkappa M + \mathfrak{M})}{nn} k^2 \cos(4\eta + 2r)$$

vbi quidem terminos, quos minimos fore facile eſt praeuidere, omiſimus.

§. 92. Terminus autem conſtans $\frac{D(3\varkappa D + 2\mathfrak{D})kk}{2n^4}$ reperitur $= 0,000175$, vnde poſito $\varkappa + 0,000285 - \frac{1}{n} = \alpha$, quoniam valorem ipſius $\frac{d\eta}{dr}$ non opus eſt tam exacte noſſe, ſumamus:

$\frac{d\eta}{dr}$

$$\frac{d\eta}{dr} = \alpha - \frac{(2\varkappa A + \mathfrak{A})}{nn}\cos 2\eta - \left(\frac{2k}{n} + \frac{\mathfrak{C}k}{nn}\right)\cos r$$
$$- \frac{(2\varkappa D + \mathfrak{D})}{nn}k\cos(2\eta - r) - \left(\frac{3kk}{2n} \mp \frac{(2\varkappa H \mp \mathfrak{H})}{nn}\right)\cos 2r$$
$$- \frac{(2\varkappa J + \mathfrak{J})}{nn}k^2\cos(2\eta - 2r)$$

Deinde vero praeter terminos iam tractatos habebitur:

$$R = \ldots 3k^2\sin 2\eta + \frac{3D}{nn}k^2\sin 4\eta + \frac{3(2D+J)}{2nn}k^2\sin 2r$$
$$+ \left(\tfrac{15}{4}k^2 + \frac{3(H-L)}{2nn}\right)\sin(2\eta - 2r) + \left(\tfrac{15}{4}k^2 + \frac{3H}{2nn}\right)\sin(2\eta + 2r)$$
$$+ \frac{3(2D+J)}{2nn}k^2\sin(4\eta - 2r)$$

atque ſimili modo:

$$\frac{ddv}{dr^2} = \tfrac{1}{2}\delta - \gamma + \frac{\mathfrak{A}a + \tfrac{1}{2}\mathfrak{D}\mathfrak{D} + 3D\mathfrak{D} + 3DD + 3Aa + 3\mathfrak{A}a + 3A\mathfrak{a}}{nn}kk$$
$$+ \left(3 + \tfrac{1}{2}(A + D + E) - a - 2\varkappa\mathfrak{a}\right)kk\cos 2\eta$$
$$+ \left(\tfrac{1}{2}(B + F + G) - b - 2\varkappa\mathfrak{b}\right)kk\cos 4\eta$$
$$+ \left(\tfrac{3}{2} - 2\varkappa\mathfrak{H} - H + \frac{\mathfrak{A}\mathfrak{J} + 3\mathfrak{A}J + 3A\mathfrak{J} + 3AJ}{nn}\right)kk\cos 2r$$
$$+ \left(\tfrac{15}{4} - 2\varkappa\mathfrak{J} - J + \tfrac{3}{4}A + \tfrac{3}{2}D\right)kk\cos(2\eta - 2r)$$
$$+ \left(\tfrac{15}{4} - 2\varkappa\mathfrak{K} - K + \tfrac{3}{4}A + \tfrac{3}{2}E\right)k^2\cos(2\eta + 2r)$$
$$+ \left(-2\varkappa\mathfrak{L} - L + \tfrac{3}{4}B + \tfrac{3}{2}F\right)kk\cos(4\eta - 2r)$$
$$+ \left(-2\varkappa\mathfrak{M} - M + \tfrac{3}{4}B + \tfrac{3}{2}G\right)k^2\cos(4\eta + 2r)$$
$$+ \frac{\mathfrak{A}\mathfrak{J} + 3\mathfrak{A}\mathfrak{J} + 3AJ + \tfrac{1}{2}\mathfrak{D}\mathfrak{D} + 3D\mathfrak{D} + 3DD)}{nn}kk\cos(4\eta - 2r)$$

§. 93. Eliciamus nunc quoque valorem ipſius R per differentiationem ex formula $\int R\,dr$, ac terminis apte diſpoſitis habebimus

 R =

$R =$

$kk \sin 2\eta$	$kk \sin 4\eta$	$kk \sin 2r$	$k^2 \sin(2\eta - 2r)$
$-2\alpha\mathfrak{a}$	$+\frac{\mathfrak{a}(2\kappa A+\mathfrak{A})}{nn}$	$+\frac{\mathfrak{A}(2\kappa J+\mathfrak{J})}{nn}$	$+\frac{3\mathfrak{A}}{2n}$
$+\frac{2\mathfrak{b}(2\kappa A+\mathfrak{A})}{nn}$	$-4\alpha\mathfrak{b}$	$-2\mathfrak{H}$	$+\frac{\mathfrak{A}(2\kappa H+\mathfrak{H})}{nn}$
$+\frac{\mathfrak{D}(2n+\mathfrak{C})}{nn}$	$+\frac{\mathfrak{E}(2\kappa D+\mathfrak{D})}{nn}$	$+\frac{\mathfrak{C}(2\kappa D+\mathfrak{D})}{nn}$	$+\frac{\mathfrak{D}(2n+\mathfrak{C})}{nn}$
$+\frac{\mathfrak{E}(2n+\mathfrak{C})}{nn}$	$+\frac{2\mathfrak{F}(2n+\mathfrak{C})}{nn}$	$-\frac{\mathfrak{J}(2\kappa A+\mathfrak{A})}{nn}$	$-2(\alpha-1)\mathfrak{J}$
$+\frac{2\mathfrak{F}(2\kappa D+\mathfrak{D})}{nn}$	$+\frac{2\mathfrak{G}(2n+\mathfrak{C})}{nn}$	$+\frac{\mathfrak{K}(2\kappa A+\mathfrak{A})}{nn}$	$+\frac{2\mathfrak{L}(2\kappa A+\mathfrak{A})}{nn}$

$kk \sin(2\eta+2r)$	$kk \sin(4\eta-2r)$	$kk \sin(4\eta+2r)$
$+\frac{3\mathfrak{A}}{2n}$	$+\frac{\mathfrak{A}(2\kappa J+\mathfrak{J})}{nn}$	
$+\frac{\mathfrak{A}(2\kappa H+\mathfrak{H})}{nn}$	$+\frac{3\mathfrak{B}}{n}$	$+\frac{3\mathfrak{B}}{n}$
$+\frac{2\mathfrak{B}(2\kappa J+\mathfrak{J})}{nn}$	$+\frac{2\mathfrak{B}(2\kappa H\dagger\mathfrak{H})}{nn}$	$+\frac{2\mathfrak{B}(2\kappa H\dagger\mathfrak{H})}{nn}$
$+\frac{\mathfrak{E}(2n+\mathfrak{C})}{nn}$	$+\frac{\mathfrak{D}(2\kappa D+\mathfrak{D})}{nn}$	$+\frac{\mathfrak{G}(2n+\mathfrak{C})}{nn}$
$+\frac{2\mathfrak{G}(2\kappa D+\mathfrak{D})}{nn}$	$+\frac{2\mathfrak{F}(2n+\mathfrak{C})}{nn}$	$+\frac{\mathfrak{K}(2\kappa A+\mathfrak{A})}{nn}$
$-2(\alpha+1)\mathfrak{K}$	$+\frac{\mathfrak{J}(2\kappa A+\mathfrak{A})}{nn}$	$-2(2\alpha+1)\mathfrak{M}$
$+\frac{2\mathfrak{M}(2\kappa A+\mathfrak{A})}{nn}$	$-2(2\alpha-1)\mathfrak{L}$	

vnde

vnde oriuntur ſequentes determinationes:

$$3 = -2\alpha\mathfrak{a} + \frac{2\mathfrak{b}(2\kappa A + \mathfrak{A}) + (\mathfrak{D} + \mathfrak{E})(2n + \mathfrak{C}) + 2\mathfrak{F}(2\kappa D + \mathfrak{D})}{nn}$$

$$\frac{3D}{nn} = -4\alpha\mathfrak{b} + \frac{\mathfrak{a}(2\kappa A + \mathfrak{A}) + \mathfrak{E}(2\kappa D + \mathfrak{D}) + 2(\mathfrak{F} + \mathfrak{G})(2n + \mathfrak{C})}{nn}$$

$$\frac{3(2D + J)}{2nn} = -2\mathfrak{H} + \frac{\mathfrak{A}(2\kappa J + \mathfrak{J}) + \mathfrak{E}(2\kappa D + \mathfrak{D}) - (\mathfrak{J} - \mathfrak{K})(2\kappa A + \mathfrak{A})}{nn}$$

$$\tfrac{15}{4} + \frac{3(H - L)}{2nn} = -2(\alpha - 1)\mathfrak{J} + \frac{3\mathfrak{A}}{nn} + \frac{\mathfrak{A}(2\kappa H + \mathfrak{H})}{nn} + \frac{\mathfrak{D}(2n + \mathfrak{C}) + 2\mathfrak{L}(2\kappa A + \mathfrak{A})}{nn}$$

$$\tfrac{5}{4} + \frac{3H}{2nn} = 2(\alpha + 1)\mathfrak{K} + \frac{3\mathfrak{A}}{2n} + \frac{\mathfrak{A}(2\kappa H + \mathfrak{H})}{nn} + \frac{2\mathfrak{B}(2\kappa J + \mathfrak{J}) + 2\mathfrak{G}(2\kappa D + \mathfrak{D}) + \mathfrak{E}(2n + \mathfrak{C}) + 2\mathfrak{M}(2\kappa A + \mathfrak{A})}{nn}$$

$$+\frac{3(2D + J)}{2nn} = -2(2\alpha - 1)\mathfrak{L} + \frac{3\mathfrak{B}}{n} + \frac{\mathfrak{A}(2\kappa J + \mathfrak{J})}{nn} + \frac{2\mathfrak{B}(2\kappa H + \mathfrak{H}) + \mathfrak{D}(2\kappa D + \mathfrak{D}) + 2\mathfrak{F}(2n + \mathfrak{C}) + \mathfrak{J}(2\kappa A + \mathfrak{A})}{nn}$$

$$0 = -2(2\alpha + 1)\mathfrak{M} + \frac{3\mathfrak{B}}{n} + \frac{2\mathfrak{B}(2\kappa H + \mathfrak{H}) + 2\mathfrak{G}(2n + \mathfrak{C})}{nn} + \frac{\mathfrak{K}(2\kappa A + \mathfrak{A})}{nn}$$

§. 94. Deinde ſimili modo ſi ponatur:

$$\frac{dv}{dr} = -A' \sin 2\eta - a'k^2 \sin 2\eta - B' \sin 4\eta - b'k^2 \sin 4\eta$$
$$- C'k \sin r - D'k \sin(2\eta - r) - F'k \sin(4\eta - r)$$
$$- E'k \sin(2\eta + r) - G'k \sin(2\eta + r)$$
$$- H'k^2 \sin 2r - J'k^2 \sin(2\eta - 2r) - L'k^2 \sin(4\eta - 2r)$$
$$- K'k^2 \sin(2\eta + 2r) - M'k^2 \sin(4\eta + 2r)$$

erit

erit praeter valores §. 81. datos:

$$a' = 2\alpha a - \frac{2b(2\kappa A \pm \mathfrak{A}) - (D \pm E)(2n + \mathfrak{C}) - 2F(2\kappa D + \mathfrak{D})}{nn}$$

$$b' = 4\alpha b - \frac{a(2\kappa A + \mathfrak{A}) - E(2\kappa D + \mathfrak{D})}{nn}$$

$$- \frac{2(F + \ddot{G})(2n + \mathfrak{C}) - D(2\kappa E + \mathfrak{E})}{nn}$$

$$H' = 2H - \frac{A(2\kappa J + \mathfrak{J}) - E(2\kappa D + \mathfrak{D})}{nn}$$

$$+ \frac{(J - K)(2\kappa A + \mathfrak{A}) + D(2\kappa E + \mathfrak{E})}{nn}$$

$$J' = 2(\alpha - 1)J - \frac{3A}{2n} - \frac{A(2\kappa H + \mathfrak{H}) - D(2n + \mathfrak{C}) - 2L(2\kappa A + \mathfrak{A})}{nn}$$

$$K' = 2(\alpha + 1)K - \frac{3A}{2n} - \frac{A(2\kappa H + \mathfrak{H}) - 2B(2\kappa J + \mathfrak{J}) - 2G(2\kappa D + \mathfrak{D})}{nn}$$

$$- \frac{E(2n + \mathfrak{C}) - 2M(2\kappa A + \mathfrak{A})}{nn}$$

$$L' = 2(2\alpha - 1)L - \frac{3B}{n} - \frac{A(2\kappa J + \mathfrak{J}) - 2B(2\kappa H + \mathfrak{H}) - D(2\kappa D + \mathfrak{D})}{nn}$$

$$- \frac{2F(2n + \mathfrak{C}) - J(2\kappa A + \mathfrak{A})}{nn}$$

$$M' = 2(2\alpha + 1)M - \frac{3B}{n} - \frac{2B(2\kappa H + \mathfrak{H}) - G(2n + \mathfrak{C}) - K(2\kappa A + \mathfrak{A})}{nn}$$

vbi quidem plures terminos, quos admodum paruos fore praeuidimus, omiſimus.

§. 95.

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§. 95. Hinc autem denuo differentiando obtinemus

valorem ipſius $\frac{ddv}{dr^2}$ ═══

kk	$kk \operatorname{coſ} 2\eta$	$kk \operatorname{coſ} 4\eta$	$kk \operatorname{coſ} 2r$
$+\frac{a'(2\varkappa A+\mathfrak{A})}{nn}$	$-\ 2\alpha a'$	$+\frac{a'(2\varkappa A+\mathfrak{A})}{nn}$	$+\frac{A'(2\varkappa J+\mathfrak{J})}{nn}$
$+\frac{D'(2\varkappa D+\mathfrak{D})}{nn}$	$+\frac{2b'(2\varkappa A+\mathfrak{A})}{nn}$	$-\ 4\alpha b'$	$+\frac{E'(2\varkappa D+\mathfrak{D})}{nn}$
	$+\frac{D'(2n+\mathfrak{C})}{nn}$	$+\frac{E'(2\varkappa D+\mathfrak{D})}{nn}$	$-\ 2H'$
	$+\frac{E'(2n+\mathfrak{C})}{nn}$	$+\frac{2F'(2n+\mathfrak{C})}{nn}$	$+\frac{J'(2\varkappa A+\mathfrak{A})}{nn}$
	$+\frac{2F'(2\varkappa D+\mathfrak{D})}{nn}$	$+\frac{2G'(2n+\mathfrak{C})}{nn}$	$+\frac{K'(2\varkappa A+\mathfrak{A})}{nn}$

$kk \operatorname{coſ}(2\eta-2r)$	$kk \operatorname{coſ}(2\eta+2r)$	$kk \operatorname{coſ}(4\eta-2r)$	$kk \operatorname{coſ}(4\eta+2r)$
$+\frac{3A'}{2n}$	$+\frac{3A'}{2n}$	$+\frac{A'(2\varkappa J+\mathfrak{J})}{nn}$	
$+\frac{A'(2\varkappa H+\mathfrak{H})}{nn}$	$+\frac{A'(2\varkappa H+\mathfrak{H})}{nn}$	$+\frac{3B'}{n}$	$+\frac{3B'}{n}$
$+\frac{D'(2n+\mathfrak{C})}{nn}$	$+\frac{2G'(2\varkappa D+\mathfrak{D})}{nn}$	$+\frac{2B'(2\varkappa H+\mathfrak{H})}{nn}$	$+\frac{2B'(2\varkappa H+\mathfrak{H})}{nn}$
$-\ 2(\alpha-1)J'$	$-\ 2(\alpha+1)K'$	$+\frac{D'(2\varkappa D+\mathfrak{D})}{nn}$	$+\frac{2G'(2n+\mathfrak{C})}{nn}$
$+\frac{2L'(2\varkappa A+\mathfrak{A})}{nn}$	$+\frac{2M'(2\varkappa A+\mathfrak{A})}{nn}$	$+\frac{2F(2n+\mathfrak{C})}{nn}$	$-\ 2(2\alpha+1)M'$
		$-\ 2(2\alpha-1)L'$	$+\frac{K'(2\varkappa A+\mathfrak{A})}{nn}$
		$+\frac{J'(2\varkappa A+\mathfrak{A})}{nn}$	

L

vnde

vnde tandem nanciſcimur has determinationes:

$$\tfrac{1}{2}\delta-\gamma+\frac{\mathfrak{A}\mathfrak{a}+\frac{3}{2}\mathfrak{D}\mathfrak{D}+3\mathrm{D}\mathfrak{D}+\frac{3}{2}\mathrm{DD}+3\mathrm{A}a+3\mathfrak{A}a+3\mathrm{A}\mathfrak{a}}{nn}kk$$
$$=\frac{a'(2\kappa\mathrm{A}+\mathfrak{A})+\mathrm{D}'(2\kappa\mathrm{D}+\mathfrak{D})}{nn}kk$$

$$3+\tfrac{3}{2}(\mathrm{A}+\mathrm{D}+\mathrm{E})-a-2\kappa\mathfrak{a}=-2\alpha a'+\frac{2b'(2\kappa\mathrm{A}+\mathfrak{A})}{nn}$$
$$+\frac{(\mathrm{D}'+\mathrm{E}')(2n+\mathfrak{C})+2\mathrm{F}'(2\kappa\mathrm{D}+\mathfrak{D})}{nn}$$

$$\tfrac{3}{2}(\mathrm{B}+\mathrm{F}+\mathrm{G})-b-2\kappa\mathfrak{b}=-4\alpha b'+\frac{a'(2\kappa\mathrm{A}+\mathfrak{A})}{nn}$$
$$+\frac{\mathrm{E}'(2\kappa\mathrm{D}+\mathfrak{D})+2(\mathrm{F}'+\mathrm{G}')(2n+\mathfrak{C})}{nn}$$

$$\tfrac{3}{2}-2\kappa\mathfrak{H}-\mathrm{H}+\frac{\mathfrak{A}\mathfrak{J}+3\mathfrak{A}\mathrm{J}+3\mathrm{A}\mathfrak{J}+3\mathrm{A}\mathrm{J}}{nn}=-2\mathrm{H}'+\frac{\mathrm{A}'(2\kappa\mathrm{J}+\mathfrak{J})}{nn}$$
$$+\frac{\mathrm{E}'(2\kappa\mathrm{D}+\mathfrak{D})+(\mathrm{J}'+\mathrm{K}')(2\kappa\mathrm{A}+\mathfrak{A})}{nn}$$

$$\tfrac{15}{4}-2\kappa\mathfrak{J}-\mathrm{J}+\tfrac{3}{4}\mathrm{A}+\tfrac{3}{2}\mathrm{D}=-2(\alpha-1)\mathrm{J}'+\frac{3\mathrm{A}'}{2n}+\frac{\mathrm{A}'(2\kappa\mathrm{H}+\mathfrak{H})}{nn}$$
$$+\frac{\mathrm{D}'(2n+\mathfrak{C})+2\mathrm{L}'(2\kappa\mathrm{A}+\mathfrak{A})}{nn}$$

$$\tfrac{15}{4}-2\kappa\mathfrak{K}-\mathrm{K}+\tfrac{3}{4}\mathrm{A}+\tfrac{3}{2}\mathrm{E}=-2(\alpha+1)\mathrm{K}'+\frac{3\mathrm{A}'}{2n}+\frac{\mathrm{A}'(2\kappa\mathrm{H}+\mathfrak{H})}{nn}$$
$$+\frac{2\mathrm{G}'(2\kappa\mathrm{D}+\mathfrak{D})+2\mathrm{M}'(2\alpha\mathrm{A}+\mathfrak{A})}{nn}$$

$$-2\kappa\mathfrak{L}-\mathrm{L}+\tfrac{3}{4}\mathrm{B}+\tfrac{3}{2}\mathrm{F}+\frac{\mathfrak{A}\mathfrak{J}+3\mathfrak{A}\mathrm{J}+3\mathrm{A}\mathfrak{J}+3\mathrm{A}\mathrm{J}+\frac{3}{2}\mathfrak{D}\mathfrak{D}+3\mathrm{D}\mathfrak{D}+3\mathrm{DD}}{nn}=$$
$$-2(2\alpha-1)\mathrm{L}'+\frac{3\mathrm{B}'}{n}+\frac{\mathrm{A}'(2\kappa\mathrm{J}+\mathfrak{J})+2\mathrm{B}'(2\kappa\mathrm{H}+\mathfrak{H})}{nn}$$
$$+\frac{\mathrm{D}'(2\kappa\mathrm{D}+\mathfrak{D})+2\mathrm{F}'(2n+\mathfrak{C})+\mathrm{J}'(2\kappa\mathrm{A}+\mathfrak{A})}{nn}$$

$$-2\kappa\mathfrak{M}-M+\tfrac{1}{2}B+\tfrac{1}{2}G=-2(2\alpha+1)M'+\frac{3B'}{n}+\frac{\alpha B'(2\kappa H+\mathfrak{H})}{nn}$$
$$+\frac{2G'(2\kappa+\mathfrak{C})+K'(2\kappa A+\mathfrak{A})}{nn}$$

§. 96. Primum autem valoribus iam cognitis ſubſtituendis, reperitur:

$2\alpha a=-3,837-0,038\,\mathfrak{b}$ et $4\alpha\mathfrak{b}=-1,073-0,019\,\mathfrak{a}$
hincque $\mathfrak{a}=-2,051$ et $\mathfrak{b}=-0,277$
ex quibus porro elicimus: $a=-12,595$ et $b=-0,086$
et $a'=-23,510$. Deinde pro reliquis litteris

$\mathfrak{J}=+32,663$ - - - $l\,\mathfrak{J}=1,514059$
$\mathfrak{K}=-1,035$ $l\text{-}\mathfrak{K}=0,014776$
$J'=-2(1-\alpha)\,J-5,060;$
$K'=2(1+\alpha)\,K+0,227;$
$J=-15,555$ $l\text{-}J=1,191891$
$K=-0,370$ $l\text{-}K=9,568589$
Porro $\mathfrak{L}=-1,453$ - - - $l\text{-}\mathfrak{L}=0,162070$
$\mathfrak{M}=-0,000$ - - -
$L'=2(2\alpha-1)\;L-12,786$
$M'=2(2\alpha+1)\;M$
$L=+6,252$ $l\,L=0,796019$
$M=-0,001$ $l\text{-}M=7,000000$
Denique $\mathfrak{H}=-0,123$ et $H=-1,033$
atque $\tfrac{1}{2}\delta-\gamma=-7,459\,kk$

§. 97. His igitur valoribus inuentis innoteſcet primum diſtantia Lunae a terra curtata, quatenus a ſola

excentricitate orbitae lunaris k pendet. Cum enim haec diſtantia poſita ſit $x = \frac{(1-kk)au}{1-k\cos r}$ ob $u = 1 + \frac{v}{nn}$, erit

$$u = 1 - 0,007161 \cos 2\eta - 0,0719\, kk \cos 2\eta$$
$$+ 0,000073 \cos 4\eta - 0,0005\, kk \cos 4\eta$$
$$+ 0,194888\, k \cos(2\eta - r) - 0,003274\, k \cos(2\eta + r)$$
$$- 0,003238\, k \cos(4\eta - r) + 0,000078\, k \cos(4\eta + r)$$
$$- 0,0059\, kk \cos 2r$$
$$- 0,0889\, kk \cos(2\eta - 2r) - 0,0021\, kk \cos(2\eta + 2r)$$
$$+ 0,0357\, kk \cos(4\eta - 2r)$$

At pro longitudine Lunae, quatenus a ſola excentricitate k pendet, prodibit $\frac{d\Phi}{dr} =$

$$u + 0,000285 + 0,019015 \cos 2\eta + 0,000076 \cos 4\eta$$
$$+ 0,1562\, kk \cos 2\eta + 0,0008\, kk \cos 4\eta$$
$$- 0,001255\, k \cos r - 0,38410\, k \cos(2\eta - r) + 0,002647\, k \cos(4\eta - r)$$
$$+ 0,01278\, k \cos(2\eta + r) - 0,000229\, k \cos(4\eta + r)$$
$$+ 0,0118\, kk \cos 2r - 0,0081\, kk \cos(2\eta - 2r) - 0,0076\, kk \cos(4\eta - 2r)$$
$$+ 0,0102\, kk \cos(2\eta + 2r)$$

§. 98. Ponatur nunc:

$$\Phi = Or + \mathfrak{A}' \sin 2\eta + \mathfrak{a}' kk \sin 2\eta + \mathfrak{B}' \sin 4\eta + \mathfrak{b}' kk \sin 4\eta$$
$$+ \mathfrak{C}' k \sin r + \mathfrak{D}' k \sin(2\eta - r) + \mathfrak{F}' k \sin(4\eta - r)$$
$$+ \mathfrak{E}' k \sin(2\eta + r) + \mathfrak{G}' k \sin(4\eta + r)$$
$$+ \mathfrak{H}' kk \sin 2r - \mathfrak{J}' kk \sin(2\eta - 2r) + \mathfrak{L}' kk \sin(4\eta - 2r)$$
$$+ \mathfrak{K}' kk \sin(2\eta + 2r) + \mathfrak{M}' kk \sin(4\eta + 2r)$$

atque

atque differentiando orientur ſequentes comparationes

$$\kappa + 0{,}000285 = O - \frac{\mathfrak{a}'(2\kappa A + \mathfrak{A}) - \mathfrak{D}'(2\alpha D + \mathfrak{D})}{nn}\, kk + 0{,}000190$$

$$+ 0{,}1562 = 2\alpha\mathfrak{a}' - \frac{(\mathfrak{D}' + \mathfrak{E}')(2n + \mathfrak{E}) - 2\mathfrak{F}'(2\kappa D + \mathfrak{D})}{nn}$$

$$+ 0{,}0008 = 4\alpha\mathfrak{b}' - \frac{\mathfrak{a}'(2\kappa A + \mathfrak{A}) - \mathfrak{E}(2\kappa D + \mathfrak{D}) - 2(\mathfrak{F}' + \mathfrak{G}')(2n + \mathfrak{E})}{nn}$$

$$+ 0{,}0118 = 2\mathfrak{H}' - \frac{\mathfrak{E}'(2\kappa D + \mathfrak{D}) - \mathfrak{A}'(2\kappa J + \mathfrak{J}) - (\mathfrak{J}' + \mathfrak{K}')(2\kappa A + \mathfrak{A})}{nn}$$

$$- 0{,}0081 = 2(\alpha - 1)\mathfrak{J}' - \frac{3\mathfrak{A}'}{2n} - \frac{\mathfrak{D}'(2n + \mathfrak{E}) - \mathfrak{A}'(2\kappa H + \mathfrak{H})}{nn} - \frac{2\mathfrak{L}'(2\kappa A + \mathfrak{A})}{nn}$$

$$+ 0{,}0102 = 2(\alpha + 1)\mathfrak{K}' - \frac{3\mathfrak{A}'}{2n} - \frac{\mathfrak{A}'(2\kappa H + \mathfrak{H}) - 2\mathfrak{G}'(2\kappa D + \mathfrak{D})}{nn}$$

$$- 0{,}0076 = 2(2\alpha - 1)\mathfrak{L}' - \frac{3\mathfrak{B}'}{2n} - \frac{\mathfrak{A}'(2\kappa J + \mathfrak{J}) - \mathfrak{J}'(2\kappa A + \mathfrak{A})}{nn} - \frac{\mathfrak{D}'(2\kappa D + \mathfrak{D}) - 2\mathfrak{F}'(2n + \mathfrak{E})}{nn}$$

$$\cdot = 2(2\alpha + 1)\mathfrak{M}' - \frac{3\mathfrak{B}'}{n} - \frac{2\mathfrak{G}'(2n + \mathfrak{E})}{nn}$$

§. 99. Ex his comparationibus elicimus:

$\mathfrak{a}' = + 0{,}0509$; $\mathfrak{b}' = 0{,}0008$
$\mathfrak{J}' = + 0{,}5385$; $\mathfrak{K}' = 0{,}0028$
$\mathfrak{L}' = - 0{,}1055$; $\mathfrak{M}' = 0{,}0000$
$\mathfrak{H}' = + 0{,}0021$; et $O = \kappa - 0{,}000429$

 poſito

poſito $k = 0,05445$. Hinc autem erit $\frac{1}{2}\delta - \gamma = -0,02302$ Cum autem iam ante inuentum eſſet $\frac{1}{2}\delta - \gamma = -0,01742$ erit reuera $\frac{1}{2}\delta - \gamma = -0,00560$. Tum vero inueniemus: $\gamma = 1,40673$, vnde erit $\frac{1}{2}\delta = 1,40113$, et $\delta = 2,80226$ hincque $2\gamma - \frac{3}{2}\delta = -1,38993$ et $\frac{2\gamma - \frac{3}{2}\delta}{nn} = -0,00794$. Verum ex cognita ratione motus medii ad motum anomaliae eſt $O = 1,0085272$, vnde $\varkappa = 1,0089562$ Verum eſſe debet $\varkappa = 1 + \frac{3 + 4\mu + \delta}{4nn}$; vnde foret $0,0089562 = 0,008289 + \frac{\mu}{nn}$; ideoque $\frac{\mu}{nn} = 0,000667$ qui valor cum ſit tam exiguus, merito dubitamus, num μ non prorſus ſit $= 0$.

CAPUT

CAPUT VII.

CORRECTIO INAEQUALITATUM LUNAE, ANTE INUENTARUM.

§. 100.

Quoniam nunc quidem valores litterarum γ et δ ita inuenimus, vt eos pro proxime veris habere queamus, ex iis coefficientes terminorum, quibus inaequalitates lunae continentur, accuratius definire poterimus. Cum enim ſit $\gamma = 1,40673$ et $\delta = 2,80226$, colligamus hic in vnum omnes formulas, quas hactenus pro inueniendis coefficientibus aſſumtis elicuimus. Poſueramus autem:

$$\int R\,dr = \mathfrak{A}\cos 2\eta + \mathfrak{a}\,kk\cos 2\eta + \mathfrak{B}\cos 4\eta + \mathfrak{b}\,kk\cos 4\eta$$
$$+ \mathfrak{C}\,k\cos r + \mathfrak{D}\,k\cos(2\eta - r) + \mathfrak{F}\,k\cos(4\eta - r)$$
$$+ \mathfrak{E}\,k\cos(2\eta + r) + \mathfrak{G}\,k\cos(4\eta + r)$$
$$+ \mathfrak{H}\,kk\cos 2r + \mathfrak{J}\,k^2\cos(2\eta - 2r) + \mathfrak{L}\,k^2\cos(4\eta - 2r)$$
$$+ \mathfrak{K}\,k^2\cos(2\eta + 2r) + \mathfrak{M}\,k^2\cos(4\eta + 2r)$$

$$v = A\cos 2\eta + a\,kk\cos 2\eta + B\cos 4\eta + b\,kk\cos 4\eta$$
$$+ D\,k\cos(2\eta - r) + F\,k\cos(4\eta - r)$$
$$+ E\,k\cos(2\eta + r) + G\,k\cos(4\eta + r)$$
$$+ H\,kk\cos 2r + J\,kk\cos(2\eta - 2r) + L\,kk\cos(4\eta - 2r)$$
$$+ K\,kk\cos(2\eta + 2r) + M\,kk\cos(4\eta + 2r)$$

§. 101.

§. 101. Hinc poſito $\varkappa = \sqrt{\left(1 + \frac{\mathfrak{Z} + 4\mu + \delta}{2nn}\right)}$ collegimus fore

$$\frac{d\Phi}{dr} = \varkappa + \frac{A(3\varkappa A + 2\mathfrak{A})}{2n^4} + \frac{D(3\varkappa D + 2\mathfrak{D})}{2n^4}kk + \frac{A(3\varkappa a + \mathfrak{a})}{2n^4}kk$$

$$+ \frac{a(3\varkappa A + 2\mathfrak{A})}{2n^4}kk - \frac{(2\varkappa A + \mathfrak{A})}{nn}\text{coſ}\,2\eta - \frac{(2\varkappa B + \mathfrak{B})}{nn}\text{coſ}\,4\eta$$

$$+ \frac{A(3\varkappa A + 2\mathfrak{A})}{2n^4}\text{coſ}\,4\eta - \frac{\mathfrak{C}}{nn}k\,\text{coſ}\,r - \frac{(2\varkappa a + \mathfrak{a})}{nn}k^2\,\text{coſ}\,2\eta$$

$$- \frac{(2\varkappa b + \mathfrak{b})}{nn}k^2\,\text{coſ}\,4\eta + \frac{D(3\varkappa A + 2\mathfrak{A}) + A(3\varkappa D + 2\mathfrak{D})}{2n^4}k\,\text{coſ}\,r$$

$$- \frac{(2\varkappa D + \mathfrak{D})}{nn}k\,\text{coſ}(2\eta - r) - \frac{(2\varkappa F + \mathfrak{F})}{nn}k\,\text{coſ}(4\eta - r)$$

$$+ \frac{D(3\varkappa A + 2\mathfrak{A}) + A(3\varkappa D + 2\mathfrak{D})}{2n^4}k\,\text{coſ}(4\eta - r)$$

$$- \frac{(2\varkappa E + \mathfrak{E})}{nn}k\,\text{coſ}(2\eta + r) - \frac{(2\varkappa G + \mathfrak{G})}{nn}k\,\text{coſ}(4\eta + r)$$

$$- \frac{(2\varkappa J + \mathfrak{J})}{nn}kk\,\text{coſ}(2\eta - 2r) - \frac{(2\varkappa H + \mathfrak{H})}{nn}kk\,\text{coſ}\,2r$$

$$+ \frac{J(3\varkappa A + 2\mathfrak{A}) + A(3\varkappa J + 2\mathfrak{J})}{2n^4}k^2\,\text{coſ}\,2r$$

$$- \frac{(2\varkappa K + \mathfrak{K})}{nn}kk\,\text{coſ}(2\eta + 2r) - \frac{(2\varkappa L + \mathfrak{L})}{nn}k^2\,\text{coſ}(4\eta - 2r)$$

$$+ \frac{D(3\varkappa D + 2\mathfrak{D}) + J(3\varkappa A + 2\mathfrak{A}) + A(3\varkappa J + 2\mathfrak{J})}{2n^4}k^2\,\text{coſ}(4\eta - 2r)$$

$$- \frac{(2\varkappa M + \mathfrak{M})}{nn}k^2\,\text{coſ}(4\eta + 2r)$$

§. 102.

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§. 102. Si iam ponamus

$$\kappa + \frac{A(3\kappa A + 2\mathfrak{A})}{2n^4} + \frac{D(3\kappa D + 2\mathfrak{D})}{2n^4}kk$$
$$+ \frac{A(3\kappa a + 2\mathfrak{a})}{2n^4}kk + \frac{a(3\kappa A + 2\mathfrak{A})}{2n^4}kk$$
$$- \frac{1}{n} = \alpha;$$

vt ſit neglectis terminis admodum exiguis

$$\frac{d\eta}{dr} = \alpha - \frac{(2\kappa A + \mathfrak{A})}{nn}\operatorname{coſ} 2\eta - \frac{(2n + \mathfrak{C})}{nn}k \operatorname{coſ} r$$
$$- \frac{(2\kappa D + \mathfrak{D})}{nn}k \operatorname{coſ}(2\eta - r) - \frac{(2\kappa E + \mathfrak{E})}{nn}k^2 \operatorname{coſ}(2\eta + r)$$

ex ſuperioribus capitibus repetimus has determinationes:

$$2\alpha\mathfrak{A} = -\tfrac{3}{2}$$
$$4\alpha\mathfrak{B} = -\frac{3A}{2nn} + \frac{\mathfrak{A}(3\kappa A + \mathfrak{A})}{nn}$$
$$\mathfrak{C} = \frac{\mathfrak{A}(2\kappa D + \mathfrak{D})}{nn} - \frac{(\mathfrak{D} - \mathfrak{E})(2n A + \mathfrak{A})}{nn} - \frac{\mathfrak{A}(2\kappa E + \mathfrak{E})}{nn} - \frac{3(D - E)}{2nn}$$
$$(2\alpha - 1)\mathfrak{D} = -3 + \frac{(2n + \mathfrak{C})}{nn}\mathfrak{A}$$
$$(2\alpha + 1)\mathfrak{E} = -3 + \frac{(2n + \mathfrak{C})}{nn}\mathfrak{A}$$
$$(4\alpha - 1)\mathfrak{F} = \frac{2(2n + \mathfrak{C})}{nn}\mathfrak{B} + \frac{\mathfrak{A}(2\kappa D + \mathfrak{D})}{nn} + \frac{\mathfrak{D}(2\kappa A + \mathfrak{A})}{nn} - \frac{3(2A + D)}{2nn}$$
$$(4\alpha + 1)\mathfrak{G} = \frac{2(2n + \mathfrak{C})}{nn}\mathfrak{B} + \frac{\mathfrak{A}(2\kappa E + \mathfrak{E})}{nn} + \frac{\mathfrak{E}(2\kappa A + \mathfrak{A})}{nn} - \frac{3(2A + E)}{2nn}$$
$$2\alpha\mathfrak{a} = -\tfrac{15}{4} + \frac{(\mathfrak{D} + \mathfrak{E})(2n + \mathfrak{C})}{nn} + \frac{2\mathfrak{b}(2\kappa A + \mathfrak{A})}{nn}$$
$$+ \frac{2\mathfrak{F}(2\kappa D + \mathfrak{D})}{nn} + \frac{\mathfrak{D}(2\kappa F + \mathfrak{F})}{nn}$$

$$4\alpha\mathfrak{b} = -\frac{3D}{nn} + \frac{2(\mathfrak{F}+\mathfrak{G})(2n+\mathfrak{C})}{nn} + \frac{\mathfrak{E}(2\kappa D+\mathfrak{D})}{nn} + \frac{\mathfrak{a}(2\kappa A+\mathfrak{A})}{nn} + \frac{\mathfrak{D}(2\kappa E+\mathfrak{E})}{nn}$$

$$2\mathfrak{H} = -\frac{3(2D+J)}{2nn} + \frac{\mathfrak{E}(2\kappa D+\mathfrak{D})}{nn} + \frac{\mathfrak{A}(2\kappa J+\mathfrak{J})}{nn} - \frac{(\mathfrak{J}-\mathfrak{K})(2\kappa A+\mathfrak{A})}{nn}$$

$$2(\alpha-1)\mathfrak{J} = -\frac{15}{4} - \frac{3(H-L)}{2nn} + \frac{3\mathfrak{A}}{2n} + \frac{\mathfrak{A}(2\kappa H+\mathfrak{H})}{nn} + \frac{\mathfrak{D}(2n+\mathfrak{C})}{nn} + \frac{2\mathfrak{L}(2\kappa A+\mathfrak{A})}{nn}$$

$$2(\alpha+1)\mathfrak{K} = -\frac{15}{4} - \frac{3H}{2nn} + \frac{3\mathfrak{A}}{2n} + \frac{\mathfrak{A}(2\kappa H+\mathfrak{H})}{nn} + \frac{\mathfrak{E}(2n+\mathfrak{C})}{nn} + \frac{2\mathfrak{G}(2\kappa D+\mathfrak{D})}{nn}$$

$$2(2\alpha-1)\mathfrak{L} = -\frac{3(2D+J)}{2nn} + \frac{3\mathfrak{B}}{n} + \frac{2\mathfrak{B}(2\kappa H+\mathfrak{H})}{nn} + \frac{2\mathfrak{F}(2n+\mathfrak{C})}{nn} + \frac{\mathfrak{A}(2\kappa J+\mathfrak{J})}{nn} + \frac{\mathfrak{D}(2\kappa D+\mathfrak{D})}{nn} + \frac{\mathfrak{J}(2\kappa A+\mathfrak{A})}{nn}$$

$$2(2\alpha+1)\mathfrak{M} = \ldots \frac{3\mathfrak{B}}{n} + \frac{2\mathfrak{B}(2\kappa H+\mathfrak{H})}{nn} + \frac{2\mathfrak{G}(2n+\mathfrak{C})}{nn} + \frac{\mathfrak{K}(2\kappa A+\mathfrak{A})}{nn}$$

§. 103. Antequam vlterius progrediamur, ſequentes notandae ſunt nouae denominationes

$$A' = 2\alpha A + \frac{A(2\kappa B+\mathfrak{B})}{nn} - \frac{B(2\kappa A+\mathfrak{A})}{nn}$$

$$B' = 4\alpha B - \frac{A(2\kappa A+\mathfrak{A})}{nn}$$

$$C' = \frac{\mathfrak{A}(D-E) - A(\mathfrak{D}-\mathfrak{E})}{nn}$$

$$D' = (2\alpha-1)D - \frac{(2n+\mathfrak{C})}{nn}A$$

E′ =

$$E' = (2\alpha+1)E - \frac{(2n+\mathfrak{C})}{nn}A$$

$$F' = (4\alpha-1)F - \frac{4B}{n} - \frac{A(2\kappa D+\mathfrak{D}) - D(2\kappa A+\mathfrak{A})}{nn}$$

$$G' = (4\alpha+1)G - \frac{4B}{n} - \frac{A(2\kappa E+\mathfrak{E}) - E(2\kappa A+\mathfrak{A})}{nn}$$

$$a' = 2\alpha a - \frac{2b(2\kappa A+\mathfrak{A})}{nn} - \frac{(D+E)(2n+\mathfrak{C})}{nn} - \frac{2F(2\kappa D+\mathfrak{D})}{nn} + \frac{D(2\kappa F+\mathfrak{F})}{nn}$$

$$b' = 4\alpha b - \frac{a(2\kappa A+\mathfrak{A})}{nn} - \frac{A(2\kappa a+\mathfrak{a})}{nn} - \frac{D(2\kappa E+\mathfrak{E})}{nn} - \frac{E(2\kappa D+\mathfrak{D})}{nn} - \frac{2(F+G)(2n+\mathfrak{C})}{nn}$$

$$H' = 2H + \frac{D\mathfrak{E} - \mathfrak{D}E - A(\mathfrak{J}-\mathfrak{K}) + \mathfrak{A}(J-K)}{nn}$$

$$J' = 2(\alpha-1)J + \frac{3A}{2n} - \frac{A(2\kappa H+\mathfrak{H})}{nn} - \frac{D(2n+\mathfrak{C})}{nn} - \frac{2\mathfrak{L}(2\kappa A+\mathfrak{A})}{nn} + \frac{A(2\kappa L+\mathfrak{L})}{nn}$$

$$K' = 2(\alpha+1)K - \frac{3A}{n} - \frac{A(2\kappa H+\mathfrak{H})}{nn} - \frac{E(2n+\mathfrak{C})}{nn} - \frac{2G(2\kappa D+\mathfrak{D})}{nn}$$

$$L' = (2\alpha-1)L - \frac{3B}{n} - \frac{2B(2\kappa H+\mathfrak{H})}{nn} - \frac{2F(2n+\mathfrak{C})}{nn} - \frac{D(2\kappa D+\mathfrak{D})}{nn} - \frac{A(2\kappa J+\mathfrak{J})}{nn} - \frac{J(2\kappa A+\mathfrak{A})}{nn}$$

$$M' = 2(\alpha+1)M - \frac{3B}{n} - \frac{2B(2\kappa H+\mathfrak{H})}{nn} - \frac{2G(2n+\mathfrak{C})}{nn} - \frac{E(2\kappa E+\mathfrak{E})}{nn}$$

§. 104. Nunc vt terminos completos obtineamus, ſaltem eos qui angulos 2η et r inuoluunt, notandum

 eſt

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eſt in noſtris aequationibus ſin 2η et coſ 2η non per $\frac{3}{2}(1+2kk)$ ſed per $\frac{3}{2}(1+2kk+\frac{9}{2}ee)$ eſſe multiplicatos. Hinc cum ſit fere $\frac{9}{4}ee = \frac{1}{4}kk$, loco kk hic ſcribi oportebit $\frac{5}{4}kk$, vnde in valore ipſius a pro 3 ſcripſi $3\cdot\frac{5}{4}$ ſeu $\frac{15}{4}$. Deinde vt in his terminis quoque rationem habeamus inclinationis orbitae, cuius medius valor ſit $= \varepsilon$, ponamus $\frac{3}{4}(nn+2+3\mu+\gamma)\,\mathrm{tang}\,\varepsilon^2 = f = \frac{3}{4}(\frac{3}{2}\kappa\kappa nn - \frac{1}{2}nn - \frac{1}{4} + \gamma - \frac{3}{4}\delta)\,\mathrm{tang}\,\varepsilon^2$ ob $\mu = \frac{1}{2}(\kappa\kappa-1)nn - \frac{3}{4} - \frac{1}{4}\delta$, eritque noſtra aequatio:

$$\frac{ddv}{dr^2} = \tfrac{1}{2}\delta - \gamma + \tfrac{1}{4}kk - \gamma k \cos r + \tfrac{3}{2}kk\cos 2r + \tfrac{3}{2}\cos 2\eta + \tfrac{15}{4}kk\cos 2\eta$$

$$+ f + \tfrac{1}{2}fkk + fk\cos r + \tfrac{1}{2}fkk\cos 2r$$

$$+ 3k\cos(2\eta - r) + 3k\cos(2\eta + r) + \tfrac{15}{4}kk\cos(2\eta - 2r) + \tfrac{15}{4}kk\cos(2\eta + 2r)$$

$$- 2n\int R\,dr - v\left(1 - \tfrac{3}{2}kk + \frac{2f}{nn} + \frac{fkk}{nn} - \frac{(2\gamma - \frac{3}{2}\delta)}{nn}\right)$$

$$+ v\left(3\kappa\kappa + \frac{3}{2nn} - \frac{2f}{nn} + \frac{(2\gamma - \frac{3}{2}\delta)}{nn}\right)k\cos r$$

$$+ v\left(\tfrac{3}{2} - \frac{f}{nn}\right)k^2 \cos 2r$$

$$+ v\left(\frac{3}{2nn}\cos 2\eta + \frac{3k}{nn}\cos(2\eta - r) + \frac{3k}{nn}\cos(2\eta + r)\right)$$

$$+ \frac{1}{nn}\left(\int R\,dr\right)^2 + \frac{6v}{nn}\int R\,dr + \frac{3vv}{nn} - \frac{3vv}{nn}k\cos r$$

§. 105. Sit breuitatis gratia:

$$1 + \frac{2f}{nn} - \frac{(2\gamma - \frac{3}{2}\delta)}{nn} = g;\quad 3\kappa\kappa + \frac{3}{2nn} - \frac{2f}{nn} + \frac{(2\gamma - \frac{3}{2}\delta)}{nn} = b$$

et $1 - \frac{3}{2}kk + \frac{2f}{nn} + \frac{fkk}{nn} - \frac{(2\gamma - \frac{3}{2}\delta)}{nn} = \mathfrak{G}$, quo

termino

termino in angulis ex 2η et r compoſitis vtemur: eritque $\frac{ddv}{dr^2} =$

$$\tfrac{1}{2}\delta - \gamma + \tfrac{1}{4}kk + f + \tfrac{1}{2}fkk + \frac{3A}{4nn} + \frac{3akk}{4nn}$$

$$+ \frac{3Dkk}{2nn} + \frac{3Ekk}{2nn} + \frac{\mathfrak{A}\mathfrak{A}}{2nn} + \frac{\mathfrak{C}\mathfrak{C}kk}{2nn} + \frac{\mathfrak{D}\mathfrak{D}kk}{2nn} + \frac{\mathfrak{E}\mathfrak{E}kk}{2nn} - \frac{3ADkk}{2nn}$$

$$+ \frac{3A\mathfrak{A}}{nn} + \frac{3D\mathfrak{D}kk}{nn} + \frac{3AA}{2nn} + \frac{3DDkk}{2nn} + \frac{3EEkk}{2nn}$$

$$+ \operatorname{coſ} 2\eta\,(\tfrac{3}{2} - 2\kappa\mathfrak{A} - \text{ϐ}A)$$

$$+ kk \operatorname{coſ} 2\eta \left\{ \tfrac{15}{4} - 2\kappa\mathfrak{a} - \text{ϐ}a + \tfrac{1}{2}bD + \tfrac{1}{2}bE + \frac{\mathfrak{C}D}{nn} + \frac{3\mathfrak{C}E}{nn} \right.$$

$$+ \operatorname{coſ} 4\eta \left\{ -2\kappa\mathfrak{B} - \text{ϐ}B + \frac{3A}{4nn} + \frac{\mathfrak{A}\mathfrak{A}}{2nn} + \frac{3A\mathfrak{A}}{nn} + \frac{3AA}{2nn} \right.$$

$$+ kk \operatorname{coſ} 4\eta \left\{ \begin{array}{l} -2\kappa\mathfrak{b} - \text{ϐ}\mathfrak{b} + \tfrac{1}{2}bF + \tfrac{1}{2}bG + \frac{3E}{2nn} + \frac{3D}{2nn} \\ + \frac{\mathfrak{A}a}{nn} + \frac{\mathfrak{D}\mathfrak{E}}{nn} + \frac{3DE}{nn} + \frac{3Aa}{nn} + \frac{3A\mathfrak{a}}{nn} + \frac{3\mathfrak{A}a}{nn} \end{array} \right.$$

$$+ k \operatorname{coſ} r \left\{ \begin{array}{l} -\gamma + f - 2\kappa\mathfrak{C} + \frac{3D}{4nn} + \frac{3E}{4nn} + \frac{3A}{nn} + \frac{\mathfrak{A}\mathfrak{D}}{nn} + \frac{\mathfrak{A}\mathfrak{E}}{nn} \\ + \frac{3A\mathfrak{D}}{nn} + \frac{3A\mathfrak{E}}{nn} + \frac{3\mathfrak{A}D}{nn} + \frac{3\mathfrak{A}E}{nn} + \frac{3AD}{nn} + \frac{3AE}{nn} - \frac{3AA}{2nn} \end{array} \right.$$

$$+ k \operatorname{coſ}(2\eta - r) \left\{ 3 - 2\kappa\mathfrak{D} - \text{ϐ}D + \tfrac{1}{2}bA + \frac{3F}{4nn} + \frac{\mathfrak{A}\mathfrak{C}}{nn} + \frac{3A\mathfrak{C}}{nn} \right.$$

$$+ k \operatorname{coſ}(2\eta + r) \left\{ 3 - 2\kappa\mathfrak{E} - \text{ϐ}E + \tfrac{1}{2}bA + \frac{3G}{4nn} + \frac{\mathfrak{A}\mathfrak{C}}{nn} + \frac{3A\mathfrak{C}}{nn} \right.$$

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$$+ kk\,\mathrm{co\text{ſ}}\,2r \left\{ \begin{array}{l} \frac{1}{2} + \frac{1}{2}f - 2\varkappa\mathfrak{H} - \beta H + \frac{3J}{4nn} + \frac{3K}{4nn} + \frac{3E}{2nn} + \frac{3D}{2nn} \\ + \frac{\mathfrak{C}\mathfrak{C}}{nn} + \frac{\mathfrak{D}\mathfrak{E}}{nn} + \frac{3D\mathfrak{E}}{nn} + \frac{3\mathfrak{D}E}{nn} + \frac{3DE}{nn} - \frac{3AD}{2nn} \end{array} \right.$$

$$+ kk\,\mathrm{co\text{ſ}}(2\eta - 2r) \left\{ \begin{array}{l} \frac{15}{4} - 2\varkappa\mathfrak{J} - \beta J + \frac{1}{2}bD + \left(\frac{3}{4} - \frac{f}{2nn}\right)A \\ + \frac{3H}{4nn} + \frac{\mathfrak{A}\mathfrak{H}}{nn} + \frac{3A\mathfrak{H}}{nn} + \frac{3\mathfrak{C}D}{nn} \end{array} \right.$$

$$+ kk\,\mathrm{co\text{ſ}}(2\eta + 2r) \left\{ \begin{array}{l} \frac{15}{4} - 2\varkappa\mathfrak{K} - \beta K + \frac{1}{2}bE + \left(\frac{3}{4} - \frac{f}{2nn}\right)A \\ + \frac{3H}{4nn} + \frac{\mathfrak{A}\mathfrak{H}}{nn} + \frac{3A\mathfrak{H}}{nn} + \frac{3\mathfrak{C}E}{nn} \end{array} \right.$$

$$+ kk\,\mathrm{co\text{ſ}}(4\eta - 2r) \left\{ \begin{array}{l} - 2\varkappa\mathfrak{L} - \beta L + \left(\frac{3}{4} - \frac{f}{2nn}\right)B \\ + \frac{3J}{4nn} + \frac{3D}{2nn} + \frac{\mathfrak{D}\mathfrak{D}}{2nn} + \frac{3DD}{2nn} + \frac{1}{2}bF \end{array} \right.$$

$$+ kk\,\mathrm{co\text{ſ}}(\eta 4 + 2r) \left\{ \begin{array}{l} - 2\varkappa\mathfrak{M} - \beta M + \left(\frac{3}{4} - \frac{f}{2nn}\right)B \\ + \frac{3K}{4nn} + \frac{3E}{2nn} + \frac{\mathfrak{E}\mathfrak{E}}{2nn} + \frac{3EE}{2nn} + \frac{1}{2}bG \end{array} \right.$$

$$+ k\,\mathrm{co\text{ſ}}(4\eta - r) \left\{ \begin{array}{l} - 2\varkappa\mathfrak{F} - \beta F + \frac{1}{2}hB + \frac{3D}{4nn} + \frac{3A}{2nn} \\ + \frac{\mathfrak{A}\mathfrak{D}}{nn} + \frac{3A\mathfrak{D}}{nn} + \frac{3\mathfrak{A}D}{nn} + \frac{3AD}{nn} - \frac{3AA}{4nn} \end{array} \right.$$

$$+ k\,\mathrm{co\text{ſ}}(4\eta + r) \left\{ \begin{array}{l} - 2\varkappa\mathfrak{G} - \beta G + \frac{1}{2}bB + \frac{3E}{4nn} + \frac{3A}{2nn} \\ + \frac{\mathfrak{A}\mathfrak{E}}{nn} + \frac{3A\mathfrak{E}}{nn} + \frac{3\mathfrak{A}E}{nn} + \frac{3AE}{nn} - \frac{3AA}{4nn} \end{array} \right.$$

§. 106.

§. 106. Hinc denique naſcentur ſequentes aequalitates.

I. $$\tfrac{1}{2}\delta-\gamma+\tfrac{1}{4}kk+\tfrac{1}{2}fkk+\frac{3A}{4nn}+\frac{\mathfrak{A}\mathfrak{A}}{2nn}+\frac{3A\mathfrak{A}}{nn}+\frac{3\cdot AA}{2nn}$$
$$+\frac{3(D+E)}{nn}kk-\frac{3ADkk}{2nn}+\frac{\mathfrak{C}\mathfrak{C}kk}{2nn}+\frac{(\mathfrak{D}\mathfrak{D}+\mathfrak{E}\mathfrak{E})}{2nn}kk$$
$$+\frac{3D\mathfrak{D}}{nn}kk+\frac{3(DD+EE)}{2nn}kk+\frac{\mathfrak{A}\mathfrak{a}}{nn}kk$$
$$+\frac{3Aa}{nn}kk+\frac{3\mathfrak{A}a}{nn}kk+\frac{3A\mathfrak{a}}{nn}kk=\frac{A'(2nA+\mathfrak{A})}{nn}+\frac{a'(2nA+\mathfrak{A})}{nn}kk$$
$$+\frac{D'(2nD+\mathfrak{D})}{nn}kk+\frac{A'(2na+\mathfrak{a})}{nn}kk$$

II. $$\tfrac{3}{2}-2n\mathfrak{A}-6A=-2\alpha A'+\frac{A'(2nB+\mathfrak{B})}{nn}+\frac{2B'(2nA+\mathfrak{A})}{nn}$$

III. $$-2n\mathfrak{B}-6B+\frac{3A}{4nn}+\frac{\mathfrak{A}\mathfrak{A}}{2nn}+\frac{3A\mathfrak{A}}{nn}+\frac{3AA}{2nn}$$
$$=-4\alpha B'+\frac{A'(2nA+\mathfrak{A})}{nn}$$

IV. $$\tfrac{\varsigma}{4}-2n\mathfrak{a}-6a+\tfrac{1}{2}b(D+E)+\frac{3\mathfrak{C}(D+E)}{nn}=-2\alpha a'$$
$$+\frac{(D'+E')(2n+\mathfrak{C})}{nn}+\frac{2b'(2nA+\mathfrak{A})}{nn}+\frac{2F'(2nD+\mathfrak{D})}{nn}$$

V. $$-2n\mathfrak{b}-6b+\tfrac{1}{2}b(F+G)+\frac{3(D+E)}{nn}+\frac{3D(E+J)}{nn}+\frac{3Aa}{nn}=-4\alpha b'$$
$$+\frac{a'(2nA+\mathfrak{A})}{nn}+\frac{E'(2nD+\mathfrak{D})}{nn}+\frac{2(F'+G')(2n+\mathfrak{C})}{nn}+\frac{D'(2nE+\mathfrak{E})}{nn}$$

VI.

VI. $-\gamma + f - 2\varkappa\mathfrak{C} + \frac{3(D+E)}{4nn} + \frac{3A}{nn} - \frac{3AA}{2nn} + \frac{\mathfrak{A}(\mathfrak{D}+\mathfrak{E})}{nn}$

$+ \frac{3A(\mathfrak{D}+\mathfrak{E})}{nn} + \frac{3A(D+E)}{nn} + \frac{3\mathfrak{A}(D+E)}{nn} =$

$-C' + \frac{A'(2\varkappa D+\mathfrak{D})}{nn} + \frac{A'(2\varkappa E+\mathfrak{E})}{nn} + \frac{(D'+E')(2\varkappa A+\mathfrak{A})}{nn}$

VII. $3 - 2\varkappa\mathfrak{D} - \beta D + \tfrac{1}{2} b A + \frac{3F}{4nn} + \frac{\mathfrak{C}(\mathfrak{A}+3A)}{nn} =$

$-(2\alpha-1)D' + \frac{A'(2n+\mathfrak{C})}{nn}$

VIII. $3 - 2\varkappa\mathfrak{E} - \beta E + \tfrac{1}{2} b A + \frac{3G}{4nn} + \frac{\mathfrak{C}(\mathfrak{A}+3A)}{nn} =$

$-(2\alpha+1)E' + \frac{A'(2n+\mathfrak{C})}{nn}$

IX. $-3\varkappa\mathfrak{F} - \beta F + \tfrac{1}{2} b B + \frac{3A}{2nn} + \frac{3D}{4nn} + \frac{\mathfrak{A}\mathfrak{D}}{nn} + \frac{3A\mathfrak{D}}{nn}$

$+ \frac{3\mathfrak{A}D}{nn} + \frac{3AD}{nn} - \frac{3AA}{4nn} = -(4\alpha-1)F + \frac{4B'}{n}$

$+ \frac{A'(2\varkappa D+\mathfrak{D})}{nn} + \frac{D'(2\varkappa A+\mathfrak{A})}{nn}$

X. $-2\varkappa\mathfrak{G} - \beta G + \tfrac{1}{2} b B + \frac{3A}{2nn} + \frac{3E}{4nn} + \frac{\mathfrak{A}\mathfrak{E}}{nn}$

$+ \frac{3A\mathfrak{E}}{nn} + \frac{3\mathfrak{A}E}{nn} + \frac{3AE}{nn} - \frac{3AA}{4nn} = -(4\alpha+1)G' + \frac{4B'}{n}$

$+ \frac{A'(2\varkappa E+\mathfrak{E})}{nn} + \frac{E'(2\varkappa A+\mathfrak{A})}{nn}$

XI.

$$\text{XI.}\quad \tfrac{3}{8}+\tfrac{1}{2}f-2\varkappa\mathfrak{H}-\beta H+\frac{3(D+E)}{2nn}+\frac{3(J+K)}{4nn}$$
$$-\frac{3AD}{2nn}+\frac{\mathfrak{C}\mathfrak{C}}{2nn}-\frac{3AE}{2nn}+\frac{\mathfrak{D}\mathfrak{E}}{nn}+\frac{3\mathfrak{D}E}{nn}+\frac{3D\mathfrak{E}}{nn}+\frac{3DE}{nn}$$
$$+\frac{\mathfrak{A}\mathfrak{J}}{nn}+\frac{3\mathfrak{A}J}{nn}+\frac{3A\mathfrak{J}}{nn}+\frac{3AJ}{nn}=-2H'+\frac{A'(2\varkappa J+\mathfrak{J})}{nn}$$
$$+\frac{E'(2\varkappa D+\mathfrak{D})}{nn}+\frac{D'(2\varkappa E+\mathfrak{E})}{nn}+\frac{(J'+K')(2\varkappa A+\mathfrak{A})}{nn}$$

$$\text{XII.}\quad \tfrac{1}{4}s-2\varkappa\mathfrak{J}-\beta J+\tfrac{1}{2}bD+\left(\tfrac{3}{4}-\frac{f}{2nn}\right)A+\frac{3H}{4nn}$$
$$+\frac{\mathfrak{H}(\mathfrak{A}+3A)}{nn}+\frac{3\mathfrak{C}D}{nn}=-2(\alpha-1)J'+\frac{3A'}{2n}$$
$$+\frac{D'(2n+\mathfrak{C})}{nn}+\frac{A'(2\varkappa H+\mathfrak{H})}{nn}+\frac{2L'(2\varkappa A+\mathfrak{A})}{nn}$$

$$\text{XIII.}\quad \tfrac{1}{4}s-2\varkappa\mathfrak{K}-\beta K+\tfrac{1}{2}bE+\left(\tfrac{3}{4}-\frac{f}{2nn}\right)A+\frac{3H}{4nn}$$
$$+\frac{\mathfrak{H}(\mathfrak{A}+3A)}{nn}+\frac{3\mathfrak{C}E}{nn}=-2(\alpha+1)K'+\frac{3A'}{2n}$$
$$+\frac{E'(2n+\mathfrak{C})}{nn}+\frac{A'(2\varkappa H+\mathfrak{H})}{nn}+\frac{2G'(2\varkappa D+\mathfrak{D})}{nn}$$

$$\text{XIV.}\quad -2\varkappa\mathfrak{C}-\beta L+\tfrac{1}{2}bF+\left(\tfrac{3}{4}-\frac{f}{2nn}\right)B-\frac{3AD}{2nn}$$
$$+\frac{3D}{2nn}+\frac{3J}{4nn}+\frac{\mathfrak{D}\mathfrak{D}}{2nn}+\frac{3\mathfrak{D}D}{nn}+\frac{3DD}{2nn}=-2(2\alpha-1)L'+\frac{3B'}{n}$$
$$+\frac{2F'(2n+\mathfrak{C})}{nn}+\frac{A'(2\varkappa J+\mathfrak{J})}{nn}+\frac{J'(2\varkappa A+\mathfrak{A})}{nn}+\frac{D'(2\varkappa D+\mathfrak{D})}{nn}$$

$$\text{XV.}\quad -2\varkappa\mathfrak{M} - \mathfrak{E}M + \tfrac{1}{2}bG + \left(\tfrac{3}{4} - \frac{f}{2nn}\right)B - \frac{3AE}{2nn}$$
$$+\frac{3E}{2nn} + \frac{3K}{4nn} + \frac{\mathfrak{C}\mathfrak{C}}{2nn} + \frac{3\mathfrak{C}E}{nn} + \frac{3EE}{2nn} =$$
$$-2(2a+1)M' + \frac{3B'}{n} + \frac{2G'(2n+\mathfrak{C})}{nn}.$$

§. 107. Nunc antequam hos valores inuenire queamus, verus valor ipſius α inueſtigari debet: quod fiet ex valore integrali ipſius φ, qui ſi vti §. 98. ponatur

$$\varphi = Or + \mathfrak{A}' \sin 2\eta + a'kk \sin 2\eta + \text{etc. obtinebitur.}$$

$$\varkappa + \frac{A(3\varkappa A + 2\mathfrak{A})}{2n^4} + \frac{D(3\varkappa D + 2\mathfrak{D})}{2n^4}kk + \frac{A(3\varkappa a + \mathfrak{a})}{2n^4}kk + \frac{a(3\varkappa A + 2\mathfrak{A})}{2n^4}kk =$$
$$= O - (\mathfrak{A}' + a'kk)\frac{(2\varkappa A + \mathfrak{A})}{nn} - \frac{\mathfrak{D}'(2\varkappa D + \mathfrak{D})}{nn}kk = \alpha + \frac{1}{n}$$

vbi ex obſeruationibus conſtat eſſe $O = 1,0085272$ Proxime autem eſſe ſupra inuenimus eſſe:

$\mathfrak{A}' = 0,01$	$\mathfrak{A} = -0,80$	$A = -1,25$
$a' = 0,05$	$\mathfrak{a} = -2,05$	$a = -12,60$
$\mathfrak{D}' = -0,44$	$\mathfrak{D} = -3,60$	$D = 34,25$

atque $\varkappa = 1,0085$; $kk = 0,003$; $nn = 175,71795$

vnde inuenimus $O + 0,000649 = \alpha + \frac{1}{n} = \varkappa + 0,000285$

§. 108. Cum nunc ſit $O = 1,0085272$, erit $\alpha + \frac{1}{n} = 1,009176$, et ob $\frac{1}{n} = 0,075438$, habebitur verus valor:

$$\alpha = 0,933738 \quad \text{et} \quad l\alpha = 9,9702255$$
atque $\varkappa = 1,008991$ et $l\varkappa = 0,0038874$

Hinc

Hinc iam primo obtinemus:

$$\mathfrak{A} = -0,80313 \qquad l{-}\mathfrak{A} = 9,9047898$$

Deinde cum ſit ſatis prope $\mathfrak{C} = -0,67465$, erit

$$\frac{(2n+\mathfrak{C})}{nn} = 0,147037 \text{ et } l\frac{2n+\mathfrak{C}}{nn} = 9,1674260$$

$$\mathfrak{D} = -3,593620 \;\ldots\; l{-}\mathfrak{D} = 0,5555310$$

$$\mathfrak{E} = -1,087320 \;\ldots\; l{-}\mathfrak{E} = 0,0363580$$

atque porro ex valore ipſius A proxime cognito erit

$$\mathfrak{B} = +0,006967 \;\ldots\; l\,\mathfrak{B} = 7,8430540$$

et quia eſt ſatis prope $B = 0,0128$, erit $A' = 2\alpha A - 0,000720$ et $B' = 4\alpha B - 0,023926$, vnde fit;

$$\tfrac{1}{4} + 1,62172 - \varepsilon A = -4\alpha\alpha A + 0,00144\alpha + 0,000374\alpha A$$
$$- 0,152\,\alpha B + 0,00091$$

$$+ 0,01247 - \varepsilon B = -16\alpha\alpha B + 0,09570\alpha - 0,038032\alpha A$$
$$+ 0,000014$$

§. 109. Nunc primum quaeri debent valores litterarum *f*, *b* et *b*: et cum ſit $\epsilon = 5°, 9'$ et $2\gamma - \tfrac{1}{2}\delta = -1,3899$ proxime, reperietur

$$f = 1,093757 \quad \text{et} \quad lf = 0,0389208$$

$$b = 3,0423 \;\ldots\; lb = 0,4832020$$

$$b = 1,01591 \;\ldots\; lb = 0,0068560$$

hincque erit

$$2,4720\, A = -3,11947 - 0,142\, B$$

$$12,9369\, B = +0,07684 - 0,0355\, A$$

unde concluditur fore:

$$A = -1,262463 \;\ldots\; l{-}A = 0,1012186$$

$$B = +0,009404 \;\ldots\; l\,B = 7,9733114$$

Porro vero eſt

$$D' = 0,867676\ D + 0,185628$$

$$E' = 2,867676\ E + 0,185628$$

et $A' = -2,35859 \quad . \quad . \quad l\text{-}A' = 0,3726530$

§. 110. Ex his valoribus aequationes VII et VIII induent has formas,

$$3 + 7,25185 - 6D - 1,92040 - 0,00244 + 0,01704 = -0,75286\ D - 0,16106 - 0,34680$$

$$3 + 2,19420 - 6E - 1,92040 + 0,00006 + 0,01704 = -8,22357\ E - 0,53232 - 0,34680$$

vnde prodibit

$$D = +33,6600 \quad . \quad . \quad . \quad l\,D = 1,5271130$$

$$E = -0,5785 \quad . \quad . \quad . \quad l\text{-}E = 9,7623410$$

ergo

$$D' = 29,39153$$

$$E' = -1,47347$$

Ex his nanciſcemur ſequentes formulas pro calculo ſequenti

$$\frac{2\varkappa A + \mathfrak{A}}{nn} = -0,01884 \qquad l\text{-}\frac{(2\varkappa A + \mathfrak{A})}{nn} = 8,275051$$

$$\frac{2\varkappa B + \mathfrak{B}}{nn} = +0,000148 \qquad l\,\frac{2\varkappa B + \mathfrak{B}}{nn} = 6,170262$$

$$\frac{2\varkappa D + \mathfrak{D}}{nn} = +0,36611 \qquad l\,\frac{2\varkappa D + \mathfrak{D}}{nn} = 9,563604$$

$$\frac{2\varkappa E + \mathfrak{E}}{nn} = -0,01283 \qquad l\text{-}\frac{(2\varkappa E + \mathfrak{E})}{nn} = 8,108292$$

§. III.

§. 111. Ex his iam porro inuenitur

$\mathfrak{C} = -0,64383$ . . . $l\text{-}\mathfrak{C} = 9,808771$

atque $C' = -0,13847$

Porro valores litterarum $\mathfrak{F}$ et $\mathfrak{G}$ determinabuntur per has aequationes

$$(4\alpha-1)\,\mathfrak{F} = 0,002049 - 0,294032 + 0,067699 + 0,021554 - 0,287335$$

$$(4\alpha+1)\,\mathfrak{G} = 0,002049 + 0,010306 + 0,020484 + 0,021554 + 0,004939$$

ex quibus reperitur

$\mathfrak{F} = -0,17957$ . . . $l\text{-}\mathfrak{F} = 9,254241$

$\mathfrak{G} = +0,01253$ . . . $l\,\mathfrak{G} = 8,097936$

atque

$$F' = (4\alpha-1)\,F - 0,00283 + 0,46219 + 0,63411 = (4\alpha-1)\,F + 1,09347$$

$$G' = (4\alpha+1)\,G - 0,00283 - 0,01620 - 0,01090 = (4\alpha+1)\,G - 0,02993$$

vnde aequationes IX et X prodibunt.

$$+0,36238 - 1,01591\,F + 0,00353 - 0,00680 - 1,00349 = -(4\alpha-1)^2\,F - 2,98415 - 0,86349 + 0,00337 - 0,55370$$

$$-0,02528 - 1,01591\,G + 0,00353 - 0,00680 + 0,04634 = -(4\alpha+1)^2\,G + 0,14473 + 0,03027 + 0,00337 + 0,02776$$

ſeu $6,43486\,F = -3,75359$

$21,40769\,G = +0,18534$

§. 110. Hinc prodeunt ſequentes valores correcti pro F et G,

$F = -0,58360$ . . . $l\text{-}F = 9,766112$

$G = +0,00866$ . . . $l\,G = 7,937400$

Ex formula autem ſexta hinc leui calculo colligitur fore:

$$\gamma - f = 1,58161 \quad \text{et} \quad \gamma = 2,67537$$

Valores autem ex F et G deriuati erunt

$$F' = -0,49919 \quad \text{et} \quad G' = 0,01107$$

$$\frac{2\kappa F + \mathfrak{F}}{nn} = -0,00772 \quad l - \frac{(2\kappa F + \mathfrak{F})}{nn} = 7,887828$$

$$\frac{2\kappa G + \mathfrak{G}}{nn} = 0,00017 \quad l\,\frac{2\kappa G + \mathfrak{G}}{nn} = 6,232305$$

§. 113. Nunc procedamus ad valores litterarum $\mathfrak{a}$ et $\mathfrak{b}$ qui erunt

$$1,867676\,\mathfrak{a} = -3,75000 - 0,68827 - 0,13148 - 0,03764\,\mathfrak{b} + 0,02776$$

$$3,735352\,\mathfrak{b} = -0,57467 - 0,04913 - 0,39807 - 0,01884\,\mathfrak{a} + 0,04611$$

vnde reperitur:

$$\mathfrak{a} = -2,42686 \quad \ldots \quad l-\mathfrak{a} = 0,385044$$

$$\mathfrak{b} = -0,24899 \quad \ldots \quad l-\mathfrak{b} = 9,396182$$

hincque porro

$$a' = 2aa + 0,03768\,b - 4,86420 + 0,16048$$

$$b' = 4ab + 0,01884\,a + 0,16887 + 0,64373 + 0,01450\,a - 0,01744$$

ſeu $a' = 2aa + 0,03768\,b - 4,70372$

$$b' = 4ab + 0,03334\,a + 0,79516$$

§. 114. Aequationes IV et V hinc induent ſequentes formas:

IV.

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$$\text{IV.} + 3{,}75000 - 1{,}01591\,a + 50{,}32200 - 0{,}36363 = -4aaa$$
$$+ 4{,}89730)$$
$$-0{,}07034\,b + 8{,}78034 + 4{,}10500 - 0{,}36551 - 0{,}00125\,a$$
$$-0{,}14067\,b \qquad\qquad -0{,}02996$$

$$\text{V.} + 0{,}50246 - 1{,}01591\,b - 0{,}87350 + 0{,}28239 - 0{,}95732$$
$$-0{,}02155\,a$$
$$= -16aab - 0{,}12447\,a - 2{,}96860 - 0{,}00142\,b$$
$$-0{,}03517\,a + 0{,}17723$$
$$-0{,}14354$$
$$-0{,}91659$$

Hinc fit

$$2{,}47355\,a = -0{,}21101\,b - 46{,}11580$$
$$12{,}93836\,b = -0{,}13809\,a - 2{,}80553$$

et $\quad a = -18{,}64200 \quad . \; . \; . \quad l\text{-}a = 1{,}270493$

$\qquad b = -0{,}01794 \quad . \; . \; . \quad l\text{-}b = 8{,}253822$

ex quibus oriuntur:

$$a' = -39{,}52164 \quad . \; . \; . \quad b' = +0{,}10663$$

et $\quad \frac{2\varkappa a + \mathfrak{a}}{nn} = -0{,}22790 \quad l\text{-}\frac{(2\varkappa a + \mathfrak{a})}{nn} = 9{,}357744$

valor autem ipſius $\frac{2\varkappa b + \mathfrak{b}}{nn}$ nullius plane erit momenti, vnde eum praetermittimus.

§. 115. Ex prima autem aequatione §. 106. colligitur

$$\tfrac{1}{2}\,\delta = \gamma - f + 0{,}02285$$

ſupra autem inuenimus eſſe $\gamma - f = 1{,}58161$, ſicque erit $\frac{1}{2}\,\delta = 1{,}60446$ atque

$$\delta = 3{,}20892 \quad . \; . \; . \quad l\delta = 0{,}506358$$

Nunc cum ſit proxime: $\mathfrak{H} = -0{,}123$; $H = -1{,}033$ ideoque $\frac{2\varkappa H + \mathfrak{H}}{nn} = -0{,}0126$; ob $\mathfrak{L} = -1{,}453$ et $L = +6{,}252$; habebimus

-0,

$$-0,132324\,\mathfrak{J} = -3,75000 + 0,00882 - 0,09088$$
$$+ 0,05336 - 0,01012$$
$$- 0,52839 + 0,05474$$
$$+3,867676\,\mathfrak{K} = -3,75000 + 0,00882 - 0,09088$$
$$+ 0,01012$$
$$- 0,15987 + 0,00917$$

Hinc reperitur

$$\mathfrak{J} = 32,05945 \quad . \quad . \quad . \quad l\,\mathfrak{J} = 1,505956$$
$$\mathfrak{K} = -1,02714 \quad . \quad . \quad . \quad l\text{-}\mathfrak{K} = 0,011629$$

§. 116. Hinc vlterius progrediendo habebimus.

$$J' = 2(\alpha-1)J + 0,28571 - 4,94924 + 0,23502 =$$
$$- 0,01591 \qquad\qquad - 0,08015$$
$$2(\alpha-1)J - 4,52457$$
$$K' = 2(\alpha+1)K + 0,28571 + 0,08507 - 0,00317 =$$
$$- 0,01591$$
$$2(\alpha+1)K + 0,35170$$

vnde aequationes XII et XIII fiunt

$$+3,75000 - 64,69550 - 1,01591\,J + 51,28180$$
$$- 0,94292 + 0,00321 \quad - 0,36999 =$$
$$- \underline{0,00441)}$$
$$-4(\alpha-1)^2 J - 0,59871 - 0,26690 + 4,32164$$
$$+ 0,02972 - 0,47004$$

$$+3,75000 + 2,07275 - 1,01591\,K - 0,88006$$
$$- 0,94292 + 0,00321 \quad + 0,00636 =$$
$$- \underline{0,00441)}$$
$$-4(\alpha+1)^2 K - 1,36026 - 0,26690 - 0,21665$$
$$+ 0,02972 + 0,00810$$

ex quibus colligitur fore

$$J = -14,09600 \quad . \quad . \quad . \quad l\text{-}J = 1,149096$$
$$K = -\ 0,41676 \quad . \quad . \quad . \quad l\text{-}K = 9,619888$$

Hinc

Hinc $J' = -2,65933$ . . $K' = -1,26020$

atque $\frac{2\varkappa J + \mathfrak{J}}{nn} = +0,02057$. $l\frac{2\varkappa J + \mathfrak{J}}{nn} = 8,313172$

$\frac{2\varkappa K + \mathfrak{K}}{nn} = -0,01063$. $l\!-\!\frac{2\varkappa K + \mathfrak{K}}{nn} = 8,026598$

§. 117. Quaeramus iam valorem ipſius $\mathfrak{H}$, ex aequatione

$$2\mathfrak{H} = -0,45434 - 0,39771 - 0,01652 + 0,62330$$

erit $\mathfrak{H} = -0,12264$ . . . $l\!-\!\mathfrak{H} = 9,088632$

hincque reperitur: $H' = 2H + 0,08011$

vnde aequatio XI praebet:

$$\begin{aligned}&2,04688 + 0,24748 - 1,01591\,H + 0,27805 - 3,06194\\&+ 0,36275 + 0,00118 - 0,00623 \quad + 0,02224 + 0,03552\\&- 0,62485 - 0,33247 - 0,83753 \quad + 0,49710 ==\\&- 4\,H - 0,16022 - 0,04851 \quad - 0,53944 - 0,37802\\&\qquad\qquad\qquad\qquad\qquad\qquad + 0,07384\end{aligned}$$

ſeu $2,98409\,H = -2,68053$

Ergo $H = -0,89829$ . . . $l\!-\!H = 9,953417$

$H' = -1,71647$; $\frac{2\varkappa H + \mathfrak{H}}{nn} = -0,01102$

§. 118. Tandem ſuperſunt litterae $\mathfrak{L}$ et $\mathfrak{M}$

$$\begin{aligned}2\,(2\alpha-1)\,\mathfrak{L} &= -0,45434 - 0,05280 - 0,01652 - 1,31570\\&\quad + 0,00158 \qquad\qquad\qquad - 0,60396\\&\quad - 0,00015\end{aligned}$$

$$\begin{aligned}2\,(2\alpha+1)\,\mathfrak{M} &= +0,00158 + 0,00368 + 0,01935\\&\quad - 0,00015\end{aligned}$$

Hinc $\mathfrak{L} = +1,40715$ . . $l\!-\!\mathfrak{L} = 0,148340$

$\mathfrak{M} = +0,00426$ . . $l\mathfrak{M} = 7,629896$

Deinde vero habebitur:

$$\begin{aligned} L' &= 2(2\alpha-1)\,L - 0{,}00213 + 0{,}17162 - 12{,}31170 - 0{,}02596 \\ &\qquad + 0{,}00013 \qquad\qquad\qquad\qquad - 0{,}26555 \\ M' &= 2(2\alpha+1)\,M - 0{,}00213 - 000254 - 0{,}00742 \\ &\qquad + 0{,}00013 \end{aligned}$$

ſeu

$$\begin{aligned} L' &= 2\,(2\alpha-1)\,L - 12{,}43359 \\ M' &= 2\,(2\alpha+1)\,M - 0{,}01196 \end{aligned}$$

§. 119. Nunc denique aggrediamur aequationes XIV et XV

$$\begin{aligned} \text{XIV.}\; & -2{,}83960 - 1{,}01591\,L - 0{,}88774 + 0{,}00702 + 0{,}36275 \\ & +0{,}28733 \qquad\qquad - 0{,}06016 + 0{,}03675 + 7{,}60650 = \\ & -3{,}01144\,L + 21{,}57666 + 0{,}00255 - 0{,}14680 + 10{,}75272 \end{aligned}$$

$$\begin{aligned} \text{XV.}\; & +0{,}00860 - 1{,}01591\,M + 0{,}01317 + 0{,}00702 - 0{,}00623 \\ & -0{,}00494 \qquad\qquad - 0{,}00176 + 0{,}00336 + 0{,}01360 = \\ & -32{,}89415\,M + 0{,}06859 + 0{,}00255 + 0{,}00325 \end{aligned}$$

ex quibus eruitur

$$L = +\;13{,}86720 \;\cdots\; l\,L = 1{,}141988$$

$$M = +\;0{,}00131 \;\cdots\; l\,M = 7{,}117165$$

hincque $L' = 11{,}3090$ et $M' = -\,0{,}00445$

$$\frac{2\varkappa L + \mathfrak{L}}{nn} = 0{,}15125 \;\cdots\; l\frac{2\varkappa L + \mathfrak{L}}{nn} = 9{,}179684$$

$$\frac{2\varkappa M + \mathfrak{M}}{nn} = 0{,}00004 \qquad l\frac{2\varkappa M + \mathfrak{M}}{nn} = 5{,}592770$$

Ex his valoribus nouae correctiones inueniri poſſent, ſed differentiae prodirent tam exiguae, vt operae pretium non ſit eas inueſtigare.

§. 120. His igitur valoribus inuentis, denotante iam a diſtantiam Lunae mediam a Terra, et eius diſtantia curtata $= x$, cum ſit $x = \frac{(1-kk)a}{1-k\cos r}u$, erit:

	log. coefficient:
$u = 1 - 0{,}0074991 \cos 2\eta$	7,875009
$+ 0{,}0000532 \cos 4\eta$	5,725912
$+ 0{,}191557k \cos(2\eta - r)$	9,282297
$- 0{,}003293k \cos(2\eta + r)$	7,517525
$- 0{,}003321k \cos(4\eta - r)$	7,521296
$+ 0{,}000049k \cos(4\eta + r)$	5,692584
$- 0{,}00511kk \cos 2r$	7,708601
$- 0{,}08022kk \cos(2\eta - 2r)$	8,904280
$- 0{,}00237kk \cos(2\eta + 2r)$	7,375072
$+ 0{,}07892kk \cos(4\eta - 2r)$	8,897172
$+ 0{,}00001kk \cos(4\eta + 2r)$	4,872349

vbi quidem in duobus primis terminis ſimul eos, qui per kk erant affecti, ſumus complexi, poſito $k = 0{,}05445$. Etiamſi enim hic valor non omnino eſſet iuſtus, tamen inde in his terminis minimis nullus error naſci poterit.

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§. 121. Porro quoque hinc ex §. 116. valorem ipſius $\frac{d\Phi}{dr}$ determinabimus, quatenus a ſola excentricitate orbitae lunaris pendet.

		log. coeff.
$\frac{d\Phi}{dr} =$	$1,009276$	0,004010
	$+ 0,0195144 \cos 2\eta$	8,290355
	$- 0,0000322 \cos 4\eta$	5,507856
	$- 0,001231k \cos r$	7,090258
	$- 0,366103k \cos(2\eta - r)$	9,563604
	$+ 0,012832k \cos(2\eta + r)$	8,108292
	$+ 0,002829k \cos(4\eta - r)$	7,451633
	$- 0,000171k \cos(4\eta + r)$	6,232305
	$+ 0,01182kk \cos 2r$	8,072618
	$- 0,02057kk \cos(2\eta - 2r)$	8,313172
	$+ 0,01063kk \cos(2\eta + 2r)$	8,026598
	$- 0,09883kk \cos(4\eta - 2r)$	8,994889
	$- 0,00004kk \cos(4\eta + 2r)$	5,592770

§. 122.

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§. 122. Cum nunc ſit $\frac{d\theta}{a\,r} = \frac{ds}{dr} = \frac{1 \mp 2ee}{n} + \frac{2}{n} k \operatorname{coſ} r + \frac{3}{2n} kk \operatorname{coſ} 2r$, erit

		log. coeff.
$\frac{d\eta}{dr} =$	0,933838	9,970272
	$+$ 0,0195144 coſ 2η	8,290355
	$-$ 0,0000322 coſ 4η	5,507856
	$-$ 0,152101 k coſ r	9,182132
	$-$ 0,366103 k coſ $(2\eta - r)$	9,563604
	$+$ 0,012829 k coſ $(2\eta + r)$	8,108292
	$+$ 0,002829 k coſ $(4\eta - r)$	7,451633
	$-$ 0,000171 k coſ $(4\eta + r)$	6,232305
	$-$ 0,10133 kk coſ $2r$	9,005738
	$-$ 0,02057 kk coſ $(2\eta - 2r)$	8,313172
	$+$ 0,01063 kk coſ $(2\eta + 2r)$	8,026598
	$-$ 0,09883 kk coſ $(4\eta - 2r)$	8,994889
	$-$ 0,00004 kk coſ $(4\eta + 2r)$	5,592770

quae formulae ad motum Lunae horarium tam abſolutum quam a ſole adhiberi poſſunt, quemadmodum illa diſtantiam definiens diametro apparenti et parallaxi horizontali inueſtigandae inſervit.

§. 23. Quaeramus nunc valorem integralem pro longitudine Lunae Φ, quatenus a ſola excentricitate orbitae lunaris pendet, ac ponamus.

$$\Phi = \mathrm{O}r + \mathfrak{A}'\sin 2\eta + \mathfrak{a}'kk\sin 2\eta + \mathfrak{B}'\sin 4\eta + \mathfrak{b}'kk\sin 4\eta$$
$$+ \mathfrak{C}'k\sin r + \mathfrak{D}'k\sin(2\eta - r) + \mathfrak{F}'k\sin(4\eta - r)$$
$$+ \mathfrak{E}'k\sin(2\eta + r) + \mathfrak{G}'k\sin(4\eta + r)$$
$$+ \mathfrak{H}'kk\sin 2r + \mathfrak{J}'kk\sin(2\eta - 2r) + \mathfrak{L}'kk\sin(4\eta - 2r)$$
$$+ \mathfrak{K}'kk\sin(2\eta + 2r) + \mathfrak{M}'kk\sin(4\eta + 2r)$$

atque ſequentes obtinebimus formulas:

$$+0,0188387 = 2\alpha\mathfrak{A}' - \frac{\mathfrak{A}'(2\varkappa\mathrm{B} + \mathfrak{B})}{nn} - \frac{2\mathfrak{B}'(2\varkappa\mathrm{A} + \mathfrak{A})}{nn}$$

$$-0,0000370 = 4\alpha\mathfrak{B}' - \frac{\mathfrak{A}'(2\varkappa\mathrm{A} + \mathfrak{A})}{nn}$$

$$-0,001231 = \mathfrak{C}' - \frac{\mathfrak{A}'(2\varkappa\mathrm{D} + \mathfrak{D})}{nn} + \frac{\mathfrak{A}'(2\varkappa\mathrm{E} + \mathfrak{E})}{nn} - \frac{(\mathfrak{D}' + \mathfrak{E}')(2\varkappa\mathrm{A} + \mathfrak{A})}{nn}$$

$$-0,366103 = (2\alpha - 1)\mathfrak{D}' - \frac{\mathfrak{A}'(2n + \mathfrak{C})}{nn}$$

$$+0,012832 = (2\alpha + 1)\mathfrak{E}' - \frac{\mathfrak{A}'(2n + \mathfrak{C})}{nn}$$

$$+0,002829 = (4\alpha - 1)\mathfrak{F}' - \frac{4\mathfrak{B}'}{n} - \frac{\mathfrak{A}'(2\varkappa\mathrm{D} + \mathfrak{D})}{nn} - \frac{\mathfrak{D}'(2\varkappa\mathrm{A} + \mathfrak{A})}{nn}$$

$$-0,000171 = (4\alpha + 1)\mathfrak{G}' - \frac{4\mathfrak{B}'}{n} - \frac{\mathfrak{A}'(2\varkappa\mathrm{E} + \mathfrak{E})}{nn} - \frac{\mathfrak{E}'(2\varkappa\mathrm{A} + \mathfrak{A})}{nn}$$

+

$$+0{,}22790 = 2\alpha\mathfrak{a}' - \frac{(\mathfrak{D}'+\mathfrak{E}')(2n+\mathfrak{C})}{nn} - \frac{2\mathfrak{F}'(2\kappa D+\mathfrak{D})}{nn}$$
$$- \frac{2\mathfrak{b}'(2\kappa A+\mathfrak{A})}{nn}$$

$$+0{,}00163 = 4\alpha\mathfrak{b}' - \frac{\mathfrak{a}'(2\kappa A+\mathfrak{A})}{nn} - \frac{\mathfrak{E}'(2\kappa D+\mathfrak{D})}{nn}$$
$$- \frac{2(\mathfrak{F}'+\mathfrak{G}')(2n+\mathfrak{C})}{nn}$$

$$+0{,}01182 = 2\mathfrak{H}' - \frac{\mathfrak{E}'(2\kappa D+\mathfrak{D})}{nn} - \frac{\mathfrak{A}'(2\kappa J+\mathfrak{J})}{nn}$$
$$- \frac{(\mathfrak{J}'+\mathfrak{K}')(2\kappa A+\mathfrak{A})}{nn} - \frac{\mathfrak{D}'(2\kappa E+\mathfrak{E})}{nn}$$

$$-0{,}02057 = 2(\alpha-1)\mathfrak{J}' - \frac{3\mathfrak{A}'}{2n} - \frac{\mathfrak{D}'(2n+\mathfrak{C})}{nn} - \frac{\mathfrak{A}'(2\kappa H+\mathfrak{H})}{nn}$$
$$- \frac{2\mathfrak{L}'(2\kappa A+\mathfrak{A})}{nn}$$

$$+0{,}01063 = 2(\alpha+1)\mathfrak{K}' - \frac{3\mathfrak{A}'}{2n} - \frac{\mathfrak{E}'(2n+\mathfrak{C})}{nn} - \frac{\mathfrak{A}'(2\kappa H+\mathfrak{H})}{nn}$$
$$- \frac{2\mathfrak{G}'(2\kappa D+\mathfrak{D})}{nn}$$

$$-0{,}09883 = 2(2\alpha-1)\mathfrak{L}' - \frac{3\mathfrak{B}'}{n} + \frac{2\mathfrak{F}'(2n+\mathfrak{C})}{nn} + \frac{\mathfrak{A}'(2\kappa J+\mathfrak{J})}{nn}$$
$$- \frac{\mathfrak{J}'(2\kappa A+\mathfrak{A})}{nn} - \frac{\mathfrak{D}'(2\kappa D+\mathfrak{D})}{nn}$$

$$-0{,}00004 = 2(2\alpha+1)\mathfrak{M}' - \frac{3\mathfrak{B}'}{n} - \frac{2\mathfrak{G}'(2n+\mathfrak{C})}{nn}$$

§. 124.

§. 124. Ex his eliciuntur valores ſequentes:

$\mathfrak{A}' = +0,0100887$	$l\,\mathfrak{A}' = 8,003837$	$\mathfrak{a}' = 0,09140$
$\mathfrak{B}' = -0,0000409$	$l{-}\mathfrak{B}' = 5,611723$	$l\,\mathfrak{a}' = 8,960934$
$\mathfrak{C}' = +0,010146$	$l\,\mathfrak{C}' = 8,006295$	$\mathfrak{b}' = 0,00089$
$\mathfrak{D}' = -0,420226$	$l{-}\mathfrak{D}' = 9,623483$	$l\,\mathfrak{b}' = 6,949340$
$\mathfrak{E}' = +0,004992$	$l\,\mathfrak{E}' = 7,698261$	$\mathfrak{A}' + \mathfrak{a}'kk = 0,0103597$
$\mathfrak{F}' = +0,005286$	$l\,\mathfrak{F}' = 7,723163$	$\mathfrak{B}' + \mathfrak{b}'kk = -0,0000382$
$\mathfrak{G}' = -0,000086$	$l{-}\mathfrak{G}' = 5,935307$	$l(\mathfrak{A}' + \mathfrak{a}'kk) = 8,015347$
$\mathfrak{H}' = +0,00420$	$l\,\mathfrak{H}' = 7,623250$	$l{-}(\mathfrak{B}' + \mathfrak{b}'kk) = 5,582063$
$\mathfrak{I}' = +0,57328$	$l\,\mathfrak{I}' = 9,758367$	
$\mathfrak{K}' = +0,00318$	$l\,\mathfrak{K}' = 7,502427$	
$\mathfrak{L}' = -0,15083$	$l{-}\mathfrak{L}' = 9,178488$	
$\mathfrak{M}' = -0,00002$	$l{-}\mathfrak{M}' = 5,301030$	

§. 125. Pro longitudine ergo Lunae habemus hactenus hanc formulam

		log. coeff.
$\Phi = \text{Conſt.}$	$+\ 1,0085272\ r$	0,003687
	$+\ 0,0103597\ \text{ſin}\ 2\eta$	8,015347
	$-\ 0,0000382\ \text{ſin}\ 4\eta$	5,582063
	$+\ 0,010146k\ \text{ſin}\ r$	8,006295
	$-\ 0,420226k\ \text{ſin}(2\eta - r)$	9,623483
	$+\ 0,004992k\ \text{ſin}(2\eta + r)$	7,698261
	$+\ 0,005286k\ \text{ſin}(4\eta - r)$	7,723163
	$-\ 0,000086k\ \text{ſin}(4\eta + r)$	5,935307
	$+\ 0,00420kk\ \text{ſin}\ 2r$	7,623250
	$+\ 0,57328kk\ \text{ſin}(2\eta - 2r)$	9,758367
	$+\ 0,00318kk\ \text{ſin}(2\eta + 2r)$	7,402427
	$-\ 0,15083kk\ \text{ſin}(4\eta - 2r)$	9,178488
	$-\ 0,00002kk\ \text{ſin}(4\eta + 2r)$	5,301030

§. 126.

§. 126. Quodſi iam ponamus $k = 0,05445$, et hos coefficientes ad minuta ſecunda cum partibus decimalibus reducamus, longitudo φ ita exprimetur vt ſit:

	log. coeff.
$\varphi =$ Conſt. $+ 1,0085272\ r$	
$+ 2136'',8 \sin 2\eta$	3,329772
$- 7,8 \sin 4\eta$	0,895488
$+ 113,9 \sin r$	2,056718
$- 4719,6 \sin(2\eta - r)$	3,673906
$+ 56,1 \sin(2\eta + r)$	1,748684
$+ 59,4 \sin(4\eta - r)$	1,773586
$- 1,0 \sin(4\eta + r)$	9,985730
$+ 2,5 \sin 2r$	0,409671
$+ 350,6 \sin(2\eta - 2r)$	2,544788
$+ 1,5 \sin(2\eta + 2r)$	0,188848
$- 92,2 \sin(4\eta - 2r)$	1,064909
$- 0,0 \sin(4\eta + 2r)$	8,087451

Hisque formulis praecipuae inaequalitates, quibus motus Lunae perturbatur, continentur.

CAPUT VIII.

DE MOTU APOGEI LUNAE.

§. 127.

His inuentis iam arduam illam de motu apogei Lunae quaeſtionem examinare, atque adeo decidere licebit. Quanquam enim in praecedentibus calculis vbique verum apogei motum, quem obſeruationes oſtendunt, introduxi, ita vt id ipſum, quod in controverſia eſt, aſſumſiſſe videar; tamen quoniam in hunc ipſum finem terrae vim, qua luna vrgetur, indefinitam ſum contemplatus, dum rationi diſtantiarum reciprocae duplicatae terminum indefinitum adiunxi, vnde littera μ in calculum eſt ingreſſa, iudicium de eo apogei motu, qui Theoriae Neutonianae eſſet conſentaneus, non erit difficile. Quodſi enim valor litterae μ nihilo aequalis reperiatur, hinc concludendum erit Theoriam Neutoni cum phaenomenis perfecte conſentire; ſin autem pro littera μ notabilis prodeat valor, Theoria iſta inſufficiens erit cenſenda.

§. 128. Motus autem apogei, quoniam huius rei in calculo nusquam mentio eſt facta, in ea continetur proportione, quam motus lunae medius ad motum anomaliae tenere eſt poſitus. Cum enim remotis lunae inaequalitatibus, quae regulae Keplerianae aduerſantur, longitudo lunae vera obtineatur, ſi eius anomalia vera r ad longitudinem apogei addatur: denotet ϖ longitudinem apogei, eritque longitudo vera $\phi = \varpi + r$,

vnde

vnde fit $\varpi = \Phi - r$. Ex quo intelligitur, ſi $\Phi - r$ quantitatem deſignet conſtantem, apogeum in quiete relinqui, ſin autem $\Phi - r$ valorem variabilem obtineat, tum apogeum quoque lunae motum eſſe habiturum.

§. 129. Cum autem terminos illos omnes, qui ſinus angulorum implicant, ideoque inaequalitates periodicas continent, quibus apogei motus non afficitur, omittimus, per integrationem deducimur ad huiusmodi formulam $\Phi = \text{Conſt.} + Or$, vnde propterea habetur longitudo apogei $\varpi = \text{Conſt.} + (O-1)\,r$. Hinc conſequimur ſequentes proportiones:

I. Vt 1 ad $O-1$, ita motus anomaliae lunae ad motum apogei.

II. Vt O ad 1, ita motus lunae medius ad motum anomaliae.

III. Vt O ad $O-1$, ita motus lunae medius ad motum apogei.

§. 130. Si obſeruationes conſulamus, valor litterae O reperitur $= 1,0085272$, quem etiam in calculo vbique adhibui; propterea quod propoſitum erat non tam in iſtum valorem a priori inquirere, quam ipſam potius Theoriam ita inſtituere, atque ſi opus fuerit, emendare, vt motus inde apogei experientiae conſentaneus reſultaret. Viciſſim autem Theoria ſtabilita, ſiue Neutoniana ſiue alia, quae ex determinato pro μ ſubſtituto valore oriatur, facile erit valorem ipſius O a priori eruere, quem deinceps cum valore vero 1,0085272 conferre licebit. Vel inuento valore ipſius O, apogeum

 lunae

lunae interuallo menſis apogiſtici progredietur per ſpatium $(O-1)\,360°$, interuallo autem menſis periodici per ſpatium $(1-\frac{1}{O})\,360°$. Secundum obſeruationes autem apogeum promouetur
vno menſe apogiſtico per ſpatium 3°, 4′, 11″
vno menſe periodico per ſpatium 3, 2, 38

§. 131. Ex calculo autem §. 107. expoſito valor litterae O ex elementis ante aſſumtis ita definitur, vt ſit $O+0{,}000649 = \varkappa + 0{,}000285$ ſiue $O = \varkappa - 0{,}000364$ Etſi enim haec exigua particula 0,000364 iam ex valore ipſius $\varkappa$ veritati conſentanee aſſumto eſt orta, tamen perſpicuum eſt, leuem differentiam nullius hic momenti futuram fuiſſe. Verum littera $\varkappa$ per Theoriam ita erat aſſumta, vt eſſet

$$\varkappa = \sqrt{\left(1+\frac{3+4\mu+\delta}{2nn}\right)}$$

vbi quidem valor ipſius nn ex motu medio lunae ad motum ſolis relato habetur, ita vt ſit ſine reſpectu ad motum apogei habito, $nn = 175{,}71795$. Ergo pro Theoria Neutoniana eſt

$$\varkappa = \sqrt{\left(1+\frac{3+\delta}{2nn}\right)} \quad \text{et} \quad O = \sqrt{\left(1+\frac{3+\delta}{2nn}\right)} - 0{,}000364.$$

§. 132. Hic igitur patet totam hanc inueſtigationem ad inuentionem litterae δ reduci, cuius valor, vti ex ſuperiori calculo manifeſtum eſt, a pluribus litteris et coefficientibus terminorum, quos ante eruere oportebat, pendet, ita vt neglecta hac littera δ motus apogei nullo modo recte definiri queat. Initio quidem vbi hanc litteram

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teram in calculum induximus, quod factum eſt §. 44. haec res leuis momenti eſt viſa; cum enim pro CC, quae erat conſtans per integrationem in calculum ingreſſa, valorem vero proximum inueniſſemus $1 + \frac{3+4\mu}{2nn}$, quoniam facile erat praeuidere, reliquis adhibitis elementis ad motum lunae pertinentibus, hunc valorem aliquantum immutari poſſe, pro vero valore ipſius $\frac{CC}{1-kk}$ poſuimus $1 + \frac{3+4\mu+\delta}{2nn}$. Deinde autem valor ipſius δ potiſſimum pendet a valore litterae γ, qua vſi ſumus ad verum valorem conſtantis $\frac{m}{nn} = 1 + \frac{2+3\mu+\gamma}{nn}$ obtinendum, cum proxime verus eſſet inuentus $= 1 + \frac{2+3\mu}{nn}$.

§. 133. Ab his ergo litteris γ et δ, quae initio nullius fere vſus eſſe videbantur, determinatio motus apogei potiſſimum pendet, quae cum ex pluribus atque adeo omnibus inaequalitatibus lunae ab excentricitate ortis determinari debeant, mirum ſane non eſt, quod legitima motus apogei deſignatio, cum tantis implicata ſit difficultatibus, tam dudum fuerit abſcondita. Plerique enim, qui motum apogei ex ſola Theoria concludere ſunt annisi, ad omnes has inaequalitates non reſpexerunt, atque calculum perinde adminiſtrauerunt, ac ſi hic litteras γ et δ neglexiſſemus. Ac ſi non defuere, qui ſibi perſuaſerunt, motum apogei cum Theoria Neutoniana conſentire, ii plerumque per errorem calculi ſeducti ad veritatem perueniſſe ſibi ſunt viſi. Quin etiam Ipſe Neu-

 tonus

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tonus Theoriae ſuae in motu apogei determinando parum tribuiſſe videtur.

§. 134. Hinc ex neglectu harum litterarum γ et δ, ſeu ex alia omiſſione eodem recidente, factum eſt, vt Theoria Neutoni obſeruationibus circa motum apogei Lunae inſtitutis plane non ſatisfacere ſit putata; quae opinio etiam ita inualuit, vt perſpicaciſſimus quisque hanc Theoriam inſufficientem pronunciaret. Atque ſagaciſſimus Clairaltius huic opinioni vehementiſſime erat addictus, antequam publice in contrarias partes diſceſſerat. Eadem ſcilicet ratione ob neglectum minutarum illarum particularum erat deceptus, qua et ego fateri cogor, me per complures annos conſtanter eſſe opinatum, ex Theoria Neutoni pro motu apogei Lunae non vltra ſemiſſem prodire, ita vt error vltra ſemiſſem exſurgens committeretur.

§. 135. Fons itaque huius erroris, qui niſi ſumma circumſpectio adhibeatur, vix euitatur, in eo latet, quod in calculo debita illa conſtantium determinatio, pro qua equidem hic litteras γ et δ adhibui, negligatur. Quemadmodum per hanc omiſſionem dimidius tantum apogei motus eliciatur, oſtendiſſe iuuabit. Sit igitur $\delta = o$, atque littera illius O ſecundum Theoriam Neutonianam, qua eſt $\mu = o$, valor erit $O = V\left(1 + \frac{3}{2nn}\right) - 0{,}000364$; qui euolutus fit: $O = 1{,}0042592 - 0{,}000364$. Quare etiamſi particula $0{,}000364$ vtpote ex profundiori indagine nata praetermittatur, tamen iſte valor pro $O = 1{,}0042592$, ſi cum vero per obſeruationes cognito $O = 1{,}0085272$ comparetur, exacte fere dimidium motum apogei praebet;

bet; atque adeo haec tam accurata medietas non parum digna videtur.

§. 136. Jam videamus, quam prope valorem litterae δ adhibendo ad veritatem perducamur. Inuenimus autem (115) $\delta = 3,20892$, vnde prodit

$$V\left(1 + \frac{3+\delta}{2nn}\right) = 1,0087947$$

qui valor iam maior eſt quam verus 1,0085272, ſed recordandum eſt inde ſubtrahi debere 0,000364, ſicque relinquetur $O = 1,0084307$, ex quo motus progreſſiuus apogei pro interuallo menſis apogiſtici prodibit $=$ $3^\circ\ 2'\ 9''$ et pro interuallo menſis periodici $=$ $3^\circ\ 0'\ 37''$, qui numeri duobus tantum minutis a vero deficiunt. Ad hunc defectum ſupplendum litterae μ tribui poterit valor conueniens ex formula $\mu = \frac{1}{4}(\kappa\kappa - 1)\, nn - \frac{3}{4} - \frac{1}{4}\delta$, vnde reperitur $\mu = 0,03782$, qui valor tantillus eſt, ut niſi de motu apogei ſit quaeſtio, ſemper pro nihilo haberi poſſit.

§. 137. Verum nullo modo affirmare poſſumus, valores illos pro γ et δ inuentos ita eſſe abſolutos, vt nulla amplius correctione indigeant. Quin potius, ſi formulas ſupra exhibitas attentius perpendamus, tantum abeſt vt eas pro completis habere poſſimus, vt potius manifeſtum ſit, omnes reliquas inaequalitates motus lunae perinde ac eas quas iam definiuimus, terminos quoque in eas ſuppeditare. Qui etſi admodum erunt parvi, tamen omnino ſufficere poterunt ad exiguum iſtud ſupplementum, quo adhuc a vero diſtamus, conficiendum.

Cum

Cum enim ſola fere inaequalitas ab angulo $2\eta - r$ pendens motum apogei a dimidio tantopere auxiſſet, vt valor ipſius O ab 1,0042592 vsque ad 1,0084307 increviſſet, nullum fere eſt dubium, quin leuis defectus huius numeri a vero valore 1,0085272 a reliquis inaequalitatibus proficiſcatur.

§. 138. Hinc igitur concludere debemus, Theoriam Neutonianam cum motu apogei obſeruato tam exacte conuenire, vt aberratio, ſi quidem vlla locum habeat, tam ſit exigua, vt merito pro nihilo reputari poſſit: neque etiam calculi ope ob ſummam paruitatem eam certo definire licebit. Cum itaque hoc pacto Theoria Neutoniana a fortiſſima obiectione ſit vindicata, gloria huius inſignis inuenti cum induſtriae tum candori excellentiſſimi Clairalti debetur, qui primus egregium hunc Theoriae conſenſum cum veritate detexit et publice eſt profeſſus: cui ea re eo maiores debemus gratias, quod ſine eius ſtudio ſummo, quod in hac inveſtigatione conſumſit, Theoria Neutoniana fortaſſe vix vnquam ab hac ſuſpicione inſufficientiae eſſet liberata. Atque nunc demum pleno lumine veritas iſtius Theoriae, cui vni Aſtronomiae Theoria vniuerſa innititur, fulgere eſt cenſenda, cum antea non mediocribus tenebris fuiſſet inuoluta.

CAPUT

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CAPUT IX.

INUESTIGATIO INAEQUALITATUM LUNAE A SOLA EXCENTRICITATE ORBITAE SOLIS PENDENTIUM.

§. 139.

Quoniam in hac inueſtigatione excentricitas orbitae lunaris non in cenſum venit, inaequalitates quas ſcrutamur partim ab anomalia vera ſolis s partim ab angulo 2η pendebunt. Cum igitur ſit

$$\frac{ds}{dr} = \frac{d\theta}{dr} = \frac{1+2ee}{n} + \frac{2}{n}k\cos r - \frac{2}{n}e\cos s - \frac{2}{n}ek\cos(r-s)$$
$$+ \frac{3}{2n}kk\cos 2r + \frac{1}{2n}ee\cos 2s - \frac{2}{n}ek\cos(r+s)$$

hinc differentiale ds ad differentiale dr reducitur. Atque hoc quidem capite, quia ad excentricitatem Lunae non attendimus, erit

$$\frac{ds}{dr} = \frac{d\theta}{dr} = \frac{1+2ee}{n} - \frac{2}{n}e\cos s + \frac{1}{2n}ee\cos 2s$$

§. 140. Incipiamus ergo a formulis $\int R\,dr$ et v, quas omiſſis terminis ab angulo r pendentibus ponamus

$$\int R\,dr = \mathfrak{A}\cos 2\eta + \mathfrak{P}e\cos s + \mathfrak{Q}e\cos(2\eta-s) + \mathfrak{R}e\cos(2\eta+s)$$
$$+ \mathfrak{S}ee\cos 2s + \mathfrak{T}ee\cos(2\eta-2s) + \mathfrak{V}ee\cos(2\eta+2s)$$
$$v = A\cos 2\eta + Pe\cos s + Qe\cos(2\eta-s) + Re\cos(2\eta+s)$$
$$+ See\cos 2s + Tee\cos(2\eta-2s) + Vee\cos(2\eta+2s)$$

vbi quidem pro $\mathfrak{A}$ et A valores ſupra inuentos completos accipi oportet, ita vt in iis termini akk et αkk ſint comprehenſi; erit ergo

$\mathfrak{A} = -0,81033$ $\quad l-\mathfrak{A} = 9,908662$

$A = -1,31773$ $\quad l-A = 0,119826$

Valores autem hinc deriuati erunt:

$\frac{2\varkappa A + \mathfrak{A}}{nn} = -0,019744, \; l\frac{-(2\varkappa A + \mathfrak{A})}{nn} = 8,295442$

$A' = -2,47576$ $\quad l-A' = 0,393708$

$\mathfrak{A}' = +0,01036$ $\quad l\,\mathfrak{A}' = 8,015347$

Terminos autem angulum quadruplum 4η inuoluentes hic ob ſummam paruitatem omiſi, quoniam in combinatione cum angulo s plane fierent imperceptibiles.

§. 141. Hinc iam primo colligitur:

$$\frac{d\Phi}{dr} = \varkappa - \frac{(2\varkappa A + \mathfrak{A})}{nn}\operatorname{coſ}2\eta - \frac{(2\varkappa P + \mathfrak{P})}{nn}e\operatorname{coſ}s$$
$$- \frac{(2\varkappa Q + \mathfrak{Q})}{nn}e\operatorname{coſ}(2\eta - s) - \frac{(2\varkappa R + \mathfrak{R})}{nn}e\operatorname{coſ}(2\eta + s)$$
$$- \frac{(2\varkappa S + \mathfrak{S})}{nn}ee\operatorname{coſ}2s$$
$$- \frac{(2\varkappa T + \mathfrak{T})}{nn}ee\operatorname{coſ}(2\eta - 2s) - \frac{(2\varkappa V + \mathfrak{V})}{nn}ee\operatorname{coſ}(2\eta + 2s)$$

atque porro

$$\frac{d\eta}{dr} = \alpha - \frac{(2\varkappa A + \mathfrak{A})}{nn}\operatorname{coſ}2\eta - \left(\frac{2\varkappa P + \mathfrak{P}}{nn} - \frac{2}{n}\right)e\operatorname{coſ}s$$
$$- \frac{(2\varkappa Q + \mathfrak{Q})}{nn}e\operatorname{coſ}(2\eta - s) + \frac{(2\varkappa R + \mathfrak{R})}{nn}e\operatorname{coſ}(2\eta + s)$$
$$- \left(\frac{(2\varkappa S + \mathfrak{S})}{nn} - \frac{1}{2n}\right)ee\operatorname{coſ}2s$$

Deinde

Deinde quia eſt proxime $kk = 9ee$, erit

$$\mathrm{R} = \tfrac{3}{4}\left(1 + \tfrac{27}{8}ee\right)\text{ſin }2\eta + \left(\frac{3\,\mathrm{Q}}{2nn} - \frac{3\,\mathrm{R}}{2nn}\right)e\text{ ſin }s$$
$$- \left(\tfrac{9}{4} - \frac{3\,\mathrm{P}}{2nn}\right)e\text{ ſin}(2\eta - s) - \left(\tfrac{3}{4} - \frac{3\,\mathrm{P}}{2nn}\right)e\text{ ſin}(2\eta + s)$$
$$+ \left(\frac{3\,\mathrm{T}}{2nn} - \frac{3\,\mathrm{V}}{2nn}\right)ee\text{ ſin }2s$$
$$+ \left(\tfrac{9}{8} + \frac{3\,\mathrm{S}}{2nn}\right)ee\text{ ſin}(2\eta - 2s) + \left(\tfrac{9}{8} + \frac{3\,\mathrm{S}}{2nn}\right)ee\text{ ſin}(2\eta + 2s)$$

atque omiſſis terminis, quibus non eſt opus

$$\frac{ddv}{dr^2} = e\text{ coſ }s\left\{\begin{array}{l} - \tfrac{3}{4} - 2\kappa\mathfrak{P} - 6\mathrm{P} - \dfrac{9\,\mathrm{A}}{4nn} + \dfrac{\mathfrak{A}\mathfrak{Q}}{nn} + \dfrac{\mathfrak{A}\mathfrak{R}}{nn} \\ + \dfrac{3\mathrm{AQ}}{nn} + \dfrac{3\mathrm{AR}}{nn} + \dfrac{3\mathfrak{A}(\mathrm{Q}+\mathrm{R})}{nn} + \dfrac{3\mathrm{A}(\mathfrak{Q}+\mathfrak{R})}{nn} \end{array}\right.$$

$$+ e\text{ coſ}(2\eta - s)\left\{\begin{array}{l} - \tfrac{9}{4} - 2\kappa\mathfrak{Q} - 6\mathrm{Q} - \dfrac{3\,\mathrm{A}}{4nn} + \dfrac{3\,\mathrm{P}}{4nn} \\ + \dfrac{\mathfrak{A}\mathfrak{P}}{nn} + \dfrac{3\mathrm{AP}}{nn} + \dfrac{3\mathfrak{A}\mathrm{P}}{nn} + \dfrac{3\mathrm{A}\mathfrak{P}}{nn} \end{array}\right.$$

$$+ e\text{ coſ}(2\eta + s)\left\{\begin{array}{l} - \tfrac{3}{4} - 2\kappa\mathfrak{R} - 6\mathrm{R} - \dfrac{3\,\mathrm{A}}{4nn} + \dfrac{3\,\mathrm{P}}{4nn} \\ + \dfrac{\mathfrak{A}\mathfrak{P}}{nn} + \dfrac{3\mathrm{AP}}{nn} + \dfrac{3\mathfrak{A}\mathrm{P}}{nn} + \dfrac{3\mathrm{A}\mathfrak{P}}{nn} \end{array}\right.$$

$$+ ee\text{ coſ }2s\left\{\begin{array}{l} + \tfrac{3}{4} - 2\kappa\mathfrak{S} - 6\mathrm{S} - \dfrac{3\,\mathrm{P}}{4nn} + \dfrac{\mathfrak{P}\mathfrak{P}}{2nn} \\ + \dfrac{3\mathrm{PP}}{2nn} + \dfrac{3\mathrm{P}\mathfrak{P}}{nn} \end{array}\right.$$

$$+ ee \cos(2\eta - 2s) \left\{ \begin{array}{l} + \tfrac{8}{8} - 2\kappa\mathfrak{T} - 6\mathrm{T} - \frac{9\mathrm{P}}{8nn} + \frac{\mathfrak{A}\mathfrak{S}}{nn} \\ + \frac{3\mathrm{AS}}{nn} + \frac{3\mathfrak{A}\mathrm{S}}{nn} + \frac{3\mathrm{A}\mathfrak{S}}{nn} \end{array} \right.$$

$$+ ee \cos(2\eta + 2s) \left\{ \begin{array}{l} + \tfrac{8}{8} - 2\kappa\mathfrak{V} - 6\mathrm{V} - \frac{9\mathrm{P}}{8nn} + \frac{\mathfrak{A}\mathfrak{S}}{nn} \\ + \frac{3\mathrm{AS}}{nn} + \frac{3\mathfrak{A}\mathrm{S}}{nn} + \frac{3\mathrm{A}\mathfrak{S}}{nn} \end{array} \right.$$

§. 142. Quodſi iam forma pro $\int R\,dr$ aſſumta differentietur, orietur:

$$\begin{aligned} R = &- 2\alpha\mathfrak{A} \sin 2\eta \\ &+ e \sin s \quad \left(-\frac{1}{n}\mathfrak{P} - \frac{(\mathfrak{Q}-\mathfrak{R})(2\kappa\mathrm{A}+\mathfrak{A})}{nn}\right) \\ &+ e \sin(2\eta - s)\left(-\frac{2\mathfrak{A}}{n} \pm \frac{\mathfrak{A}(2\kappa\mathrm{P}+\mathfrak{P})}{nn} - (2\alpha - \frac{1}{n})\mathfrak{Q}\right) \\ &+ e \sin(2\eta + s)\left(-\frac{2\mathfrak{A}}{n} + \frac{\mathfrak{A}(2\kappa\mathrm{P}+\mathfrak{P})}{nn} - (2\alpha + \frac{1}{n})\mathfrak{R}\right) \\ &+ ee \sin 2s \quad \left(\frac{1}{n}\mathfrak{P} - \frac{2}{n}\mathfrak{S} - \frac{(\mathfrak{T}-\mathfrak{V})(2\kappa\mathrm{A}+\mathfrak{A})}{nn}\right) \\ &+ ee \sin(2\eta - 2s)\left(\frac{1}{2n}\mathfrak{A} - \frac{3}{n}\mathfrak{Q} - (2\alpha - \frac{2}{n})\mathfrak{T}\right) \\ &+ ee \sin(2\eta + 2s)\left(\frac{1}{2n}\mathfrak{A} - \frac{1}{n}\mathfrak{R} - (2\alpha + \frac{2}{n})\mathfrak{V}\right) \end{aligned}$$

§. 143. Comparatione ergo inſtituta habebitur

$$-\frac{1}{n}\mathfrak{P} - \frac{(\mathfrak{Q}-\mathfrak{R})(2\kappa\mathrm{A}+\mathfrak{A})}{nn} = \frac{3(\mathrm{Q}-\mathrm{R})}{2nn}$$

—

$$-\frac{2}{n}\mathfrak{A}+\frac{\mathfrak{A}(2\kappa P+\mathfrak{P})}{nn}-\left(2\alpha-\frac{1}{n}\right)\mathfrak{Q}=-\frac{e}{4}+\frac{3P}{2nn}$$

$$-\frac{2}{n}\mathfrak{A}+\frac{\mathfrak{A}(2\kappa P+\mathfrak{P})}{nn}-\left(2\alpha+\frac{1}{n}\right)\mathfrak{R}=-\frac{e}{4}+\frac{3P}{2nn}$$

$$\frac{1}{n}\mathfrak{P}-\frac{2}{n}\mathfrak{S}-\frac{(\mathfrak{T}-\mathfrak{V})(2\kappa A\mp\mathfrak{A})}{nn}=\frac{3(T-V)}{2nn}$$

$$\frac{1}{2n}\mathfrak{A}-\frac{3}{n}\mathfrak{Q}-\left(2\alpha-\frac{2}{n}\right)\mathfrak{T}=\frac{e}{8}+\frac{3S}{2nn}$$

$$\frac{1}{2n}\mathfrak{A}-\frac{1}{n}R-\left(2\alpha+\frac{2}{n}\right)\mathfrak{V}=\frac{e}{8}+\frac{3S}{2nn}$$

vnde deinceps valores litterarum germanicarum $\mathfrak{P}$, $\mathfrak{Q}$, $\mathfrak{R}$, $\mathfrak{S}$, $\mathfrak{T}$, $\mathfrak{V}$ ſumus inueſtigaturi.

§. 144. Differentietur ſimili modo quantitas v, ac ponatur:

$$\frac{dv}{dr}=-A'\,\text{ſn}\,2\eta-P'e\,\text{ſn}\,s-Q'e\,\text{ſn}(2\eta-s)-S'ee\,\text{ſn}\,2s-T'ee\,\text{ſn}(2\eta-2s)$$
$$-R'e\,\text{ſn}(2\eta+s)\qquad\qquad -V'ee\,\text{ſn}(2\eta+2s)$$

eritque

$A'=2\alpha\mathfrak{A}$, cuius quidem valor iam ſupra habetur

$$P'=\frac{1}{n}P+\frac{(Q-R)(2\kappa A+\mathfrak{A})}{nn}$$

$$Q'=\left(2\alpha-\frac{1}{n}\right)Q+\frac{2}{n}A-\frac{A(2\kappa P+\mathfrak{P})}{nn}$$

$$R'=\left(2\alpha+\frac{1}{n}\right)R+\frac{2}{n}A-\frac{A(2\kappa P+\mathfrak{P})}{nn}$$

$$S'=\frac{2}{n}S-\frac{1}{n}P+\frac{(T-V)(2\kappa A+\mathfrak{A})}{nn}$$

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$$T' = (2\alpha - \frac{2}{n})\,T + \frac{3}{n}\,Q - \frac{1}{2n}\,A$$

$$V' = (2\alpha + \frac{2}{n})\,V + \frac{1}{n}\,R - \frac{1}{2n}\,A$$

vnde denuo differentiando eruitur.

$$\frac{ddv}{dr^2} = e\,\mathrm{coſ}\,s\;\left(-\frac{1}{n}\,P' + \frac{(Q'+R')\,(2\kappa A+\mathfrak{A})}{nn}\right.$$

$$e\,\mathrm{coſ}\,(2\eta - s)\;\left(-(2\alpha - \frac{1}{n})\,Q' - \frac{2}{n}\,A' + \frac{A'(2\kappa P+\mathfrak{P})}{nn}\right.$$

$$e\,\mathrm{coſ}\,(2\eta + s)\;\left(-(2\alpha + \frac{1}{n})\,R' - \frac{2}{n}\,A' + \frac{A'(2\kappa P+\mathfrak{P})}{nn}\right.$$

$$ee\,\mathrm{coſ}\,2s\;\left(-\frac{2}{n}\,S' + \frac{1}{n}\,P' + \frac{(T'+V')\,(2\kappa A+\mathfrak{A})}{nn}\right.$$

$$ee\,\mathrm{coſ}(2\eta - 2s)\;\left(-(2\alpha - \frac{2}{n})\,T' + \frac{1}{2n}\,A' - \frac{3}{n}\,Q'\right.$$

$$ee\,\mathrm{coſ}(2\eta + 2s)\;\left(-(2\alpha + \frac{2}{n})\,V' + \frac{1}{2n}\,A' - \frac{1}{n}\,R'\right.$$

§. 145. Sequentes ergo aequationes reſoluendae occurrent

$$-\frac{3}{2} - 2\kappa\mathfrak{P} - 6P - \frac{9A}{4nn} + \frac{(\mathfrak{A}+3A)(Q+R)}{nn} + \frac{(3\mathfrak{A}+3A)\,(Q+R)}{nn} =$$

$$-\frac{1}{n}\,P' + \frac{(Q'+R')\,(2\kappa A+\mathfrak{A})}{nn}$$

$$-\frac{2}{4} - 2\kappa\mathfrak{Q} - 6Q - \frac{3A}{4nn} + \frac{3P}{4nn} + \frac{(\mathfrak{A}+3A)\mathfrak{P}}{nn} + \frac{(3\mathfrak{A}+3A)\,P}{nn} =$$

$$-(2\alpha - \frac{1}{n})\,Q' - \frac{2}{n}\,A' + \frac{A'(2\kappa P+\mathfrak{P})}{nn}$$

—

$$-\tfrac{3}{4} - 2\varkappa\mathfrak{R} - \varepsilon R - \frac{3A}{4nn} + \frac{3P}{4nn} + \frac{(\mathfrak{A}+3A)\mathfrak{P}}{nn} + \frac{(3\mathfrak{A}+3A)P}{nn} =$$

$$-\left(2\alpha + \frac{1}{n}\right)R' - \frac{2}{n}A' + \frac{A'(2\varkappa P+\mathfrak{P})}{nn}$$

$$+\tfrac{3}{4} - 2\varkappa\mathfrak{S} - \varepsilon S - \frac{3P}{4nn} + \frac{\mathfrak{P}\mathfrak{P}+6P\mathfrak{P}+3PP}{2nn} =$$

$$-\frac{2}{n}S' + \frac{1}{n}P' + \frac{(T'+V')(2\varkappa A+\mathfrak{A})}{nn}$$

$$+\tfrac{9}{8} - 2\varkappa\mathfrak{T} - \varepsilon T - \frac{9P}{8nn} + \frac{(\mathfrak{A}+3A)\mathfrak{S}}{nn} + \frac{(3\mathfrak{A}+3A)S}{nn} =$$

$$-\left(2\alpha - \frac{2}{n}\right)T' + \frac{1}{2n}A' - \frac{3}{n}Q'$$

$$+\tfrac{9}{8} - 2\varkappa\mathfrak{V} - \varepsilon V - \frac{9P}{8nn} + \frac{(\mathfrak{A}+3A)\mathfrak{S}}{nn} + \frac{(3\mathfrak{A}+3A)S}{nn} =$$

$$-\left(2\alpha + \frac{2}{n}\right)V' + \frac{1}{2n}A' - \frac{1}{n}R'$$

Neglectis primo terminis minimis, qui adhuc ſunt incogniti, reperitur:

$\mathfrak{Q} = +1,3238$	$Q = +2,5714$	$Q' = 4,40924$
$\mathfrak{R} = +1,2210$	$R = +2,0571$	$R' = 3,79793$
$\mathfrak{P} = -0,0313$	$P = -1,4807$	$P' = -0,12185$

§. 146. Ex his autem accuratius ita definientur vt ſit:

$\mathfrak{Q} = +1,33859$ . . . . $l\mathfrak{Q} = 0,126649$

$\mathfrak{R} = +1,23468$ . . . . $l\mathfrak{R} = 0,091545$

$Q = +2,60087$ . . . . $lQ = 0,415119$

$R = +2,00590$ . . . . $lR = 0,302308$

$Q' = 4,44802$; $R' = 3,67581$

et

et $\mathfrak{P} = -0{,}04010$ . . . $l\text{-}\mathfrak{P} = 8{,}603133$
$P = -1{,}46488$ . . . $l\text{-}P = 0{,}165801$
$P' = -0{,}12222$

Deinde reperitur

$\mathfrak{S} = +0{,}03911$ . . . $l\mathfrak{S} = 8{,}592230$
$S = +0{,}60882$ . . . $lS = 9{,}784486$
$\mathfrak{T} = -0{,}85267$ . . . $l\text{-}\mathfrak{T} = 9{,}930827$
$\mathfrak{V} = -0{,}62125$ . . . $l\text{-}\mathfrak{V} = 9{,}793266$
$T = -2{,}60380$ . . . $l\text{-}T = 0{,}465615$
$V = -1{,}02720$ . . . $l\text{-}V = 0{,}011672$

Pro ſequentibus vero calculis eſt

$\frac{2\kappa P+\mathfrak{P}}{nn} = -0{,}017051$ . . . $l\text{-}\frac{(2\kappa P+\mathfrak{P})}{nn} = 8{,}231755$

$\frac{2\kappa Q+\mathfrak{Q}}{nn} = +0{,}037487$ . . . $l+\frac{2\kappa Q+\mathfrak{Q}}{nn} = 8{,}573878$

$\frac{2\kappa R+\mathfrak{R}}{nn} = +0{,}030062$ . . . $l+\frac{2\kappa R+\mathfrak{R}}{nn} = 8{,}478023$

§. 147. Hinc igitur pro diſtantia lunae a terra curtata $x = \frac{(1-kk)au}{1-k\,\text{coſ}\,r}$, pars quantitatis u ab excentricitate orbitae ſolaris tantum pendens erit

		log. coeff.
$u =$ Praeced. $-\ 0{,}008336e$	$\text{coſ}\,s$	7,920985
$+\ 0{,}014801e$	$\text{coſ}(2\eta-s)$	8,170303
$+\ 0{,}011415e$	$\text{coſ}(2\eta+s)$	8,057492
$+\ 0{,}00364ee$	$\text{coſ}\,2s$	7,539670
$-\ 0{,}01482ee$	$\text{coſ}(2\eta-2s)$	8,170799
$-\ 0{,}00584ee$	$\text{coſ}(2\eta+2s)$	7,766856

Deinde

Deinde vero erit

	log. coeff.
$\frac{d\Phi}{dr} = \text{Praec.} + 0{,}017041\, e \cos s$	8,231755
$- 0{,}037487\, e \cos(2\eta - s)$	8,573878
$- 0{,}030062\, e \cos(2\eta + s)$	8,478023
$- 0{,}00722\, ee \cos 2s$	7,858166
$+ 0{,}03470\, ee \cos(2\eta - 2s)$	8,540319
$+ 0{,}01533\, ee \cos(2\eta + 2s)$	8,185614

§. 148. Ponatur nunc integrale

$$\begin{aligned}\Phi = \text{Praec.} &+ \mathfrak{A}' \sin 2\eta + \mathfrak{P}' e \sin s \\ &+ \mathfrak{Q}' e \sin(2\eta - s) + \mathfrak{R}' e \sin(2\eta + r) \\ &+ \mathfrak{S}' ee \sin 2s + \mathfrak{T}' ee \sin(2\eta - 2s) \\ &+ \mathfrak{V}' ee \sin(2\eta + 2s)\end{aligned}$$

erit differentiatione peracta:

$$+ 0{,}017051 = \frac{1}{n}\mathfrak{P}' - \frac{(\mathfrak{Q}' + \mathfrak{R}')(2\kappa A + \mathfrak{A})}{nn}$$

$$- 0{,}037489 = \left(2\alpha - \frac{1}{n}\right)\mathfrak{Q}' + \frac{2}{n}\mathfrak{A}' - \frac{\mathfrak{A}'(2\kappa P + \mathfrak{P})}{nn}$$

$$- 0{,}030062 = \left(2\alpha + \frac{1}{n}\right)\mathfrak{R}' + \frac{2}{n}\mathfrak{A}' - \frac{\mathfrak{A}'(2\kappa P + \mathfrak{P})}{nn}$$

$$- 0{,}00722 = \frac{2}{n}\mathfrak{S}' - \frac{1}{n}\mathfrak{P}' - \frac{(\mathfrak{T}' + \mathfrak{V}')(2\kappa A + \mathfrak{A})}{nn}$$

$$+ 0{,}03470 = \left(2\alpha - \frac{2}{n}\right)\mathfrak{T}' - \frac{1}{2n}\mathfrak{A}' + \frac{3}{n}\mathfrak{Q}'$$

$$+ 0{,}01533 = \left(2\alpha + \frac{2}{n}\right)\mathfrak{V}' - \frac{1}{2n}\mathfrak{A}' + \frac{1}{n}\mathfrak{R}'$$

fietque his valoribus determinatis

		log. coeff.
$\Phi =$ Praec.	$+\ 0{,}236034\, e\ \text{ſin}\, s$	9,372974
	$-\ 0{,}021889\, e\ \text{ſin}(2\eta - s)$	8,340237
	$-\ 0{,}016368\, e\ \text{ſin}(2\eta + s)$	8,214002
	$+\ 0{,}06615\, ee\ \text{ſin}\, 2s$	8,820508
	$+\ 0{,}02332\, ee\ \text{ſin}(2\eta - 2s)$	8,367825
	$+\ 0{,}00840\, ee\ \text{ſin}(2\eta + 2s)$	7,924429

§. 149. Reducamus has inaequalitates etiam ad minuta ſecunda, ponendo excentricitatem orbitae ſolaris $e = 0{,}01680$, atque habebimus

		log. coeff.
$\Phi =$ Praec.	$+\ 817'',9\ \text{ſin}\, s$	2,912708
	$-\ 75,\ 8\ \text{ſin}(2\eta - s)$	1,879971
	$-\ 56,\ 7\ \text{ſin}(2\eta + s)$	1,753736
	$+\ 3,\ 8\ \text{ſin}\, 2s$	0,585551
	$+\ 1,\ 4\ \text{ſin}(2\eta - 2s)$	0,132868
	$+\ 0,\ 5\ \text{ſin}(2\eta + 2s)$	9,689472

Denotat hic s anomaliam veram ſolis; vnde patet eam Lunae inaequalitatem, quae ſinui huius anomaliae eſt proportionalis, admodum eſſe notabilem, dum ad 13′, 38″ exſurgit. Tabulae autem Aſtronomicae, vbi haec inaequalitas aequatio ſolaris nominatur, eam multo minorem faciunt, cuius rei cauſam inueſtigari adhuc conveniet.

§. 150.

§. 150. Quodſi enim litteram $\mathfrak{P}'$ accuratius definire velimus, habebimus has formulas reſoluendas:

$$-\frac{1}{n}\mathfrak{P}+\frac{\mathfrak{A}(2\kappa Q+\mathfrak{Q})}{nn}-\frac{\mathfrak{A}(2\kappa R+\mathfrak{R})}{nn}-\frac{(\mathfrak{Q}-\mathfrak{R})(2\kappa A+\mathfrak{A})}{nn}=\frac{3(Q-R)}{nn}$$

$$P'=\frac{1}{n}P-\frac{A(2\kappa Q+\mathfrak{Q})}{nn}+\frac{A(2\kappa R+\mathfrak{R})}{nn}+\frac{(Q-R)(2\kappa A+\mathfrak{A})}{nn}$$

$$-\frac{3}{2}-2\kappa\mathfrak{P}-6P-\frac{9A}{4nn}+\frac{(\mathfrak{A}+3A)(\mathfrak{Q}+\mathfrak{R})}{nn}+\frac{(3\mathfrak{A}+3A)(Q+R)}{nn}=$$

$$-\frac{1}{n}P'+\frac{A'(2\kappa Q+\mathfrak{Q})}{nn}+\frac{A'(2\kappa R+\mathfrak{R})}{nn}+\frac{(Q'+R')(2\kappa A+\mathfrak{A})}{nn}$$

vnde elicimus:

$$\mathfrak{P}=-0,1183;\quad P=-1,1356;\quad \frac{P}{nn}=-0,0064$$

atque $\frac{2\kappa P+\mathfrak{P}}{nn}=-0,01376$, fierique iam oportet

$$+0,01376=\frac{1}{n}\mathfrak{P}'-\frac{\mathfrak{A}'(2\kappa Q+\mathfrak{Q})}{nn}-\frac{\mathfrak{A}'(2\kappa R+\mathfrak{R})}{nn}-\frac{(\mathfrak{Q}'+\mathfrak{R}')(2\kappa A+\mathfrak{A})}{nn}$$

vnde oritur $\mathfrak{P}'=+0,201385$. Quare accuratius habemus

u = Praec. — 0,006400 e coſ s	7,806180
$\frac{d\Phi}{dr}$ = Praec. + 0,013760 e coſ s	8,138618
Φ = Praec. + 0,201385 e ſin s	9,304026
ſeu	
Φ = Praec. + 701″, 1 ſin s	2,845780

Ergo aequatio ſinui Anguli s proportionalis tantum eſt 11′, 41″.

CAPUT X.

INUESTIGATIO INAEQUALITATVM LUNAE AB VTRIUSQUE ORBITAE EXCENTRICITATE SIMUL PENDENTIUM.

§. 151.

Quoniam praeuidemus inaequalitates huius generis, quae altiores litterarum k et e poteſtates ſimul complectuntur, minimas eſſe futuras, alios terminos non ſcrutabimur, niſi qui producto ſimplici ek ſint affecti. Habebimus ergo

$$\frac{ds}{dr} = \frac{d\theta}{dr} = \frac{1}{n} + \frac{2}{n} k \cos r - \frac{2}{n} e \cos s - \frac{2}{n} ek \cos(r-s) - \frac{2}{n} ek \cos(r+s)$$

Cum igitur ad hanc inueſtigationem opus non ſit illis terminis ex praecedentibus, qui vel per k^2 vel per e^2 erant affecti, quia litterae alphabethi deficere incipiunt, litteris S, T et ſequentibus denuo utemur; quare cavendum, ne iſtae litterae cum ante adhibitis confundantur.

§. 152. Aſſumtis ergo ex terminis iam ante definitis, iis qui in eos, quos iam inueſtigamus, vim exſerunt, ponamus

$$\int R dr = \mathfrak{A} \cos 2\eta + \mathfrak{C} k \cos r + \mathfrak{D} k \cos(2\eta - r) + \mathfrak{P} e \cos s + \mathfrak{Q} e \cos(2\eta - s)$$
$$+ \mathfrak{E} k \cos(2\eta + r) \qquad + \mathfrak{R} e \cos(2\eta + s)$$
$$+ \mathfrak{S} ek \cos(r-s) + \mathfrak{V} ek \cos(2\eta - r + s) + \mathfrak{Y} ek \cos(2\eta - r - s)$$
$$+ \mathfrak{T} ek \cos(r+s) + \mathfrak{X} ek \cos(2\eta + r - s) + \mathfrak{Z} ek \cos(2\eta + r + s)$$

$v =$

$$v = A\cos 2\eta \;.\;.\; + Dk\cos(2r-r) + Pe\cos s + Qe\cos(2\eta - s)$$
$$+ Ek\cos(2\eta + r) \qquad\qquad + Re\cos(2\eta + s)$$
$$+ Sek\cos(r-s) + Vek\cos(2\eta - r + s) + Yek\cos(2\eta - r - s)$$
$$+ Tek\cos(r+s) + Xek\cos(2\eta + r - s) + Zek\cos(2\eta + r + s)$$

eritque ex praecedentibus

𝔄 = −0,81033	;	l−𝔄 = 9,908662
ℭ = −0,64383	;	l−ℭ = 9,808771
𝔇 = −3,59362	;	l−𝔇 = 0,555531
𝔈 = −1,08732	;	l−𝔈 = 0,036358
𝔓 = −0,11830	;	l−𝔓 = 9,072985
𝔔 = +1,33859	;	l 𝔔 = 0,126649
ℜ = +1,23468	;	l ℜ = 0,091545
A = −1,31773	;	l−A = 0,119826
. .	.	. .
D = +33,6600	;	l D = 1,527113
E = −0,5785	;	l−E = 9,762341
P = −1,1356	;	l−P = 0,055225
Q = +2,60087	;	l−Q = 0,415119
R = +2,00590	;	l R = 0,302308

§. 153. Reliqui vero valores hinc deriuati, quibus opus habemus, ſunt:

A′ = −2,47576	;	l−A′ = 0,393708
C′ = −0,13847	;	l−C′ = 9,141356
D′ = 29,39153	;	l D′ = 1,468222
E′ = −1,47347	;	l−E′ = 0,168341
P′ = −0,0260	;	l−P′ = 8,414973
Q′ = 4,40924	;	l Q′ = 0,644363
R′ = 3,79793	;	l R′ = 0,579548

$$\mathfrak{A}' = +0{,}01036 \quad ; \quad l\,\mathfrak{A}' = 8{,}015347$$
$$\mathfrak{C}' = +0{,}01015 \quad ; \quad l\,\mathfrak{C}' = 8{,}006295$$
$$\mathfrak{D}' = -0{,}42023 \quad ; \quad l\text{-}\mathfrak{D}' = 9{,}623483$$
$$\mathfrak{E}' = +0{,}00499 \quad ; \quad l\text{-}\mathfrak{E}' = 7{,}698261$$
$$\mathfrak{P}' = +0{,}20138 \quad ; \quad l\,\mathfrak{P}' = 9{,}304016$$
$$\mathfrak{Q}' = -0{,}02189 \quad ; \quad l\text{-}\mathfrak{Q}' = 8{,}340237$$
$$\mathfrak{R}' = -0{,}01637 \quad ; \quad l\text{-}\mathfrak{R}' = 8{,}214002$$

Sit breuitatis gratia

$$a' = \frac{2\varkappa A + \mathfrak{A}}{nn} = -0{,}019744 \quad ; \quad l\text{-}\frac{(2\varkappa A + \mathfrak{A})}{nn} = 8{,}295442$$
$$d' = \frac{2\varkappa D + \mathfrak{D}}{nn} = +0{,}36611 \quad ; \quad l\,\frac{2\varkappa D + \mathfrak{D}}{nn} = 9{,}563604$$
$$e' = \frac{2\varkappa E + \mathfrak{E}}{nn} = -0{,}01283 \quad ; \quad l\text{-}\frac{(2\varkappa E + \mathfrak{E})}{nn} = 8{,}108292$$
$$p' = \frac{2\varkappa P + \mathfrak{P}}{nn} = -0{,}01376 \quad ; \quad l\text{-}\frac{(2\varkappa P + \mathfrak{P})}{nn} = 8{,}138618$$
$$q' = \frac{2\varkappa Q + \mathfrak{Q}}{nn} = +0{,}03749 \quad ; \quad l\,\frac{2\varkappa Q + \mathfrak{Q}}{nn} = 8{,}573878$$
$$r' = \frac{2\varkappa R + \mathfrak{R}}{nn} = +0{,}03006 \quad ; \quad l\,\frac{2\varkappa R + \mathfrak{R}}{nn} = 8{,}478023$$

§. 154. Si ſimili modo vlterius ponatur:

$$s' = \frac{2\varkappa S + \mathfrak{S}}{nn}; \quad t' = \frac{2\varkappa T + \mathfrak{T}}{nn}; \quad v' = \frac{2\varkappa V + \mathfrak{V}}{nn};$$
$$x' = \frac{2\varkappa X + \mathfrak{X}}{nn}; \quad y' = \frac{2\varkappa Y + \mathfrak{Y}}{nn}; \quad z' = \frac{2\varkappa Z + \mathfrak{Z}}{nn}$$

habe-

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habebimus:

$$\frac{d\Phi}{dr} = \text{Praec.} - a'\,\text{cſ}\,2\eta - \frac{\mathfrak{C}}{nn}\,k\,\text{cſ}\,r - d'k\,\text{cſ}(2\eta-r) - p'e\,\text{cſ}\,s - q'e\,\text{cſ}(2\eta-s)$$
$$- e'k\,\text{cſ}(2\eta+r) \qquad - r'e\,\text{cſ}(2\eta+s)$$
$$- s'ek\,\text{coſ}(r-s) - v'ek\,\text{coſ}(2\eta-r+s) - y'ek\,\text{coſ}(2\eta-r-s)$$
$$- t'ek\,\text{coſ}(r+s) - x'ek\,\text{coſ}(2\eta+r-s) - z'ek\,\text{coſ}(2\eta+r+s)$$

atque poſito $\frac{2}{n} + \frac{\mathfrak{C}}{nn} = c' = 0,147197$; $l\,c' = 9,167900$

$$\frac{d\eta}{dr} = a - a'\,\text{cſ}\,2\eta - c'k\,\text{cſ}\,r - d'k\,\text{cſ}(2\eta-r) + \left(\frac{2}{n} - p'\right)e\,\text{cſ}\,s - q'e\,\text{cſ}(2\eta-s)$$
$$- e'k\,\text{cſ}(2\eta+r) \qquad - r'e\,\text{cſ}(2\eta+s)$$
$$+ \left(\frac{2}{n} - s'\right)ek\,\text{cſ}(r-s) - v'ek\,\text{cſ}(2\eta-r+s) - y'ek\,\text{cſ}(2\eta-r-s)$$
$$+ \left(\frac{2}{n} - t'\right)ek\,\text{cſ}(r+s) - x'ek\,\text{cſ}(2\eta+r-s) - z'ek\,\text{cſ}(2\eta+r+s)$$

vbi cum ſit $\frac{2}{n} = 0,150876$, erit $\frac{2}{n} - p' = 0,16464$

§. 155. Nunc termini coefficiente ek affecti, qui in formis R $=$ et $\frac{ddv}{dr^2}$ inſunt, colligantur: eritque

$$R = ek\,\text{ſin}(r-s)\left(+\frac{3}{2nn}V - \frac{3}{2nn}X - \frac{3Q}{nn} + \frac{3R}{nn} - \frac{9D}{4nn} + \frac{9E}{4nn}\right)$$
$$ek\,\text{ſin}(r+s)\left(+\frac{3}{2nn}Y - \frac{3}{2nn}Z - \frac{3R}{nn} + \frac{3Q}{nn} + \frac{9E}{4nn} - \frac{9D}{4nn}\right)$$
$$ek\,\text{ſin}(2\eta-r+s)\left(-\frac{3}{2} + \frac{3}{2nn}S + \frac{3P}{nn}\right)$$
$$ek\,\text{ſin}(2\eta+r-s)\left(-\frac{3}{2} + \frac{3}{2nn}S + \frac{3P}{nn}\right)$$

ek

$$ek \sin(2\eta - r - s)\left(-\tfrac{1}{2} + \frac{3}{2nn}T + \frac{3P}{nn}\right)$$

$$ek \sin(2\eta + r + s)\left(-\tfrac{1}{2} + \frac{3}{2nn}T + \frac{3P}{nn}\right)$$

et $\dfrac{ddv}{dr^2} =$

$$ek \cos(r-s)\left\{\begin{aligned}&-3-2\kappa\mathfrak{S}-\epsilon S+\tfrac{1}{2}bP+\frac{3V}{4nn}+\frac{3X}{4nn}+\frac{3Q}{2nn}+\frac{3R}{2nn}\\&-\frac{9D}{8nn}-\frac{9E}{8nn}+\frac{\mathfrak{A}\mathfrak{V}}{nn}+\frac{\mathfrak{A}\mathfrak{X}}{nn}+\frac{\mathfrak{C}\mathfrak{P}}{nn}+\frac{\mathfrak{D}\mathfrak{Q}}{nn}+\frac{\mathfrak{E}\mathfrak{R}}{nn}\\&+\frac{3AV}{nn}+\frac{3AX}{nn}+\frac{3DQ}{nn}+\frac{3ER}{nn}-\frac{3AQ}{2nn}-\frac{3AR}{2nn}\\&+\frac{3\mathfrak{A}V}{nn}+\frac{3A\mathfrak{V}}{nn}+\frac{3\mathfrak{A}X}{nn}+\frac{3A\mathfrak{X}}{nn}+\frac{3\mathfrak{C}P}{nn}\\&+\frac{3\mathfrak{D}Q}{nn}+\frac{3D\mathfrak{Q}}{nn}+\frac{3\mathfrak{E}R}{nn}+\frac{3E\mathfrak{R}}{nn}\end{aligned}\right.$$

$$ek \cos(r+s)\left\{\begin{aligned}&-3-2\kappa\mathfrak{T}-\epsilon T+\tfrac{1}{2}bP+\frac{3Y}{4nn}-\frac{3Z}{4nn}+\frac{3Q}{2nn}+\frac{3R}{2nn}\\&-\frac{9D}{8nn}-\frac{9E}{8nn}+\frac{\mathfrak{A}\mathfrak{Y}}{nn}+\frac{\mathfrak{A}\mathfrak{Z}}{nn}+\frac{\mathfrak{C}\mathfrak{P}}{nn}+\frac{\mathfrak{D}\mathfrak{R}}{nn}+\frac{\mathfrak{E}\mathfrak{Q}}{nn}\\&+\frac{3AY}{nn}+\frac{3AZ}{nn}+\frac{3DR}{nn}+\frac{3EQ}{nn}-\frac{3AQ}{2nn}-\frac{3AR}{2nn}\\&+\frac{3\mathfrak{A}Y}{nn}+\frac{3A\mathfrak{Y}}{nn}+\frac{3\mathfrak{A}Z}{nn}+\frac{3A\mathfrak{Z}}{nn}+\frac{3\mathfrak{C}P}{nn}\\&+\frac{3\mathfrak{C}Q}{nn}+\frac{3E\mathfrak{Q}}{nn}+\frac{3\mathfrak{D}R}{nn}+\frac{3D\mathfrak{R}}{nn}\end{aligned}\right.$$

ek

$$ek\cos(2\eta-r+s)\left\{\begin{array}{l}-\frac{g}{2}-2\kappa\mathfrak{V}-6V+\frac{1}{2}bR+\frac{3S}{4nn}-\frac{3D}{4nn}+\frac{3P}{2nn}\\+\frac{\mathfrak{A}\mathfrak{S}}{nn}+\frac{\mathfrak{C}\mathfrak{R}}{nn}+\frac{\mathfrak{D}\mathfrak{P}}{nn}+\frac{3AS}{nn}+\frac{3DP}{nn}-\frac{3AP}{2nn}\\+\frac{3\mathfrak{A}S}{nn}+\frac{3A\mathfrak{S}}{nn}+\frac{3\mathfrak{C}R}{nn}+\frac{3\mathfrak{D}P}{nn}+\frac{3D\mathfrak{P}}{nn}\end{array}\right.$$

$$ek\cos(2\eta+r-s)\left\{\begin{array}{l}-\frac{g}{2}-2\kappa\mathfrak{X}-6X+\frac{1}{2}bQ+\frac{3S}{4nn}-\frac{3E}{4nn}+\frac{3P}{2nn}\\+\frac{\mathfrak{A}\mathfrak{S}}{nn}+\frac{\mathfrak{C}\mathfrak{Q}}{nn}+\frac{\mathfrak{E}\mathfrak{P}}{nn}+\frac{3AS}{nn}+\frac{3EP}{nn}-\frac{3AP}{2nn}\\+\frac{3\mathfrak{A}S}{nn}+\frac{3A\mathfrak{S}}{nn}+\frac{3\mathfrak{C}Q}{nn}+\frac{3\mathfrak{E}P}{nn}+\frac{3E\mathfrak{P}}{nn}\end{array}\right.$$

$$ek\cos(2\eta-r-s)\left\{\begin{array}{l}-\frac{g}{2}-2\kappa\mathfrak{Y}-6Y+\frac{1}{2}hQ+\frac{3T}{4nn}-\frac{3D}{4nn}+\frac{3P}{2nn}\\+\frac{\mathfrak{A}\mathfrak{C}}{nn}+\frac{\mathfrak{C}\mathfrak{Q}}{nn}+\frac{\mathfrak{D}\mathfrak{P}}{nn}+\frac{3AT}{nn}+\frac{3DP}{nn}-\frac{3AP}{2nn}\\+\frac{3\mathfrak{A}T}{nn}+\frac{3A\mathfrak{T}}{nn}+\frac{3\mathfrak{C}Q}{nn}+\frac{3\mathfrak{D}P}{nn}+\frac{3D\mathfrak{P}}{nn}\end{array}\right.$$

$$ek\cos(2\eta+r+s)\left\{\begin{array}{l}-\frac{g}{2}-2\kappa\mathfrak{Z}-6Z+\frac{1}{2}bR+\frac{3T}{4nn}-\frac{3E}{4nn}+\frac{3P}{2nn}\\+\frac{\mathfrak{A}\mathfrak{T}}{nn}+\frac{\mathfrak{C}\mathfrak{R}}{nn}+\frac{\mathfrak{E}\mathfrak{P}}{nn}+\frac{3AT}{nn}+\frac{3EP}{nn}-\frac{3AP}{2nn}\\+\frac{3\mathfrak{A}T}{nn}+\frac{3A\mathfrak{T}}{nn}+\frac{3\mathfrak{C}R}{nn}+\frac{3\mathfrak{E}P}{nn}+\frac{3E\mathfrak{P}}{nn}\end{array}\right.$$

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§. 156. Quaeramus ergo quoque ex formula aſſumta $\int R\,dr$ differentiale; quod erit

$$R =\!=\!=\!=\!=$$

$$ek\sin(r-s)\left(+\mathfrak{A}v'-\mathfrak{A}x'-\mathfrak{D}q'+\mathfrak{E}r'+\frac{1}{n}\mathfrak{P}+\mathfrak{Q}d'-\mathfrak{R}e'-\left(1-\frac{1}{n}\right)\mathfrak{S}-\mathfrak{V}a'+\mathfrak{X}a'\right)$$

$$ek\sin(r+s)\left(+\mathfrak{A}y'-\mathfrak{A}z'-\mathfrak{D}r'+\mathfrak{E}q'-\frac{1}{n}\mathfrak{P}-\mathfrak{Q}e'+\mathfrak{R}d'-\left(1+\frac{1}{n}\right)\mathfrak{T}-\mathfrak{Y}a'+\mathfrak{Z}a'\right)$$

$$ek\sin(2\eta-r+s)\left(-\mathfrak{A}\left(\frac{2}{n}-s'\right)-\mathfrak{D}\left(\frac{2}{n}-p'\right)+\mathfrak{R}c'-\frac{1}{n}\mathfrak{R}-\left(2a-1+\frac{1}{n}\right)\mathfrak{V}\right)$$

$$ek\sin(2\eta+r-s)\left(-\mathfrak{A}\left(\frac{2}{n}-s'\right)-\mathfrak{E}\left(\frac{2}{n}-p'\right)+\mathfrak{Q}c'+\frac{1}{n}\mathfrak{Q}-\left(2a+1-\frac{1}{n}\right)\mathfrak{X}\right)$$

$$ek\sin(2\eta-r-s)\left(-\mathfrak{A}\left(\frac{2}{n}-t'\right)-\mathfrak{D}\left(\frac{2}{n}-p'\right)+\mathfrak{Q}c'+\frac{1}{n}\mathfrak{Q}-\left(2a-1-\frac{1}{n}\right)\mathfrak{Y}\right)$$

$$ek\sin(2\eta+r+s)\left(-\mathfrak{A}\left(\frac{2}{n}-t'\right)-\mathfrak{E}\left(\frac{2}{n}-p'\right)+\mathfrak{R}c'-\frac{1}{n}\mathfrak{R}-\left(2a+1+\frac{1}{n}\right)\mathfrak{Z}\right)$$

§. 157. Ponatur ex differentiatione formae v;

$$S' = \left(1-\frac{1}{n}\right)S-A(v'-x')+Dq'-Er'-\frac{1}{n}P-Qd'+Re'+(V-X)a'$$

$$T' = \left(1+\frac{1}{n}\right)T-A(y'-z')+Dr'-Eq'+\frac{1}{n}P+Qe'-Rd'+(Y-Z)a'$$

$$V' = \left(2a-1+\frac{1}{n}\right)V+A\left(\frac{2}{n}-s'\right)+D\left(\frac{2}{n}-p'\right)-Rc'+\frac{1}{n}R$$

$$X' = \left(2a+1-\frac{1}{n}\right)X+A\left(\frac{2}{n}-s'\right)+E\left(\frac{2}{n}-p'\right)-Qc'-\frac{1}{n}Q$$

$$Y' = \left(2a-1-\frac{1}{n}\right)Y+A\left(\frac{2}{n}-t'\right)+D\left(\frac{2}{n}-p'\right)-Qc'-\frac{1}{n}Q$$

$$Z' = \left(2a+1+\frac{1}{n}\right)Z+A\left(\frac{2}{n}-t'\right)+E\left(\frac{2}{n}-p'\right)-Rc'+\frac{1}{n}R$$

vt

vt habeatur

$$\frac{dv}{dr} = -A'\sin 2\eta - C'k\sin r - D'k\sin(2\eta-r) - E'k\sin(2\eta+r)$$
$$- P'e\sin s - Q'e\sin(2\eta-s) - R'e\sin(2\eta+s)$$
$$- S'ek\sin(r-s) - V'ek\sin(2\eta-r+s) - Y'ek\sin(2\eta-r-s)$$
$$- T'ek\sin(r+s) - X'ek\sin(2\eta+r-s) - Z'ek\sin(2\eta+r+s)$$

§. 158. Haec iam forma denuo differentiata dabit

$$\frac{ddv}{dr^2} = \text{Praec.}$$

$$+ek\cos(r-s)(+A'(v'+x')+D'q'+E'r'-\frac{1}{n}P'+Q'd'+R'e'-(1-\frac{1}{n})S'+(V'+X')a'$$
$$+ek\cos(r+s)(+A'(y'+z')+D'r'+E'q'-\frac{1}{n}P'+Q'e'+R'd'-(1+\frac{1}{n})T'+(Y+Z')a'$$
$$+ek\cos(2\eta-r+s)(-A'(\frac{2}{n}-s')-D'(\frac{2}{n}-p')+R'c'-\frac{1}{n}R'-(2\alpha-1+\frac{1}{n})V')$$
$$+ek\cos(2\eta+r-s)(-A'(\frac{2}{n}-s')-E'(\frac{2}{n}-p')+Q'c'+\frac{1}{n}Q'-(2\alpha+1-\frac{1}{n})X')$$
$$+ek\cos(2\eta-r-s)(-A'(\frac{2}{n}-s')-D'(\frac{2}{n}-p')+Q'c'+\frac{1}{n}Q'-(2\alpha-1-\frac{1}{n})Y')$$
$$+ek\cos(2\eta+r+s)(-A'(\frac{2}{n}-s')-E'(\frac{2}{n}-p')+R'c'-\frac{1}{n}R'-(2\alpha+1+\frac{1}{n})Z')$$

§. 159. Priores autem expreſſiones, ſi litterarum cognitarum valores ſubſtituantur, ſequenti modo prodibunt.

$$R = ek\sin(r-s)(-0{,}44856 + 0{,}00854(V-X))$$
$$ek\sin(r+s)(-0{,}42826 + 0{,}00854(Y-Z))$$
$$ek\sin(2\eta-r+s)(-4{,}51938 + 9{,}00854\,S)$$
$$ek\sin(2\eta+r-s)(-4{,}51938 + 0{,}00854\,S)$$
$$ek\sin(2\eta-r-s)(-4{,}51939 + 0{,}00854\,T)$$
$$ek\sin(2\eta+r+s)(-4{,}51938 + 0{,}00854\,T)$$

§. 160. Altera vero forma pro $\frac{ddv}{dr^2}$ fit

$$\frac{ddv}{dr^2} =$$

$$ek\,\text{cſ}(r-s)(-2{,}83505-2k\mathfrak{S}-\mathfrak{C}S-0{,}02711(\mathfrak{V}+\mathfrak{X})-0{,}03207(V+X))$$
$$ek\,\text{cſ}(r+s)(-3{,}21669-2\varkappa\mathfrak{T}-\mathfrak{C}T-2{,}02711(\mathfrak{Y}+\mathfrak{Z})-0{,}03207(Y+Z))$$
$$ek\,\text{cſ}(2\eta-r+s)(-2{,}26927-2\varkappa\mathfrak{V}-\mathfrak{C}V-0{,}02711\,\mathfrak{S}-0{,}03207\,S)$$
$$ek\,\text{cſ}(2\eta+r-s)(-0{,}53441-2\varkappa\mathfrak{X}-\mathfrak{C}X-0{,}02711\,\mathfrak{S}-0{,}03207\,S)$$
$$ek\,\text{cſ}(2\eta-r-s)(-1{,}36456-2\varkappa\mathfrak{Y}-\mathfrak{C}Y-0{,}02711\,\mathfrak{T}-0{,}03207\,T)$$
$$ek\,\text{cſ}(2\eta+r+s)(-1{,}43912-2\varkappa\mathfrak{Z}-\mathfrak{C}Z-0{,}02711\,\mathfrak{T}-0{,}03207\,T)$$

§. 161. Deinde ſimili modo alterae formulae per differentiationem erutae, ſubſtitutis valoribus cognitis ita ſe habebunt.

$$R =$$

$$ek\,\text{ſin}(r-s)\left(+0{,}59883+\mathfrak{A}(v'-x')\cdot\left(1-\frac{1}{n}\right)\mathfrak{S}\right)+0{,}01974(\mathfrak{V}-\mathfrak{X})$$
$$ek\,\text{ſin}(r+s)\left(+0{,}54556+\mathfrak{A}(y'-z')-\left(1+\frac{1}{n}\right)\mathfrak{T}\right)+0{,}01974(\mathfrak{Y}-\mathfrak{Z})$$
$$ek\,\text{ſin}(2\eta-r+s)\left(+0{,}80251+\mathfrak{A}s'-\left(2\alpha-1+\frac{1}{n}\right)\mathfrak{V}\right)$$
$$ek\,\text{ſin}(2\eta+r-s)\left(+0{,}59936+\mathfrak{A}s'-\left(2\alpha+1-\frac{1}{n}\right)\mathfrak{X}\right)$$
$$ek\,\text{ſin}(2\eta-r-s)\left(+1{,}01199+\mathfrak{A}s'-\left(2\alpha-1-\frac{1}{n}\right)\mathfrak{Y}\right)$$
$$ek\,\text{ſin}(2\eta+r+s)\left(+0{,}38988+\mathfrak{A}s'-\left(\alpha+1+\frac{1}{n}\right)\mathfrak{Z}\right)$$

Porro

Porro reperiemus ſequentes valores

$$S' = (1 - \frac{1}{n}) S - A (v' - x') + 0{,}38693 - 0{,}01974 (V - X)$$

$$T' = (1 + \frac{1}{n}) T - A (y' - z') + 0{,}18018 - 0{,}01974 (Y - Z)$$

$$V' = (2\alpha - 1 + \frac{1}{n})\ V - A s' + 5{,}19902$$

$$X' = (2\alpha + 1 - \frac{1}{n})\ X - A s' - 0{,}87311$$

$$Y' = (2\alpha - 1 - \frac{1}{n})\ Y - A t' + 4{,}76391$$

$$Z' = (2\alpha + 1 + \frac{1}{n})\ Z - A t' - 0{,}43800$$

ac denique $\frac{ddv}{dr^2} = \text{Praec.}$

$$+ ek\,\mathrm{cſ}(r - s)(-(1 - \frac{1}{n})S' + A'(v' + x') + 2{,}62592 - 0{,}01974 (V' + X'))$$

$$+ ek\,\mathrm{cſ}(r + s)(-(1 + \frac{1}{n})T' + A'(y' + z') + 2{,}16417 - 0{,}01974 (Y' + Z'))$$

$$+ ek \,\mathrm{coſ}(2\eta - r + s)(-(2\alpha - 1 + \frac{1}{n})\ V' + A' s' - 4{,}19296)$$

$$+ ek \,\mathrm{coſ}(2\eta + r - s)(-(2\alpha + 1 - \frac{1}{n})\ X' + A' s' + 1{,}59777)$$

$$+ ek \,\mathrm{coſ}(2\eta - r - s)(-(2\alpha - 1 - \frac{1}{n})\ Y' + A' t' - 3{,}48384)$$

$$+ ek \,\mathrm{coſ}(2\eta + r + s)(-(2\alpha + 1 + \frac{1}{n})\ Z' + A' t' + 0{,}88865)$$

§. 162. Hinc ergo pro determinandis coefficientibus ſequentes obtinemus aequationes

$$\left(1-\frac{1}{n}\right)\mathfrak{S}=1{,}04739-0{,}00854(V-X)+\mathfrak{A}(v'-x')+0{,}01974(\mathfrak{V}-\mathfrak{X})$$

$$\left(1+\frac{1}{n}\right)\mathfrak{T}=0{,}97382-0{,}00858(Y-Z)+\mathfrak{A}(y'-z')+0{,}01974(\mathfrak{Y}-\mathfrak{Z})$$

$$\left(2a-1+\frac{1}{n}\right)\mathfrak{V}=5{,}32189-0{,}00854\,S+\mathfrak{A}\,s'$$

$$\left(2a+1-\frac{1}{n}\right)\mathfrak{X}=5{,}11874-0{,}00854\,S+\mathfrak{A}\,s'$$

$$\left(2a-1-\frac{1}{n}\right)\mathfrak{Y}=5{,}53137-0{,}00854\,T+\mathfrak{A}\,t'$$

$$\left(2a+1+\frac{1}{n}\right)\mathfrak{Z}=4{,}90926-0{,}00854\,T+\mathfrak{A}\,t'$$

Deinde

$$+5{,}46097=\left(1-\frac{1}{n}\right)S'-2\varkappa\mathfrak{S}-\beta S-0{,}02711(\mathfrak{V}+\mathfrak{X})-0{,}03207(V+X)$$
$$-A'(v'+x')+0{,}01974(V'+X')$$

$$+5{,}38086=\left(1+\frac{1}{n}\right)T'-2\varkappa\mathfrak{T}-\beta\Gamma-0{,}02711(\mathfrak{Y}+\mathfrak{Z})-0{,}03207(Y+Z)$$
$$-A'(y'+z')+0{,}01974(Y'+Z')$$

$$-1{,}92371=\left(2a-1+\frac{1}{n}\right)V'-2\varkappa\mathfrak{V}-\beta V-0{,}02711\mathfrak{S}-0{,}03207\,S-A's'$$

$$+2{,}13218=\left(2a+1-\frac{1}{n}\right)X'-2\varkappa\mathfrak{X}-\beta X-0{,}02711\mathfrak{S}-0{,}03207\,S-A's'$$

$$-2{,}11928=\left(2a-1-\frac{1}{n}\right)Y'-2\varkappa\mathfrak{Y}-\beta Y-0{,}02711\mathfrak{T}\;0{,}03207\,T-A't'$$

$$+2{,}32777=\left(2a+1+\frac{1}{n}\right)Z'-2\varkappa\mathfrak{Z}-\beta Z-0{,}02711\mathfrak{T}-0{,}03207\,T-A't'$$

§. 163.

§. 163, Pro vlteriori calculo eſt

$$1-\frac{1}{n}=0{,}924562 \quad \ldots \quad l\left(1-\frac{1}{n}\right)=9{,}965935$$

$$1+\frac{1}{n}=1{,}075438 \quad \ldots \quad l\left(1+\frac{1}{n}\right)=0{,}031570$$

$$2\alpha-1+\frac{1}{n}=0{,}942914 \quad \ldots \quad l\left(2\alpha-1+\frac{1}{n}\right)=9{,}965935$$

$$2\alpha+1-\frac{1}{n}=2{,}792038 \quad \ldots \quad l\left(2\alpha+1-\frac{1}{n}\right)=0{,}445915$$

$$2\alpha-1-\frac{1}{n}=0{,}792038 \quad \ldots \quad l\left(2\alpha-1-\frac{1}{n}\right)=9{,}898747$$

$$2\alpha+1+\frac{1}{n}=2{,}942914 \quad \ldots \quad l\left(2\alpha+1+\frac{1}{n}\right)=0{,}468775$$

Hinc in aequationibus poſterioribus valores litterarum S', T', V' ſubſtituantur, et ob $\varepsilon=1{,}01591$, erit

$$\begin{aligned}0{,}16110\,S = & -5{,}10323-2\kappa\mathfrak{S}-A'(v'+x')-0{,}02711(\mathfrak{V}+\mathfrak{X})\\ & +0{,}01974(V'+X')\\ & +1{,}21835(v'-x')-0{,}03207(V+X)\\ & -0{,}01825(V-X)\end{aligned}$$

$$\begin{aligned}0{,}14059\,T = & +5{,}18709+2\kappa\mathfrak{T}+A'(y'+z')+0{,}02711(\mathfrak{Y}+\mathfrak{Z})\\ & -0{,}01974(Y'+Z')\\ & -1{,}41710(y'-z')+0{,}03207(Y+Z)\\ & +0{,}02124(Y-Z)\end{aligned}$$

$$\begin{aligned}0{,}12683\,V = & +6{,}82591-2\kappa\mathfrak{V}+3{,}718\,s'-0{,}02711\,\mathfrak{S}\\ & -0{,}03207\,S\end{aligned}$$

$$\begin{aligned}6{,}77934\,X = & +4{,}56988+2\kappa\mathfrak{X}-6{,}1548\,s'+0{,}02711\,\mathfrak{S}\\ & +0{,}03207\,S\end{aligned}$$

$$\begin{aligned}0{,}38858\,Y = & +5{,}89253-2\kappa\mathfrak{Y}+3{,}5194\,t'-0{,}02711\,\mathfrak{T}\\ & -0{,}03207\,T\end{aligned}$$

$$\begin{aligned}7{,}64474\,Z = & +3{,}61676+2\kappa\mathfrak{Z}-6{,}3536\,t'+0{,}02711\,\mathfrak{T}\\ & +0{,}03207\,T\end{aligned}$$

§. 164.

§. 164. Commodiſſime hi coefficientes inueniri videntur, ſi primo $\mathfrak{V}$, $\mathfrak{X}$, $\mathfrak{Y}$, $\mathfrak{Z}$ et V, X, Y, Z proxime quaerantur, quod fiet terminos minimos negligendo:

$$\mathfrak{V} = +\,5{,}7560 \quad \ldots \quad l\,\mathfrak{V} = 0{,}760125$$
$$\mathfrak{X} = +\,1{,}8334 \quad \ldots \quad l\,\mathfrak{X} = 0{,}263245$$
$$\mathfrak{Y} = +\,6{,}9837 \quad \ldots \quad l\,\mathfrak{Y} = 0{,}844088$$
$$\mathfrak{Z} = +\,1{,}6643 \quad \ldots \quad l\,\mathfrak{Z} = 0{,}221240$$
$$V = -37{,}7650 \quad \ldots \quad l{-}V = 1{,}577086$$
$$X = +\,1{,}2198 \quad \ldots \quad l\,X = 0{,}086293$$
$$Y = -21{,}1040 \quad \ldots \quad l{-}Y = 1{,}324360$$
$$Z = +\,0{,}9125 \quad \ldots \quad l\,Z = 9{,}960209$$
$$V' = -29{,}7170 \quad \ldots \quad v' = -\,0{,}398$$
$$X' = +\,2{,}5326 \quad \ldots \quad x' = +\,0{,}024$$
$$Y' = -11{,}9510 \quad \ldots \quad y' = -\,0{,}201$$
$$Z' = +\,2{,}2472 \quad \ldots \quad z' = +\,0{,}020$$

§. 165. Hic autem valores pro V' et Y' tam fiunt magni, vt viciſſim poſt inuentas litteras S, T nimium valores modo erutos afficiant, vnde neceſſe erit reſolutionem harum aequationum ordinario modo inſtruere. Reperitur ergo

$$\mathfrak{V} = 5{,}7560 - 0{,}0193\,S - 0{,}0050\,\mathfrak{S}$$
$$\mathfrak{X} = 1{,}8333 - 0{,}0064\,S - 0{,}0016\,\mathfrak{S}$$
$$\mathfrak{Y} = 6{,}9837 - 0{,}0225\,T - 0{,}0058\,\mathfrak{T}$$
$$\mathfrak{Z} = 1{,}6643 - 0{,}0061\,T - 0{,}0016\,\mathfrak{T}$$
$$2x\,\mathfrak{V} = 11{,}6155 - 0{,}0390\,S - 0{,}0100\,\mathfrak{S}$$
$$2x\,\mathfrak{X} = 3{,}6996 - 0{,}0129\,S - 0{,}0033\,\mathfrak{S}$$
$$2x\,\mathfrak{Y} = 14{,}0930 - 0{,}0455\,T - 0{,}0118\,\mathfrak{T}$$
$$2x\,\mathfrak{Z} = 3{,}3586 - 0{,}0122\,T - 0{,}0032\,\mathfrak{T}$$

qui

qui valores ſubſtituti dant:

$$V = -37,7647 + 0,3912\,S + 0,0031\,\mathfrak{S}$$
$$X = +\ 1,2198 - 0,0076\,S - 0,0016\,\mathfrak{S}$$
$$Y = -21,1038 + 0,1385\,T + 0,0121\,\mathfrak{T}$$
$$Z = +\ 0,9125 - 0,0069\,T - 0,0016\,\mathfrak{T}$$

$$2\kappa V = -76,2085 + 0,7893\,S + 0,0064\,\mathfrak{S}$$
$$2\kappa X = +\ 2,4616 - 0,0153\,S - 0,0033\,\mathfrak{S}$$
$$2\kappa Y = -42,5870 + 0,2794\,T + 0,0244\,\mathfrak{T}$$
$$2\kappa Z = +\ 1,8413 - 0,0140\,T - 0,0033\,\mathfrak{T}$$

§. 166. Hinc porro valores deriuati erunt

$$V' = -29,7167 + 0,3767\,S + 0,0094\,\mathfrak{S}$$
$$X' = +\ 2,5326 - 0,0061\,S + 0,0029\,\mathfrak{S}$$
$$Y' = -11,9511 + 0,1248\,T + 0,0161\,\mathfrak{T}$$
$$Z' = +\ 2,2472 - 0,0054\,T + 0,0028\,\mathfrak{T}$$
$$v' = -0,4009 + 0,0044\,S + 0,0001\,\mathfrak{S}$$
$$x' = +0,0244$$
$$y' = -0,2026 + 0,0015\,T + 0,0001\,\mathfrak{T}$$
$$z' = +0,0200$$

ac porro

$$\mathfrak{V} - \mathfrak{X} = 3,9227 - 0,0129\,S - 0,0034\,\mathfrak{S}$$
$$\mathfrak{Y} - \mathfrak{Z} = 5,3194 - 0,0164\,T - 0,0042\,\mathfrak{T}$$
$$V - X = -38,9845 + 0,3988\,S + 0,0047\,\mathfrak{S}$$
$$Y - Z = -22,0163 + 0,1454\,T + 0,0137\,\mathfrak{T}$$

$$\mathfrak{V} + \mathfrak{X} = 7,5893 - 0,0257\,S - 0,0066\,\mathfrak{S}$$
$$\mathfrak{Y} + \mathfrak{Z} = 8,6480 - 0,0286\,T - 0,0074\,\mathfrak{T}$$
$$V + X = -36,5449 + 0,3836\,S + 0,0015\,\mathfrak{S}$$
$$Y + Z = -20,1913 + 0,1316\,T + 0,0105\,\mathfrak{T}$$

T V'+X'

$$V'+X' = -27,1841 + 0,3706\,S + 0,0123\,\mathfrak{S}$$
$$Y'+Z' = -9,7039 + 0,1194\,T + 0,0189\,\mathfrak{T}$$
$$v'-x' = -0,4253 + 0,0044\,S + 0,0001\,\mathfrak{S}$$
$$y'-z' = -0,2226 + 0,0015\,T + 0,0001\,\mathfrak{T}$$
$$v'+x' = -0,3765 + 0,0044\,S + 0,0001\,\mathfrak{S}$$
$$y'+z' = -0,1826 + 0,0015\,T + 0,0001\,\mathfrak{T}$$

§. 167. His valoribus ſubſtitutis reperitur

$$\left(1-\frac{1}{n}\right)\mathfrak{S} = +1,8024 - 0,0070\,S$$
$$\left(1+\frac{1}{n}\right)\mathfrak{T} = +1,4453 - 0,0027\,T$$

vnde concluditur

$\mathfrak{S} = 1,9495 - 0,0075\,S$	$2n\mathfrak{S} = 3,9340 - 0,0160\,S$
$\mathfrak{T} = 1,3440 - 0,0025\,T$	$2n\mathfrak{T} = 2,7121 - 0,0053\,T$
$0,1401\,S = -9,2444$	$0,1479\,T = +7,9767$

§. 168. Nunc igitur habebimus

$S = -66,6980$	. . .	$l\text{-}S = 1,824113$
$T = +53,9330$	. . .	$l\,T = 1,731855$
$\mathfrak{S} = +2,4497$	. . .	$l\,\mathfrak{S} = 0,389113$
$\mathfrak{T} = +1,2092$	. . .	$l\,\mathfrak{T} = 0,082498$
$s' = -0,75204$	. . .	$l\text{-}s' = 9,876241$
$t' = +0,62626$	. . .	$l\,t' = 9,796755$
$v' = -0,6942$	. . .	$l\text{-}v' = 9,841484$
$x' = +0,0244$	. . .	$l\,x' = 8,387390$
$y' = -0,1216$	. . .	$l\text{-}y' = 9,084933$
$z' = +0,0200$	. . .	$l\,z' = 9,301030$

$\mathfrak{V} =$

$\mathfrak{V} = +\,7{,}0311 \quad . \;.\; . \quad l\,\mathfrak{V} = 0{,}847029$

$\mathfrak{X} = +\,2{,}2561 \quad . \;.\; . \quad l\,\mathfrak{X} = 0{,}353358$

$\mathfrak{Y} = +\,5{,}7659 \quad . \;.\; . \quad l\,\mathfrak{Y} = 0{,}760867$

$\mathfrak{Z} = +\,1{,}3339 \quad . \;.\; . \quad l\,\mathfrak{Z} = 0{,}125123$

$V = -\,63{,}8498 \quad . \;.\; . \quad l\text{-}V = 1{,}805160$

$X = +\,1{,}7451 \quad . \;.\; . \quad l\,X = 0{,}241820$

$Y = -\,13{,}6222 \quad . \;.\; . \quad l\text{-}Y = 1{,}134241$

$Z = +\,0{,}5356 \quad . \;.\; . \quad l\,Z = 9{,}728840$

§. 169. His iam valoribus inuentis pro diſtantia lunae $x = \frac{(1-kk)\,au}{1-k\cos r}$ erit valoris ipſius u portio ab his terminis pendens:

	Log. coeff.
$u = \text{Praec.} - 0{,}3796\; ek \cos(r-s)$	9,579297
$+\,0{,}3069\; ek \cos(r+s)$	9,487093
$-\,0{,}3634\; ek \cos(2\eta-r+s)$	9,560344
$+\,0{,}0099\; ek \cos(2\eta+r-s)$	7,997004
$-\,0{,}0775\; ek \cos(2\eta-r-s)$	8,889425
$+\,0{,}0030\; ek \cos(2\eta+r+s)$	7,484024

et pro longitudine lunae

$$\frac{d\Phi}{dr} = \text{Praec.} + 0{,}7520\; ek \cos(r-s)$$
$$-\,0{,}6263\; ek \cos(r+s)$$
$$+\,0{,}6942\; ek \cos(2\eta-r+s)$$
$$-\,0{,}0244\; ek \cos(2\eta+r-s)$$
$$+\,0{,}1216\; ek \cos(2\eta-r-s)$$
$$-\,0{,}0200\; ek \cos(2\eta+r+s)$$

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cuius integrale ſi ponatur:

$$\varphi = \text{Praec.} + \mathfrak{S}'ek\,\text{ſn}(r-s) + \mathfrak{V}'ek\,\text{ſn}(2\eta-r+s) + \mathfrak{Y}'ek\,\text{ſn}(2\eta-r-s)$$
$$+ \mathfrak{T}'ek\,\text{ſn}(r+s) + \mathfrak{X}'ek\,\text{ſn}(2\eta+r-s) + \mathfrak{Z}ek\,\text{ſn}(2\eta+r+s)$$

erit

$$+0{,}7520 = \left(1-\frac{1}{n}\right)\mathfrak{S}' - \mathfrak{A}'(v'+x') - \mathfrak{D}'q' - \mathfrak{C}'r' + \frac{1}{n}\mathfrak{P}' - \mathfrak{Q}'d' - \mathfrak{R}'c' - (\mathfrak{V}'+\mathfrak{X}')a'$$

$$-0{,}6263 = \left(1+\frac{1}{n}\right)\mathfrak{T}' - \mathfrak{A}'(y'+z') - \mathfrak{D}'r' - \mathfrak{C}'q' + \frac{1}{n}\mathfrak{P}' - \mathfrak{Q}'c' - \mathfrak{R}'d' - (\mathfrak{Y}'+\mathfrak{Z}')a'$$

$$+0{,}6942 = \left(2a-1+\frac{1}{n}\right)\mathfrak{V}' + \mathfrak{A}'\left(\frac{2}{n}-s'\right) + \mathfrak{D}'\left(\frac{2}{n}-p'\right) - \mathfrak{R}'\left(c'-\frac{1}{n}\right)$$

$$-0{,}0244 = \left(2a+1-\frac{1}{n}\right)\mathfrak{X}' + \mathfrak{A}'\left(\frac{2}{n}-r'\right) + \mathfrak{C}'\left(\frac{2}{n}-p'\right) - \mathfrak{Q}'\left(c'+\frac{1}{n}\right)$$

$$+0{,}1216 = \left(2a-1-\frac{1}{n}\right)\mathfrak{Y} + \mathfrak{A}'\left(\frac{2}{n}-s'\right) + \mathfrak{D}'\left(\frac{2}{n}-p'\right) - \mathfrak{Q}'\left(c'+\frac{1}{n}\right)$$

$$-0{,}0200 = \left(2a+1+\frac{1}{n}\right)\mathfrak{Z} + \mathfrak{A}'\left(\frac{2}{n}-r'\right) + \mathfrak{C}'\left(\frac{2}{n}-p'\right) - \mathfrak{R}'\left(c'-\frac{1}{n}\right)$$

§. 170. Hinc autem reperitur

$\mathfrak{S}' = +0{,}7467$	. . .	$l\,\mathfrak{S}' = 9{,}873165$
$\mathfrak{T}' = -0{,}6185$	. . .	$l-\mathfrak{T}' = 9{,}791317$
$\mathfrak{V}' = +0{,}8143$	. . .	$l\,\mathfrak{V}' = 9{,}910800$
$\mathfrak{X}' = -0{,}0142$	. . .	$l-\mathfrak{X}' = 8{,}150690$
$\mathfrak{Y}' = +0{,}2396$	. . .	$l\,\mathfrak{Y}' = 9{,}379550$
$\mathfrak{Z}' = -0{,}0061$	. . .	$l-\mathfrak{Z}' = 7{,}788910$

§. 171.

§. 171. Quatenus ergo longitudo Lunae ab excentricitate orbitae ſolis pendet, erit

		log coeff.
$\varphi =$ Praec.	$+\ 0{,}201385\,e\ \text{ſin}\ s$	9,304026
	$-\ 0{,}021889\,e\ \text{ſin}\,(2\eta - s)$	8,340237
	$-\ 0{,}016368\,e\ \text{ſin}\,(2\eta + s)$	8,214002
	$+\ 0{,}06615\,ee\ \text{ſin}\ 2s$	8,820508
	$+\ 0{,}02332\,ee\ \text{ſin}\,(2\eta - 2s)$	8,367825
	$+\ 0{,}00840\,ee\ \text{ſin}\,(2\eta + 2s)$	7,924429
	$+\ 0{,}7476\,e\,k\ \text{ſin}\,(r - s)$	9,873165
	$-\ 0{,}6185\,e\,k\ \text{ſin}\,(r + s)$	9,791317
	$+\ 0{,}8143\,e\,k\ \text{ſin}(2\eta - r + s)$	9,910800
	$-\ 0{,}0142\,e\,k\ \text{ſin}(2\eta + r - s)$	8,150690
	$+\ 0{,}2396\,e\,k\ \text{ſin}\,(2\eta - r - s)$	9,379550
	$-\ 0{,}0061\,e\,k\ \text{ſin}(2\eta + r + s)$	7,788910

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§. 172. Hae autem ſingulae inaequalitates ad numerum minutorum ſecundorum reductae dabunt:

		log. coeff.
$\Phi =$ Praec.	$+ 701'', 1 \sin s$	2,845780
	$- 75, 8 \sin(2\eta - s)$	1,879971
	$- 56, 7 \sin(2\eta + s)$	1,753736
	$+ 3, 8 \sin 2s$	0,585551
	$+ 1, 4 \sin(2\eta - 2s)$	0,132862
	$+ 0, 5 \sin(2\eta + 2s)$	9,689472
	$+ 140, 9 \sin(r - s)$	2,148800
	$- 116, 7 \sin(r + s)$	2,067000
	$+ 153, 7 \sin(2\eta - r + s)$	2,186530
	$- 2, 7 \sin(2\eta + r - s)$	0,426300
	$+ 45, 2 \sin(2\eta - r - s)$	1,655200
	$+ 1, 2 \sin(2\eta + r + s)$	0,064600

Hic ſcilicet et inaequalitates, quas in capite praecedente inuenimus, et iſtas in hoc capite erutas ſimul ſum complexus, vt coniunctim conſpectui exponerentur.

CAPUT

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CAPUT XI.

INUESTIGATIO INAEQUALITATUM LUNAE A PARALLAXI SOLIS PENDENTIUM.

§. 173.

Jam in formulis noſtris primariis ad eos quoque terminos progrediamur, qui littera ν ſunt affecti, et quoniam eſt $1:\nu$ vt diſtantia Solis media ad diſtantiam Lunae mediam a Terra, erit $1:\nu$ vt parallaxis Lunae media ad parallaxin ſolis: ex quo inaequalitates Lunae, quae hinc oriuntur, a parallaxi ſolis pendere dicuntur. Quoniam vero valor ipſius ν eſt valde paruus, quippe $\frac{1}{380}$ propemodum, alios terminos non contemplabimur, niſi qui per ν ac per νk et νe ſunt multiplicati, propterea quod magis compoſiti fiant minimi.

§. 174. Ex terminis ergo iam inuentis hic retineamus eos, qui ſunt alicuius momenti, et cum iis novos determinandos coniungamus; ſit ergo:

$$\begin{aligned}\int R\,dr = \mathfrak{A}\,\text{cſ}\,2\eta &+ \mathfrak{C}k\,\text{cſ}\,r + \mathfrak{D}k\,\text{cſ}(2\eta-r) + \mathfrak{P}e\,\text{cſ}\,s + \mathfrak{Q}e\,\text{cſ}(2\eta-s)\\ &+ \mathfrak{E}k\,\text{cſ}(2\eta+r) \qquad + \mathfrak{R}e\,\text{cſ}(2\eta+s)\\ &+ \mathfrak{F}\nu\,\text{coſ}\,\nu \quad + \mathfrak{H}\nu k\,\text{coſ}(\eta-r) \quad + \mathfrak{K}\nu e\,\text{coſ}(\eta-s)\\ &+ \mathfrak{G}\nu\,\text{coſ}\,3\eta \quad + \mathfrak{J}\nu k\,\text{coſ}(\eta+r) \quad + \mathfrak{L}\nu e\,\text{coſ}(\eta+s)\end{aligned}$$

$$\begin{aligned}v = A\,\text{coſ}\,2\eta \;.\;. &+ Dk\,\text{coſ}(2\eta-r) + Pe\,\text{coſ}\,s + Qe\,\text{coſ}(2\eta-s)\\ &+ Ek\,\text{coſ}(2\eta+r) \qquad + Re\,\text{coſ}(2\eta+s)\\ &+ F\nu\,\text{coſ}\,\eta \quad + H\nu k\,\text{coſ}(\eta-r) \quad + K\nu e\,\text{coſ}(\eta-s)\\ &+ G\nu\,\text{coſ}\,3\eta \quad + J\nu k\,\text{coſ}(\eta+r) \quad + L\nu e\,\text{coſ}(\eta+s)\end{aligned}$$

Non

Non difficulter enim praeuidere licet, terminos, qui angulos r et s cum angulo 3η habeant coniunctos, fore tam exiguos, vt ſine errore praetermitti queant.

§. 175. Quodſi iam retentis litterarum §. 153. inductarum valoribus, praeterea ponamus:

$$f' = \frac{2\varkappa F + \mathfrak{F}}{nn} \;;\; g' = \frac{2\varkappa G + \mathfrak{G}}{nn} \;;\; h' = \frac{2\varkappa H + \mathfrak{H}}{nn}$$

$$i' = \frac{2\varkappa J + \mathfrak{J}}{nn} \;;\; k' = \frac{2\varkappa K + \mathfrak{K}}{nn} \;;\; l' = \frac{2\varkappa L + \mathfrak{L}}{nn}$$

habebimus:

$$\begin{aligned}\frac{d\Phi}{dr} = \text{Praec.} - a'\,\text{cſ}\,2\eta - \frac{\mathfrak{C}}{nn}k\,\text{cſ}\,r - d'k\,\text{cſ}(2\eta - r) - p'e\,\text{cſ}\,s - q'e\,\text{cſ}(2\eta - s)\\ - e'k\,\text{cſ}(2\eta + r) \qquad - r'e\,\text{cſ}(2\eta + s)\\ - f'v\,\text{coſ}\,\eta - h'vk\,\text{coſ}(\eta - r) - k've\,\text{coſ}(\eta - s)\\ - g'v\,\text{coſ}\,3\eta - i'vk\,\text{coſ}(\eta + r) - l've\,\text{coſ}(\eta + s)\end{aligned}$$

atque

$$\begin{aligned}\frac{d\eta}{dr} = \alpha - a'\,\text{cſ}\,2\eta - c'k\,\text{cſ}\,r - d'k\,\text{cſ}(2\eta - r) + \left(\frac{2}{n} - p'\right)e\,\text{cſ}\,s - q'e\,\text{cſ}(2\eta - s)\\ - e'k\,\text{cſ}(2\eta + r) \qquad - r'e\,\text{cſ}(2\eta + s)\\ - f'v\,\text{coſ}\,\eta - h'vk\,\text{coſ}(\eta - r) - k've\,\text{coſ}(\eta - s)\\ - g'v\,\text{coſ}\,\eta - i'vk\,\text{coſ}(\eta + s) - l've\,\text{coſ}(\eta + s)\end{aligned}$$

§. 176. Jam vero pro his terminis ab v pendentibus ſequentes colligemus aequationes.

$$R = v\,\text{ſin}\,\eta\left(\tfrac{3}{8} + \frac{3F - 3G}{2nn}\right)$$

$$v\,\text{ſin}\,3\eta\left(\tfrac{15}{8} + \frac{3F}{2nn}\right)$$

vk

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$$v\,k\,\text{ſin}\,(\eta-r)\left(\tfrac{1}{2}\tfrac{f}{g}+\frac{3J}{2nn}+\frac{3F}{nn}-\frac{3G}{nn}\right)$$

$$v\,k\,\text{ſin}\,(\eta+r)\left(\tfrac{1}{2}\tfrac{f}{g}+\frac{3H}{2nn}-\frac{3G}{nn}+\frac{3F}{nn}\right)$$

$$v\,\varepsilon\,\text{ſin}\,(\eta-s)\left(-\tfrac{3}{4}+\frac{3L}{2nn}-\frac{9F}{4nn}+\frac{9G}{4nn}\right)$$

$$v\,\varepsilon\,\text{ſin}\,(\eta+s)\left(-\tfrac{3}{4}+\frac{3K}{2nn}+\frac{9G}{4nn}-\frac{9F}{4nn}\right)$$

$$\frac{ddv}{dr^2}=\!=\!=$$

$$v\,\text{coſ}\,\eta\quad\left\{\tfrac{e}{g}-\mathfrak{C}F+\frac{3F}{4nn}+\frac{3G}{4nn}-2\varkappa\mathfrak{F}+\frac{\mathfrak{A}\mathfrak{F}}{nn}+\frac{\mathfrak{A}\mathfrak{G}}{nn}\right.$$

$$v\,\text{coſ}\,3\eta\quad\left\{\tfrac{\gamma}{g}-\mathfrak{C}G+\frac{3F}{4nn}-2\varkappa\mathfrak{G}+\frac{\mathfrak{A}\mathfrak{F}}{nn}\right.$$

$$v\,k\,\text{coſ}\,(\eta-r)\left\{\begin{array}{l}\tfrac{4}{1}\tfrac{f}{g}-\mathfrak{C}H+\tfrac{1}{2}bF+\dfrac{3J}{4nn}+\dfrac{3F}{2nn}+\dfrac{3G}{2nn}-2\varkappa\mathfrak{H}\\[2mm] +\dfrac{\mathfrak{A}\mathfrak{J}}{nn}+\dfrac{\mathfrak{C}\mathfrak{F}}{nn}+\dfrac{\mathfrak{D}\mathfrak{F}}{nn}+\dfrac{\mathfrak{E}\mathfrak{G}}{nn}\end{array}\right.$$

$$v\,k\,\text{coſ}\,(\eta+r)\left\{\begin{array}{l}\tfrac{4}{1}\tfrac{f}{g}-\mathfrak{C}J+\tfrac{1}{2}bF+\dfrac{3H}{4nn}+\dfrac{3G}{2nn}+\dfrac{3F}{2nn}-2\varkappa\mathfrak{J}\\[2mm] +\dfrac{\mathfrak{A}\mathfrak{H}}{nn}+\dfrac{\mathfrak{C}\mathfrak{F}}{nn}+\dfrac{\mathfrak{D}\mathfrak{G}}{nn}+\dfrac{\mathfrak{E}\mathfrak{F}}{nn}\end{array}\right.$$

$$v\,e\cos(\eta - s)\left\{\begin{array}{l} -\frac{9}{4} - 6K + \frac{3L}{4nn} - \frac{3F}{4nn} - \frac{9F}{8nn} - \frac{9G}{8nn} - 2x\mathfrak{K} \\ + \frac{\mathfrak{A}\mathfrak{L}}{nn} + \frac{\mathfrak{F}\mathfrak{P}}{nn} + \frac{\mathfrak{F}\mathfrak{Q}}{nn} + \frac{\mathfrak{G}\mathfrak{R}}{nn} \end{array}\right.$$

$$v\,e\cos(\eta + s)\left\{\begin{array}{l} -\frac{9}{4} - 6L + \frac{3K}{4nn} - \frac{3F}{4nn} - \frac{9F}{8nn} - \frac{9G}{8nn} - 2x\mathfrak{L} \\ + \frac{\mathfrak{A}\mathfrak{K}}{nn} + \frac{\mathfrak{F}\mathfrak{P}}{nn} + \frac{\mathfrak{F}\mathfrak{R}}{nn} + \frac{\mathfrak{G}\mathfrak{Q}}{nn} \end{array}\right.$$

ſequentes partes ſeorſim exponamus:

$$v\cos\eta\left\{\begin{array}{l} + \frac{3\mathfrak{A}F}{nn} + \frac{3\mathfrak{A}G}{nn} + \frac{3A\mathfrak{F}}{nn} + \frac{3A\mathfrak{G}}{nn} + \frac{3AF}{nn} + \frac{3AG}{nn} \\ + \frac{9A}{8nn} + \frac{15A}{8nn} \end{array}\right.$$

$$v\cos 3\eta\left\{ + \frac{3\mathfrak{A}F}{nn} + \frac{3\mathfrak{A}\mathfrak{F}}{nn} + \frac{3AF}{nn} + \frac{9A}{8nn}\right.$$

$$v\,k\cos(\eta - r)\left\{\begin{array}{l} \frac{3\mathfrak{A}J}{nn} + \frac{3\mathfrak{C}F}{nn} + \frac{3\mathfrak{D}F}{nn} + \frac{3\mathfrak{E}G}{nn} + \frac{3A\mathfrak{J}}{nn} + \frac{3D\mathfrak{F}}{nn} \\ + \frac{3E\mathfrak{G}}{nn} + \frac{3AJ}{nn} + \frac{3DF}{nn} + \frac{3EG}{nn} - \frac{3AF}{nn} - \frac{3AG}{nn} \\ + \frac{9D}{8nn} + \frac{15E}{8nn} \end{array}\right.$$

$$v\,k\cos(\eta + r)\left\{\begin{array}{l} \frac{3\mathfrak{A}H}{nn} + \frac{3\mathfrak{C}F}{nn} + \frac{3\mathfrak{D}\mathfrak{G}}{nn} + \frac{3\mathfrak{E}F}{nn} + \frac{3A\mathfrak{H}}{nn} \\ + \frac{3D\mathfrak{G}}{nn} + \frac{3E\mathfrak{F}}{nn} + \frac{3AH}{nn} + \frac{3DG}{nn} + \frac{3EF}{nn} \\ - \frac{3AF}{2nn} - \frac{3AG}{2nn} + \frac{9E}{8nn} + \frac{15D}{8nn} \end{array}\right.$$

ve coſ

$$v\,e\cos(\eta-s)\left\{\begin{array}{l}\frac{3\mathfrak{A}L}{nn}+\frac{3\mathfrak{P}F}{nn}+\frac{3\mathfrak{Q}F}{nn}+\frac{3\mathfrak{R}G}{nn}+\frac{3A\mathfrak{L}}{nn}\\+\frac{3P\mathfrak{F}}{nn}+\frac{3Q\mathfrak{F}}{nn}+\frac{3R\mathfrak{G}}{nn}+\frac{3AL}{nn}+\frac{3PF}{nn}+\frac{3QF}{nn}\\+\frac{3RG}{nn}+\frac{9P}{8nn}+\frac{9Q}{8nn}+\frac{15R}{8nn}\end{array}\right.$$

$$v\,e\cos(\eta+s)\left\{\begin{array}{l}\frac{3\mathfrak{A}K}{nn}+\frac{3\mathfrak{P}F}{nn}+\frac{3\mathfrak{Q}G}{nn}+\frac{3\mathfrak{R}F}{nn}+\frac{3A\mathfrak{K}}{nn}+\frac{3PF}{nn}\\+\frac{3Q\mathfrak{G}}{nn}+\frac{3R\mathfrak{F}}{nn}+\frac{3AK}{nn}+\frac{3PF}{nn}+\frac{3QF}{nn}+\frac{3QG}{nn}\\+\frac{9P}{8nn}+\frac{9R}{8nn}+\frac{15Q}{8nn}\end{array}\right.$$

§. 177. Verum differentiando nanciſcemur

$$R =$$

$$v\sin\eta\quad\{\mathfrak{A}f'-\mathfrak{A}g'-\alpha\mathfrak{F}-\tfrac{1}{2}\mathfrak{F}a'+\tfrac{1}{2}\mathfrak{G}a'$$

$$v\sin 3\eta\quad\{\mathfrak{A}f'+\tfrac{1}{2}\mathfrak{F}a'-3\alpha\mathfrak{G}$$

$$v\,k\sin(\eta-r)\{\mathfrak{A}e'+\mathfrak{D}f'-\mathfrak{E}g'+\tfrac{1}{2}\mathfrak{F}c'-\tfrac{1}{2}\mathfrak{F}d'+\tfrac{1}{2}\mathfrak{G}c'-(\alpha-1)\mathfrak{H}-\tfrac{1}{2}\mathfrak{I}a'$$

$$v\,k\sin(\eta+r)\left\{\begin{array}{l}\mathfrak{A}h'-\mathfrak{D}g'+\mathfrak{E}f'+\tfrac{1}{2}\mathfrak{F}c'-\tfrac{1}{2}\mathfrak{F}e'+\tfrac{1}{2}\mathfrak{G}d'-\tfrac{1}{2}\mathfrak{H}a'\\\qquad\qquad\qquad\qquad\qquad\qquad-(\alpha+1)\mathfrak{I}\end{array}\right.$$

$$v\,e\sin(\eta-s)\left\{\begin{array}{l}+\mathfrak{A}i'+\mathfrak{Q}f'-\mathfrak{R}g'-\tfrac{1}{2}(\frac{2}{n}-p')\mathfrak{F}-\tfrac{1}{2}\mathfrak{F}q'+\tfrac{1}{2}\mathfrak{G}r'\\\qquad\qquad\qquad\qquad\qquad-(\alpha-\frac{1}{n})\mathfrak{K}-\tfrac{1}{2}\mathfrak{L}a'\end{array}\right.$$

$$v\,e\sin(\eta+s)\left\{\begin{array}{l}+\mathfrak{A}k'-\mathfrak{Q}g'+\mathfrak{R}f'-\tfrac{1}{2}(\frac{2}{n}-p')\mathfrak{F}-\tfrac{1}{2}\mathfrak{F}r'+\tfrac{1}{2}\mathfrak{G}q'\\\qquad\qquad\qquad\qquad\qquad-(\alpha+\frac{1}{n})\mathfrak{L}-\tfrac{1}{2}\mathfrak{K}a'\end{array}\right.$$

 Deinde

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Deinde poſito

$$F' = \alpha\ F - A(f' - g') + \tfrac{1}{2} F a' - \tfrac{3}{2} G a'$$
$$G' = 3\alpha\ G - A f' - \tfrac{1}{2} F a'$$
$$H' = (\alpha - 1) H - A i' - D f' + E g' - \tfrac{1}{2} F c' + \tfrac{1}{2} F d' - \tfrac{3}{2} G e' + \tfrac{1}{2} J a'$$
$$J' = (\alpha + 1) J - A b' + D g' - E f' - \tfrac{1}{2} F c' + \tfrac{1}{2} F e' - \tfrac{3}{2} G d' + \tfrac{1}{2} H a'$$
$$K' = (\alpha - 1) K - A l' - Q f' + R g' + \tfrac{1}{2} F\left(\frac{2}{n} - p'\right) + \tfrac{1}{2} F q' - \tfrac{3}{2} G r' + \tfrac{1}{2} L a'$$
$$L' = (\alpha + 1) L - A k' + Q g' - R f' + \tfrac{1}{2} F\left(\frac{2}{n} - p'\right) + \tfrac{1}{2} F r' - \tfrac{3}{2} G q' + \tfrac{1}{2} K a'$$

erit

$$\begin{aligned}\frac{dv}{dr} = & - A' \text{ſin}\, 2\eta - D' k\, \text{ſin}(2\eta - r) - P' e\, \text{ſin}\, s - Q' e\, \text{ſin}(2\eta - s) \\ & - E' k\, \text{ſin}(2\eta + r) \qquad\qquad - R' e\, \text{ſin}(2\eta + s) \\ & - F' v\, \text{ſin}\, \eta - H' v k\, \text{ſin}(\eta - r) - K' v e\, \text{ſin}(\eta - s) \\ & - G' v\, \text{ſin}\, 3\eta - J' v k\, \text{ſin}(\eta + r) - L' v e\, \text{ſin}(\eta + s)\end{aligned}$$

§. 178. Hinc iam denuo differentiando conſequemur: $\frac{ddv}{dr^2} = \text{Praec.}$

$$\begin{aligned}& + v \cos\, \eta\ \left[+ A' f' + A' g' - \alpha F' + \tfrac{1}{2} F' a' + \tfrac{3}{2} G' a' \right. \\ & + v \cos\, 3\eta\ \left[+ A' f' + \tfrac{1}{2} F' a' - 3\alpha G' \right. \\ & + v k \cos(\eta - r) \left\{ \begin{array}{l} + A' i' + D' f' + E' g' + \tfrac{1}{2} F' c' + \tfrac{1}{2} F' d' + \tfrac{3}{2} G' e' \\ \qquad - (\alpha - 1) H' + \tfrac{1}{2} J' a' \end{array} \right. \\ & + v k \cos(\eta + r) \left\{ \begin{array}{l} + A' b' + E' f' + D' g' + \tfrac{1}{2} F' c' + \tfrac{1}{2} F' e' + \tfrac{3}{2} G' d' \\ \qquad + \tfrac{1}{2} H' a' - (\alpha + 1) J' \end{array} \right. \\ & + v e \cos(\eta - s) \left\{ \begin{array}{l} + A' l' + Q' f' + R' g' - \tfrac{1}{2}\left(\frac{2}{n} - p'\right) F' + \tfrac{1}{2} F' g' + \tfrac{3}{2} G' r' \\ \qquad - \left(\alpha - \frac{1}{n}\right) K' + \tfrac{1}{2} L' a' \end{array} \right.\end{aligned}$$

+

$$+\nu e\cos(\eta+s)\left\{\begin{array}{l}+A'k'+R'f'+Q'g'-\tfrac{1}{2}\left(\tfrac{2}{n}-p'\right)F'+\tfrac{1}{2}F'r'+\tfrac{1}{2}G'q'\\ \qquad\qquad\qquad\qquad +\tfrac{1}{2}K's'-\left(\alpha+\tfrac{1}{n}\right)L'\end{array}\right.$$

qui valores cum antecedentibus comparari debent, vt inde valores coefficientium eliciantur.

§. 179. Sumamus primo duos valores ab initio poſitos, quoniam hi a ſequentibus non pendent, atque habebimus,

$$0=\alpha\mathfrak{F}-\mathfrak{A}f'+\mathfrak{A}g'+\tfrac{1}{2}\mathfrak{F}a'-\tfrac{1}{2}\mathfrak{G}a'+\tfrac{3}{8}+\frac{3F-3G}{2nn}$$

$$0=3\alpha\mathfrak{G}-\mathfrak{A}f'-\tfrac{1}{2}\mathfrak{F}a'+\mathfrak{V}'+\frac{3F}{2nn}$$

$$0=\alpha F'-A'f'-A'g'-\tfrac{1}{2}F'a'-\tfrac{1}{2}G'a'+\tfrac{3}{8}-\mathfrak{E}F-2\varkappa\mathfrak{F}$$
$$+\frac{3(F+G)}{4nn}+\frac{(\mathfrak{A}+3A)}{nn}(\mathfrak{F}+\mathfrak{G})+\frac{(3A+3A)}{nn}(F+G)+\frac{3A}{nn}$$

$$0=3\alpha G'-A'f'-\tfrac{1}{2}F'a'+\mathfrak{V}-\mathfrak{E}G-2\varkappa G$$
$$+\frac{3F}{4nn}+\frac{(\mathfrak{A}+3A)}{nn}\mathfrak{F}+\frac{(3\mathfrak{A}+3A)}{nn}F+\frac{9A}{8nn}$$

et $F'=\alpha F-A(f'-g')+\tfrac{1}{2}Fa'-\tfrac{1}{2}Ga'$

$G'=3\alpha G-Af'-\tfrac{1}{2}Fa'$

at eſt $f'=\dfrac{2\varkappa F+\mathfrak{F}}{nn}$ et $g'=\dfrac{2\varkappa G+\mathfrak{G}}{nn}$.

Hic igitur primum litterarum, quae ſunt cognitae, valores in numeris ſubſtituantur, eritque

$F' = 0{,}93905\,F + 0{,}00753\,\mathfrak{F} + 0{,}01443\,G - 0{,}00753\,\mathfrak{G}$
$G' = 2{,}80121\,G + 0{,}02505\,F + 0{,}00753\,\mathfrak{F}$
$0 = 0{,}92847\,\mathfrak{F} + 0{,}01776\,F - 0{,}01776\,G - 0{,}02501\,\mathfrak{G} + 0{,}37500$
$0 = 2{,}80121\,\mathfrak{G} + 0{,}01447\,\mathfrak{F} + 0{,}01776\,F + 1{,}87500$.
hincque

$$\mathfrak{F} = -0{,}01930\,F + 0{,}01913\,G - 0{,}42145$$
$$\mathfrak{G} = -0{,}00623\,F - 0{,}00010\,G - 0{,}66717$$

§. 180. Inde porro colligemus

$$F' = 0{,}93895\,F + 0{,}01457\,G + 0{,}00185$$
$$G' = 0{,}02491\,F + 2{,}80135\,G - 0{,}00317$$
$$f' = 0{,}01138\,F + 0{,}00011\,G - 0{,}00240$$
$$g' = -0{,}00004\,F + 0{,}01148\,G - 0{,}00380$$

quibus valoribus ſubſtitutis peruenimus ad has aequationes:

$$0{,}09337\,F = +0{,}05421\,G + 1{,}96867$$
$$6{,}83135\,G = -0{,}08830\,F - 3{,}20944$$

vnde fit:

$F = 20{,}65700$ . . . $l\,F = 1{,}315067$
$G = -0{,}73681$ . . . $l{-}G = 9{,}867356$
$\mathfrak{F} = -0{,}83418$ . . . $l{-}\mathfrak{F} = 9{,}921260$
$\mathfrak{G} = -0{,}79580$ . . . $l{-}\mathfrak{G} = 9{,}900604$
$F' = +19{,}38712$ . . . $l\,F' = 1{,}287512$
$G' = -1{,}55266$ . . . $l{-}G' = 0{,}191075$
$f' = +0{,}23259$ . . . $l\,f' = 9{,}366501$
$g' = -0{,}01309$ . . . $l{-}g' = 8{,}116940$

§. 181.

§. 181. His valoribus, qui ad inaequalitates abſolutas pertinent, expeditis, progrediamur ad eos, qui ab excentricitate orbitae lunaris pendent, ac his aequationibus continentur:

$$(\alpha-1)\mathfrak{H} - \mathfrak{A}i' + \tfrac{1}{2}\mathfrak{J}a' - \mathfrak{D}f' + \mathfrak{E}g' - \tfrac{1}{2}\mathfrak{F}(c'-d') - \tfrac{3}{2}\mathfrak{G}e'$$
$$+ \tfrac{3}{2}\tfrac{\zeta}{\delta} + \frac{3J}{2nn} + \frac{3(F-G)}{nn} = 0$$

$$(\alpha+1)\mathfrak{J} - \mathfrak{A}b' + \tfrac{1}{2}\mathfrak{H}a' + \mathfrak{D}g' - \mathfrak{E}f' - \tfrac{1}{2}\mathfrak{F}(c'-e') - \tfrac{3}{2}\mathfrak{G}d'$$
$$+ \tfrac{3}{2}\tfrac{\zeta}{\delta} + \frac{3H}{2nn} + \frac{3(F-G)}{nn} = 0$$

$$H' = (\alpha-1)H - Ai' + \tfrac{1}{2}Ja' - Df' + Eg' - \tfrac{1}{2}F(c'-d') - \tfrac{3}{2}Ge'$$
$$J' = (\alpha+1)J - Ab' + \tfrac{1}{2}Ha' + Dg' - Ef' - \tfrac{1}{2}F(c'-e') - \tfrac{3}{2}Gd'$$

$$(\alpha-1)H' - A'i' - \tfrac{1}{2}J'a' - D'f' - E'g' - \tfrac{1}{2}F'(c'+d') - \tfrac{3}{2}G'e'$$
$$+ \tfrac{4}{3}\tfrac{\zeta}{\delta} - 6H - 2n\mathfrak{H} + \tfrac{1}{2}bF + \frac{3J}{4nn} + \frac{3(F+G)}{2nn} + \frac{\mathfrak{E}\mathfrak{F}}{nn} + \frac{3\mathfrak{E}F}{nn}$$
$$+ \frac{(\mathfrak{A}+3A)}{nn}\mathfrak{J} + \frac{(3\mathfrak{A}+3A)}{nn}J + \frac{(\mathfrak{D}+3D)}{nn}\mathfrak{F} + \frac{(3\mathfrak{D}+3D)}{nn}F$$
$$+ \frac{(\mathfrak{E}+3E)}{nn}\mathfrak{G} + \frac{(3\mathfrak{E}+3E)}{nn}G - \frac{3A(F+G)}{2nn} + \frac{9D+15E}{8nn} = 0$$

$$(\alpha+1)J' - A'b' - \tfrac{1}{2}H'a' - E'f' - D'g' - \tfrac{1}{2}F'(c'+e') - \tfrac{3}{2}G'd'$$
$$+ \tfrac{4}{3}\tfrac{\zeta}{\delta} - 6J - 2n\mathfrak{J} + \tfrac{1}{2}bF + \frac{3H}{4nn} + \frac{3(F+G)}{2nn} + \frac{\mathfrak{E}\mathfrak{F}}{nn} + \frac{3\mathfrak{E}F}{nn}$$
$$+ \frac{(\mathfrak{A}+3A)}{nn}\mathfrak{H} + \frac{(3\mathfrak{A}+3A)}{nn}H + \frac{(\mathfrak{D}+3D)}{nn}\mathfrak{G} + \frac{(3\mathfrak{D}+3D)}{nn}G$$
$$+ \frac{(\mathfrak{E}+3E)}{nn}\mathfrak{F} + \frac{(3\mathfrak{E}+3E)}{nn}F - \frac{3A(F+G)}{2nn} + \frac{9E+15D}{8nn} = 0$$

§. 182.

§. 182. In his aequationibus ſubſtituantur valores iam cogniti, atque obtinebimus,

$$-0{,}06626\,\mathfrak{H} + 0{,}81033\,i' - 0{,}00987\,\mathfrak{J} + 0{,}00854\,J \pm 2{,}04620 = 0$$

$$1{,}83374\,\mathfrak{J} + 0{,}81033\,b' - 0{,}00987\,\mathfrak{H} + 0{,}00854\,H + 2{,}10646 = 0$$

$$H' = -0{,}06626\,H + 1{,}31773\,i' - 0{,}00937\,J - 5{,}57458$$

$$J' = 1{,}83374\,J + 1{,}31773\,b' - 0{,}00987\,H - 1{,}55427$$

$$\left.\begin{array}{l} -0{,}06626\,H' + 2{,}47576\,i' + 0{,}00987\,J' - 11{,}86106 \\ -1{,}01591\,H + 0{,}00427\,J - 0{,}02711\,\mathfrak{J} + 2{,}81250 \\ -2{,}01798\,\mathfrak{H} - 0{,}03634\,J \qquad + 31{,}31044 \\ \qquad\qquad + 10{,}60834 \end{array}\right\} = 0$$

$$\left.\begin{array}{l} 1{,}83374\,J' + 2{,}47576\,b' + 0{,}00987\,H' + 0{,}27762 \\ -1{,}01591\,J + 0{,}00427\,H - 0{,}02711\,\mathfrak{H} + 2{,}81250 \\ -2{,}01798\,\mathfrak{J} - 0{,}03634\,H \qquad + 31{,}81380 \\ \qquad\qquad - 0{,}81380 \end{array}\right\} = 0$$

§. 183. Subſtituamus primo loco b' et i' valores, atque noſtrae aequationes reducentur ad formas ſequentes,

$$-0{,}06626\,\mathfrak{H} + 0{,}01785\,J - 0{,}00526\,\mathfrak{J} + 2{,}04620 = 0$$

$$+1{,}83374\,\mathfrak{J} + 0{,}01785\,H - 0{,}00526\,\mathfrak{H} + 2{,}10646 = 0$$

$$H' = -0{,}06626\,H + 0{,}00526\,J + 0{,}00750\,\mathfrak{J} - 5{,}57458$$

$$J' = +1{,}83374\,J + 0{,}00526\,H + 0{,}00750\,\mathfrak{H} - 1{,}55427$$

qui valores in ſequentibus ſubſtituti dant

$$-1{,}01147\,H - 2{,}01791\,\mathfrak{H} + 0{,}01417\,J - 0{,}01352\,\mathfrak{J} + 33{,}22429 = 0$$

$$+2{,}34674\,J - 2{,}01791\,\mathfrak{J} + 0{,}00535\,H + 0{,}00073\,\mathfrak{H} + 30{,}68146 = 0$$

vnde elicimus:

$$\mathfrak{H} = \pm 0{,}00077\,H + 0{,}26935\,J + 30{,}96780$$

$$\mathfrak{J} = -0{,}00974\,H + 0{,}00077\,J - 1{,}05989$$

§. 184.

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§. 184. Hi autem valores in poſterioribus aequationibus ſubſtituti producent

$$1,01290\, H + 0,52937\, J + 29,25137 = 0$$

$$2,34539\, J + 0,02500\, H + 32,84282 = 0$$

hincque tandem concluditur:

$H = -21,68110$	. . .	$l\text{-}H = 1,336081$
$J = -13,77206$	. . .	$l\text{-}J = 1,138999$
$\mathfrak{H} = +27,24155$	. . .	$l\,\mathfrak{H} = 1,435232$
$\mathfrak{J} = -1,25933$	. . .	$l\text{-}\mathfrak{J} = 0,100140$
$b' = -0,09396$	. . .	$l\text{-}b' = 8,972943$
$i' = -0,16533$	. . .	$l\text{-}i' = 9,218352$

§. 185. Nunc pro excentricitate orbitae ſolaris hae reſtant aequationes,

$$(\alpha-\tfrac{1}{n})\mathfrak{K}-\mathfrak{A}l'+\tfrac{1}{2}\mathfrak{L}a'-\mathfrak{Q}f'+\mathfrak{R}g'+\tfrac{1}{2}(\tfrac{2}{n}-p')\mathfrak{F}+\tfrac{1}{2}\mathfrak{F}q'-\tfrac{3}{2}\mathfrak{G}r'$$

$$-\tfrac{3}{4}+\frac{3\mathfrak{L}}{2nn}-\frac{9F}{4nn}+\frac{9G}{4nn}=0$$

$$(\alpha+\tfrac{1}{n})\mathfrak{L}-\mathfrak{A}k'+\tfrac{1}{2}\mathfrak{K}a'+\mathfrak{Q}g'-\mathfrak{R}f'+\tfrac{1}{2}(\tfrac{2}{n}-p')\mathfrak{F}+\tfrac{1}{2}\mathfrak{F}r'-\tfrac{3}{2}\mathfrak{G}q'$$

$$-\tfrac{3}{4}+\frac{3K}{2nn}-\frac{9F}{4nn}+\frac{9G}{4nn}=0$$

$$K'=(\alpha-\tfrac{1}{n})K-Al'+\tfrac{1}{2}La'-Qf'+Rg'+\tfrac{1}{2}(\tfrac{2}{n}-p')F+\tfrac{1}{2}Fq'-\tfrac{3}{2}Gr'$$

$$L'=(\alpha+\tfrac{1}{n})L-Ak'+\tfrac{1}{2}Ka'+Qg'-Rf'+\tfrac{1}{2}(\tfrac{2}{n}-p')F+\tfrac{1}{2}Fr'-\tfrac{3}{2}Gq'$$

$$(\alpha-\tfrac{1}{n})K'-A'l'-\tfrac{1}{2}L'a'-Q'f'-R'g'+\tfrac{1}{2}(\tfrac{2}{n}-p')F'-\tfrac{1}{2}F'q'-\tfrac{3}{2}G'r'$$

$$-\tfrac{9}{4}-6K-2\alpha\mathfrak{K}-\frac{15F}{8nn}-\frac{9G}{8nn}+\frac{3L}{4nn}+\frac{(\mathfrak{A}+3A)}{nn}\mathfrak{L}+\frac{(3\mathfrak{A}+3A)L}{nn}$$

$$+\frac{(\mathfrak{F}+3F)}{nn}(\mathfrak{P}+\mathfrak{Q})+\frac{(3\mathfrak{F}+3F)}{nn}(P+Q)+\frac{(\mathfrak{G}+3G)}{nn}\mathfrak{R}+\frac{(3\mathfrak{G}+3G)}{nn}R$$
$$+\frac{9P}{8nn}+\frac{9Q}{8nn}+\frac{15R}{8nn}=0$$

$$(\alpha+\frac{1}{n})L'-A'k'-\tfrac{1}{2}K'a'-R'f'-Q'g'+\tfrac{1}{2}(\frac{2}{n}-p')F'-\tfrac{3}{4}F'r-\tfrac{1}{2}G'q'$$
$$-\tfrac{2}{4}\mathfrak{L}L-2n\mathfrak{L}+\frac{3K}{4nn}-\frac{15F}{8nn}-\frac{9G}{8nn}+\frac{(\mathfrak{A}+3A)}{nn}\mathfrak{K}+\frac{(3\mathfrak{A}+3A)}{nn}K$$
$$+\frac{(\mathfrak{F}+3F)}{nn}(\mathfrak{P}+\mathfrak{R})+\frac{(3\mathfrak{F}+3F)}{nn}(P+R)+\frac{(\mathfrak{G}+3G)}{nn}\mathfrak{Q}+\frac{(3\mathfrak{G}+3G)}{nn}Q$$
$$+\frac{9P}{8nn}+\frac{9R}{8nn}+\frac{15Q}{8nn}=0$$

§. 186. Hic autem obſeruo, hanc determinationem maxime eſſe lubricam, cum coefficiens litterae L, quem poſtremo eſt habitura, admodum fiat paruus; vnde is a terminis, quos omiſimus, non mediocrem mutationem perpeti poſſet. Hanc ob cauſam conſultum iudico, in calculum quoque terminos $3\eta-s$ et $3\eta+s$ introducere, quia praeuideo ab iis coefficientes terminorum, quos quaerimus, non leuiter affici. Sequenti ergo modo calculum redintegro.

§. 187. In hunc finem quoque rationem habeamus angulorum $3\eta-s$ et $3\eta+s$, ſitque

$$\int R\,dr = \mathfrak{A}\,\mathrm{coſ}\,2\eta + \mathfrak{P}e\,\mathrm{coſ}\,s + \mathfrak{Q}e\,\mathrm{coſ}(2\eta-s)$$
$$+ \mathfrak{R}e\,\mathrm{coſ}(2\eta+s)$$
$$+ \mathfrak{F}v\,\mathrm{coſ}\,\eta + \mathfrak{K}ve\,\mathrm{coſ}(\eta-s) + \mathfrak{M}ve\,\mathrm{coſ}(3\eta-s)$$
$$+ \mathfrak{G}v\,\mathrm{coſ}\,\eta + \mathfrak{L}ve\,\mathrm{coſ}(\eta+s) + \mathfrak{N}ve\,\mathrm{coſ}(3\eta+s)$$

$v =$

$$v = A\cos 2\eta + Pe\cos s + Qe\cos(2\eta - s) + Re\cos(2\eta + s) + Fv\cos\eta + Kve\cos(\eta - s) + Mve\cos(3\eta - s) + Gv\cos 3\eta + Lve\cos(\eta + s) + Nve\cos(3\eta + s)$$

§. 188. Quodſi iam ponamus:

$$\frac{2\kappa K + \mathfrak{K}}{nn} = k' \;;\quad \frac{2\kappa L + \mathfrak{L}}{nn} = l'$$

$$\frac{2\kappa M + \mathfrak{M}}{nn} = m' \quad \frac{2\kappa N + \mathfrak{N}}{nn} = n'$$

erit

$$\frac{d\Phi}{dr} = \text{Praec.} - a'\cos 2\eta - p'e\cos s - q'e\cos(2\eta - s) - r'e\cos(2\eta + s) - f'v\cos\eta - k've\cos(\eta - s) - m've\cos(3\eta - s) - g'v\cos 3\eta - l've\cos(\eta + s) - n've\cos(3\eta + s)$$

atque ob $\frac{ds}{dr} = \frac{1}{n} - \frac{2}{n}e\cos s$ erit

$$\frac{d\eta}{dr} = \alpha - a'\cos 2\eta + \left(\frac{2}{n} - p'\right)e\cos s - q'e\cos(2\eta - s) - r'e\cos(2\eta + s) - f'v\cos\eta - k've\cos(\eta - s) - m've\cos(3\eta - s) - g'v\cos 3\eta - l've\cos(\eta + s) - n've\cos(3\eta + s)$$

§. 189. Formulas nunc aſſumtas differentiemus, ſolosque terminos, quibus opus habemus, in calculo exprimamus ac reperiemus:

$$R = \text{Praec.}$$

$$+ ve\sin(\eta - s)\left(+\mathfrak{A}l' - \mathfrak{A}m' + \mathfrak{Q}f' - \mathfrak{R}g' - \tfrac{1}{2}\mathfrak{F}\left(\frac{2}{n} - p'\right) - \tfrac{1}{2}\mathfrak{F}q' + \tfrac{1}{2}\mathfrak{G}r' - \mathfrak{K}\left(\alpha - \frac{1}{n}\right)\right.$$

$$- \tfrac{1}{2}\mathfrak{L}a' + \tfrac{1}{2}\mathfrak{M}a'$$

$$+ve\sin(\eta+s)\left(+\mathfrak{A}k' - \mathfrak{A}n' + \mathfrak{R}f' - \mathfrak{Q}g' - \tfrac{1}{2}\mathfrak{F}\left(\tfrac{2}{n}-p'\right) - \tfrac{1}{2}\mathfrak{F}r' + \tfrac{3}{2}\mathfrak{G}q' - \mathfrak{L}\left(\alpha-\tfrac{1}{n}\right)\right.$$
$$\left. - \tfrac{1}{2}\mathfrak{K}a' + \tfrac{3}{2}\mathfrak{N}a'\right)$$
$$+ve\sin(3\eta-s)\left(+\mathfrak{A}k' + \mathfrak{Q}f' + \tfrac{1}{2}\mathfrak{F}q' - \tfrac{3}{2}\mathfrak{G}\left(\tfrac{2}{n}-p'\right) + \tfrac{1}{2}\mathfrak{K}a' - \mathfrak{M}\left(3\alpha-\tfrac{1}{n}\right)\right)$$
$$+ve\sin(3\eta-s)\left(+\mathfrak{A}l' + \mathfrak{R}f' + \tfrac{1}{2}\mathfrak{F}r' - \tfrac{3}{2}\mathfrak{G}\left(\tfrac{2}{n}-p'\right) + \tfrac{1}{2}\mathfrak{L}a' - \mathfrak{N}\left(3\alpha+\tfrac{1}{n}\right)\right)$$

Ac ſi breuitatis gratia ponamus:

$$K' = \left(\alpha-\tfrac{1}{n}\right)K - A(l'-m') - Qf' + Rg' + \tfrac{1}{2}F\left(\tfrac{2}{n}-p'\right) + \tfrac{1}{2}Fq' - \tfrac{3}{2}Gr'$$
$$+ \tfrac{1}{2}La' - \tfrac{3}{2}Ma'$$
$$L' = \left(\alpha+\tfrac{1}{n}\right)L - A(k'-n') - Rf' + Qg' + \tfrac{1}{2}F\left(\tfrac{2}{n}-p'\right) + \tfrac{1}{2}Fr' - \tfrac{3}{2}Gq'$$
$$+ \tfrac{1}{2}Ka' - \tfrac{3}{2}Na'$$
$$M' = \left(3\alpha-\tfrac{1}{n}\right)M - Ak' - Qf' - \tfrac{1}{2}Fq' + \tfrac{3}{2}G\left(\tfrac{2}{n}-p'\right) - \tfrac{1}{2}Ka'$$
$$N' = \left(3\alpha+\tfrac{1}{n}\right)N - Al' - Rf' - \tfrac{1}{2}Fr' + \tfrac{3}{2}G\left(\tfrac{2}{n}-p'\right) - \tfrac{1}{2}La'$$

erit

$$\frac{dv}{dr} = -A'\sin 2\eta - P'e\sin s - Q'e\sin(2\eta-s)$$
$$- R'e\sin(2\eta+s)$$
$$- F'v\sin\eta - K've\sin(\eta-s) - M've\sin(3\eta-s)$$
$$- G'v\sin 3\eta - L've\sin(\eta+s) - N've\sin(3\eta+s)$$

§. 190. Hinc iam denuo differentiando nanciſcemur

$$\frac{ddv}{dr^2} = \text{Praec.}$$

$$+ve\cos(\eta-s)\left\{\begin{array}{l} A'l' + A'm' + Q'f' + R'g' - \tfrac{1}{2}F'\left(\tfrac{2}{n}-p'\right) + \tfrac{1}{2}F'q' \\ - + \tfrac{3}{2}G'r' + \tfrac{1}{2}L'a' + \tfrac{3}{2}M'a' - \left(\alpha-\tfrac{1}{n}\right)K' \end{array}\right.$$

+

$$+ v e \operatorname{coſ}(\eta + s) \left\{ \begin{array}{l} A'k' + A'n' + R'f' + Q'g' - \tfrac{1}{2} F'(\tfrac{2}{n} - p') + \tfrac{1}{2} F'r' \\ \qquad + \tfrac{1}{2} G'q' + \tfrac{1}{2} K'a' + \tfrac{1}{2} N'a' - (\alpha + \tfrac{1}{n}) L' \end{array} \right.$$

$$+ v e \operatorname{coſ}(3\eta - s) \left\{ \begin{array}{l} A'k' + Q'f' + \tfrac{1}{2} F'q' - \tfrac{1}{2} G'(\tfrac{2}{n} - p') + \tfrac{1}{2} K'a' \\ \qquad - (3\alpha - \tfrac{1}{n}) M' \end{array} \right.$$

$$+ v e \operatorname{coſ}(3\eta + s) \left\{ \begin{array}{l} A'l' + R'f' + \tfrac{1}{2} F'r' - \tfrac{1}{2} G'(\tfrac{2}{n} - p') + \tfrac{1}{2} L'a' \\ \qquad - (3\alpha + \tfrac{1}{n}) N' \end{array} \right.$$

§. 191. Quodſi autem valores iam inuenti ſubſtituantur, habebitur

$$R = \text{Praec.}$$

$$+ v e \operatorname{ſn}(\eta - s) \left\{ \begin{array}{l} -0{,}85830\,\mathfrak{K} + 0{,}00526\,\mathfrak{L} - 0{,}02500\,\mathfrak{M} + 0{,}37592 \\ \qquad - 0{,}00931\,L + 0{,}00931\,M \end{array} \right]$$

$$+ v e \operatorname{ſn}(\eta + s) \left\{ \begin{array}{l} -1{,}00918\,\mathfrak{L} + 0{,}00526\,\mathfrak{K} - 0{,}02500\,\mathfrak{N} + 0{,}34115 \\ \qquad - 0{,}00931\,K + 0{,}00931\,N \end{array} \right]$$

$$+ v e \operatorname{ſn}(3\eta - s) \left\{ \begin{array}{l} -2{,}72578\,\mathfrak{M} - 0{,}01448\,\mathfrak{K} + 0{,}49223 \\ \qquad - 0{,}00931\,K \end{array} \right]$$

$$+ v e \operatorname{ſn}(3\eta + s) \left\{ \begin{array}{l} -2{,}87666\,\mathfrak{N} - 0{,}01448\,\mathfrak{L} + 0{,}47116 \\ \qquad - 0{,}00931\,L \end{array} \right]$$

 $K' =$

$$K' = 0{,}85830\,K + 0{,}00527\,L + 0{,}01447\,M + 1{,}48970 + 0{,}00750\,\mathfrak{L} - 0{,}00750\,\mathfrak{M}$$

$$L' = 1{,}00916\,L + 0{,}00527\,K + 0{,}01447\,N + 1{,}55183 + 0{,}00750\,\mathfrak{K} - 0{,}00750\,\mathfrak{N}$$

$$M' = 2{,}72528\,M + 0{,}02501\,K - 1{,}17408 + 0{,}00750\,\mathfrak{K}$$

$$N' = 2{,}87666\,N + 0{,}02501\,L - 0{,}95901 + 0{,}00750\,\mathfrak{L}$$

$$\frac{ddv}{dr^2} = \text{Praec.}$$

$$+ v\,e\,\text{cſ}(\eta - s)\left\{\begin{array}{l} -0{,}85830\,K' - 0{,}02843\,L - 0{,}02843\,M - 0{,}32674 \\ -0{,}00987\,L' - 0{,}01409\,\mathfrak{L} - 0{,}01409\,\mathfrak{M} \\ -0{,}02961\,M' \end{array}\right]$$

$$+ v\,e\,\text{cſ}(\eta + s)\left\{\begin{array}{l} -1{,}00918\,L' - 0{,}02843\,K - 0{,}02843\,N - 0{,}56620 \\ -0{,}00987\,K' - 0{,}01409\,\mathfrak{K} - 0{,}01409\,\mathfrak{N} \\ -0{,}02981\,N' \end{array}\right]$$

$$+ v\,e\,\text{cſ}(3\eta - s)\left\{\begin{array}{l} -2{,}72578\,M' - 0{,}02843\,K + 0{,}74683 \\ -0{,}00987\,K' - 0{,}01409\,\mathfrak{K} \end{array}\right]$$

$$+ v\,e\,\text{cſ}(3\eta + s)\left\{\begin{array}{l} -2{,}87661\,N' - 0{,}02853\,L + 0{,}67486 \\ -0{,}00987\,L' - 0{,}01409\,\mathfrak{L} \end{array}\right]$$

§. 192. Valores autem litterarum commate notatarum hic ſubſtitutae dabunt

$$\frac{ddv}{dr^2} =$$

$$v\,e\,\text{cſ}(\eta - s)\left\{\begin{array}{l} -0{,}73747\,K - 0{,}04264\,L - 0{,}12156\,M - 0{,}00014\,N \\ -0{,}00029\,\mathfrak{K} - 0{,}02053\,\mathfrak{L} - 0{,}00765\,\mathfrak{M} + 0{,}00008\,\mathfrak{N} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad -1{,}58590 \end{array}\right]$$

ve

$$ve\cos(\eta+s)\left\{\begin{array}{l}-1{,}01923L-0{,}04222K-0{,}12821N-0{,}00014M\\ -0{,}00029\mathfrak{L}-0{,}02166\mathfrak{K}-0{,}00642\mathfrak{N}+0{,}00008\mathfrak{M}\\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad -2{,}11858\end{array}\right]$$

$$ve\cos(3\eta-s)\left\{\begin{array}{l}-7{,}43058M-0{,}09507K-0{,}00005L+3{,}93257\\ +0{,}00008\mathfrak{M}-0{,}03453\mathfrak{K}-0{,}00007\mathfrak{L}\end{array}\right]$$

$$ve\cos(3\eta+s)\left\{\begin{array}{l}-8{,}27529N-0{,}11338L-0{,}00005K+3{,}41830\\ +0{,}00008\mathfrak{N}-0{,}03566\mathfrak{L}-0{,}00007\mathfrak{K}\end{array}\right]$$

§. 193. His expreſſionibus ita euolutis atque ad calculum numericum praeparatis, quaeramus easdem expreſſiones ex formulis ſupra traditis pro R et $\frac{ddv}{dr^2}$, quae continentur in §. 52 et 54. Inde autem omittendis terminis, quos iam tractauimus, conſequemur.

$$R = \text{Pr.} + ve\sin(\eta-s)\left(-\tfrac{3}{4}+\frac{3L}{2nn}-\frac{3M}{2nn}-\frac{9F}{4nn}+\frac{9G}{4nn}\right)$$

$$+ve\sin(\eta+r)\left(-\tfrac{3}{4}+\frac{3K}{2nn}-\frac{3N}{2nn}+\frac{9G}{4nn}-\frac{9F}{4nn}\right)$$

$$+ve\sin(3\eta-s)\left(-\tfrac{15}{4}+\frac{3K}{2nn}-\frac{9F}{4nn}\right)$$

$$+ve\sin(3\eta+s)\left(-\tfrac{15}{4}+\frac{3L}{2nn}-\frac{9F}{4nn}\right)$$

ddv

$$v\,e\,\mathrm{coſ}(\eta-s)\left\{\begin{array}{l}-\tfrac{9}{4}-6K+\frac{3L}{4nn}-\frac{3F}{4nn}-\frac{9F}{8nn}-\frac{9G}{8nn}-2x\mathfrak{K}\\ +\frac{\mathfrak{A}\mathfrak{L}}{nn}+\frac{\mathfrak{F}\mathfrak{P}}{nn}+\frac{\mathfrak{F}\mathfrak{Q}}{nn}+\frac{\mathfrak{G}\mathfrak{R}}{nn}\end{array}\right.$$

$$v\,e\,\mathrm{coſ}(\eta+s)\left\{\begin{array}{l}-\tfrac{9}{4}-6L+\frac{3K}{4nn}-\frac{3F}{4nn}-\frac{9F}{8nn}-\frac{9G}{8nn}-2x\mathfrak{L}\\ +\frac{\mathfrak{A}\mathfrak{K}}{nn}+\frac{\mathfrak{F}\mathfrak{P}}{nn}+\frac{\mathfrak{F}\mathfrak{R}}{nn}+\frac{\mathfrak{G}\mathfrak{Q}}{nn}\end{array}\right.$$

ſequentes partes ſeorſim exponamus:

$$v\,\mathrm{coſ}\,\eta\left\{\begin{array}{l}+\frac{3\mathfrak{A}F}{nn}+\frac{3\mathfrak{A}G}{nn}+\frac{3A\mathfrak{F}}{nn}+\frac{3A\mathfrak{G}}{nn}+\frac{3AF}{nn}+\frac{3AG}{nn}\\ +\frac{9A}{8nn}+\frac{15A}{8nn}\end{array}\right.$$

$$v\,\mathrm{coſ}\,3\eta\left\{+\frac{3\mathfrak{A}F}{nn}+\frac{3\mathfrak{A}\mathfrak{F}}{nn}+\frac{3AF}{nn}+\frac{9A}{8nn}\right.$$

$$v\,k\,\mathrm{coſ}(\eta-r)\left\{\begin{array}{l}\frac{3\mathfrak{A}J}{nn}+\frac{3\mathfrak{C}F}{nn}+\frac{3\mathfrak{D}F}{nn}+\frac{3\mathfrak{E}G}{nn}+\frac{3A\mathfrak{J}}{nn}+\frac{3D\mathfrak{F}}{nn}\\ +\frac{3E\mathfrak{G}}{nn}+\frac{3AJ}{nn}+\frac{3DF}{nn}+\frac{3EG}{nn}-\frac{3AF}{nn}-\frac{3AG}{nn}\\ +\frac{9D}{8nn}+\frac{15E}{8nn}\end{array}\right.$$

$$v\,k\,\mathrm{coſ}(\eta+r)\left\{\begin{array}{l}\frac{3\mathfrak{A}H}{nn}+\frac{3\mathfrak{C}F}{nn}+\frac{3\mathfrak{D}\mathfrak{G}}{nn}+\frac{3\mathfrak{E}F}{nn}+\frac{3A\mathfrak{H}}{nn}\\ +\frac{3D\mathfrak{G}}{nn}+\frac{3E\mathfrak{F}}{nn}+\frac{3AH}{nn}+\frac{3DG}{nn}+\frac{3EF}{nn}\\ -\frac{3AF}{2nn}-\frac{3AG}{2nn}+\frac{9E}{8nn}+\frac{15D}{8nn}\end{array}\right.$$

$$v\,e\,\text{coſ}\,(\eta-s)\left\{\begin{array}{l}\frac{3\mathfrak{A}L}{nn}+\frac{3\mathfrak{P}F}{nn}+\frac{3\mathfrak{Q}F}{nn}+\frac{3\mathfrak{R}G}{nn}+\frac{3A\mathfrak{L}}{nn}\\+\frac{3P\mathfrak{F}}{nn}+\frac{3Q\mathfrak{F}}{nn}+\frac{3R\mathfrak{G}}{nn}+\frac{3AL}{nn}+\frac{3PF}{nn}+\frac{3QF}{nn}\\+\frac{3RG}{nn}+\frac{9P}{8nn}+\frac{9Q}{8nn}+\frac{15R}{8nn}\end{array}\right.$$

$$v\,e\,\text{coſ}\,(\eta+s)\left\{\begin{array}{l}\frac{3\mathfrak{A}K}{nn}+\frac{3\mathfrak{P}F}{nn}+\frac{3\mathfrak{Q}G}{nn}+\frac{3\mathfrak{R}F}{nn}+\frac{3A\mathfrak{K}}{nn}+\frac{3PF}{nn}\\+\frac{3Q\mathfrak{G}}{nn}+\frac{3R\mathfrak{F}}{nn}+\frac{3AK}{nn}+\frac{3PF}{nn}+\frac{3QF}{nn}+\frac{3QG}{nn}\\+\frac{9P}{8nn}+\frac{9R}{8nn}+\frac{15Q}{8nn}\end{array}\right.$$

§. 177. Verum differentiando nanciſcemur

$$R =\!=\!=$$

$$v\,\text{ſin}\,\eta \quad \{\mathfrak{A}f' - \mathfrak{A}g' - \alpha\mathfrak{F} - \tfrac{1}{2}\mathfrak{F}a' + \tfrac{3}{2}\mathfrak{G}a'$$

$$v\,\text{ſin}\,3\eta \quad \{\mathfrak{A}f' + \tfrac{1}{2}\mathfrak{F}a' - 3\alpha\mathfrak{G}$$

$$v\,k\,\text{ſin}\,(\eta-r)\,\{\mathfrak{A}e' + \mathfrak{D}f' - \mathfrak{E}g' + \tfrac{1}{2}\mathfrak{F}c' - \tfrac{1}{2}\mathfrak{F}d' + \tfrac{3}{2}\mathfrak{G}c' - (\alpha-1)\mathfrak{H} - \tfrac{1}{2}\mathfrak{J}a'$$

$$v\,k\,\text{ſin}\,(\eta+r)\left\{\begin{array}{l}\mathfrak{A}h' - \mathfrak{D}g' + \mathfrak{E}f' + \tfrac{1}{2}\mathfrak{F}c' - \tfrac{1}{2}\mathfrak{F}c' + \tfrac{3}{2}\mathfrak{G}d' - \tfrac{1}{2}\mathfrak{H}a'\\ \qquad - (\alpha+1)\mathfrak{J}\end{array}\right.$$

$$v\,e\,\text{ſin}\,(\eta-s)\left\{\begin{array}{l}+\mathfrak{A}i' + \mathfrak{Q}f' - \mathfrak{R}g' - \tfrac{1}{2}\left(\frac{2}{n}-p'\right)\mathfrak{F} - \tfrac{1}{2}\mathfrak{F}q' + \tfrac{3}{2}\mathfrak{G}r'\\ \qquad -\left(\alpha-\frac{1}{n}\right)\mathfrak{K} - \tfrac{1}{2}\mathfrak{L}a'\end{array}\right.$$

$$v\,e\,\text{ſin}\,(\eta+s)\left\{\begin{array}{l}+\mathfrak{A}k' - \mathfrak{Q}g' + \mathfrak{R}f' - \tfrac{1}{2}\left(\frac{2}{n}-p'\right)\mathfrak{F} - \tfrac{1}{2}\mathfrak{F}r' + \tfrac{3}{2}\mathfrak{G}q'\\ \qquad -\left(\alpha+\frac{1}{n}\right)\mathfrak{L} - \tfrac{1}{2}\mathfrak{K}a'\end{array}\right.$$

 Deinde

Deinde poſito

$$F' = \alpha F - A(f' - g') + \tfrac{1}{2} F a' - \tfrac{3}{2} G a'$$
$$G' = 3\alpha G - A f' - \tfrac{1}{2} F a'$$
$$H' = (\alpha - 1) H - A i' - D f' + E g' - \tfrac{1}{2} F c' + \tfrac{1}{2} F d' - \tfrac{3}{2} G e' + \tfrac{1}{2} J a'$$
$$J' = (\alpha + 1) J - A b' + D g' - E f' - \tfrac{1}{2} F c' + \tfrac{1}{2} F e' - \tfrac{3}{2} G d' + \tfrac{1}{2} H a'$$
$$K' = (\alpha - 1) K - A l' - Q f' + R g' + \tfrac{1}{2} F\left(\frac{2}{n} - p'\right) + \tfrac{1}{2} F q' - \tfrac{3}{2} G r' + \tfrac{1}{2} L a'$$
$$L' = (\alpha + 1) L - A k' + Q g' - R f' + \tfrac{1}{2} F\left(\frac{2}{n} - p'\right) + \tfrac{1}{2} F r' - \tfrac{3}{2} G q' + \tfrac{1}{2} K a'$$

erit

$$\begin{aligned}\frac{dv}{dr} = &- A' \sin 2\eta - D' k \sin(2\eta - r) - P' e \sin s - Q' e \sin(2\eta - s) \\ & - E' k \sin(2\eta + r) - R' e \sin(2\eta + s) \\ &- F' v \sin\eta - H' v k \sin(\eta - r) - K' v e \sin(\eta - s) \\ &- G' v \sin 3\eta - J' v k \sin(\eta + r) - L' v e \sin(\eta + s)\end{aligned}$$

§. 178. Hinc iam denuo differentiando conſequemur:

$$\frac{ddv}{dr^2} = \text{Praec.}$$

$$\begin{aligned}
&+ v \cos\eta \; \{ + A' f' + A' g' - \alpha F' + \tfrac{1}{2} F' a' + \tfrac{3}{2} G' a' \\
&+ v \cos 3\eta \; \{ + A' f' + \tfrac{1}{2} F' a' - 3\alpha G' \\
&+ v k \cos(\eta - r) \left\{ \begin{array}{r} + A' i' + D' f' + E' g' + \tfrac{1}{2} F' c' + \tfrac{1}{2} F' d' + \tfrac{3}{2} G' e' \\ - (\alpha - 1) H' + \tfrac{1}{2} J' a' \end{array} \right. \\
&+ v k \cos(\eta + r) \left\{ \begin{array}{r} + A' b' + E' f' + D' g' + \tfrac{1}{2} F' c' + \tfrac{1}{2} F' e' + \tfrac{3}{2} G' d' \\ + \tfrac{1}{2} H' a' - (\alpha + 1) J' \end{array} \right. \\
&+ v e \cos(\eta - s) \left\{ \begin{array}{r} + A' l' + Q' f' + R' g' - \tfrac{1}{2}\left(\frac{2}{n} - p'\right) F' + \tfrac{1}{2} F' g' + \tfrac{2}{3} G' r' \\ - \left(\alpha - \frac{1}{n}\right) K' + \tfrac{1}{2} L' a' \end{array} \right.
\end{aligned}$$

+

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$$+ \nu \epsilon \operatorname{cof}(\eta + s) \left\{ \begin{array}{r} + A'k' + R'f' + Q'g' - \frac{1}{2}\left(\frac{2}{n} - p'\right)F' + \frac{1}{2}F'r' + \frac{3}{2}G'q' \\ + \frac{1}{2}K'a' - \left(\alpha + \frac{1}{n}\right)L' \end{array} \right.$$

qui valores cum antecedentibus comparari debent, vt inde valores coefficientium eliciantur.

§. 179. Sumamus primo duos valores ab initio poſitos, quoniam hi a ſequentibus non pendent, atque habebimus,

$$o = \alpha\mathfrak{F} - \mathfrak{A}f' + \mathfrak{A}g' + \tfrac{1}{2}\mathfrak{F}a' - \tfrac{3}{2}\mathfrak{G}a' + \tfrac{3}{8} + \frac{3F - 3G}{2nn}$$

$$o = 3\alpha\mathfrak{G} - \mathfrak{A}f' - \tfrac{1}{2}\mathfrak{F}a' + \tfrac{1}{8} + \frac{3F}{2nn}$$

$$o = \alpha F' - A'f' - A'g' - \tfrac{1}{2}F'a' - \tfrac{3}{2}G'a' + \tfrac{9}{8} - 6F - 2\kappa\mathfrak{F}$$
$$+ \frac{3(F+G)}{4nn} + \frac{(\mathfrak{A}+3A)}{nn}(\mathfrak{F}+\mathfrak{G}) + \frac{(3A+3A)}{nn}(F+G) + \frac{3A}{nn}$$

$$o = 3\alpha G' - A'f' - \tfrac{1}{2}F'a' + \tfrac{1}{8} - 6G - 2\kappa G$$
$$+ \frac{3F}{4nn} + \frac{(\mathfrak{A}+3A)}{nn}\mathfrak{F} + \frac{(3\mathfrak{A}+3A)}{nn}F + \frac{9A}{8nn}$$

et $F' = \alpha F - A(f' - g') + \tfrac{1}{2}Fa' - \tfrac{3}{2}Ga'$

$G' = 3\alpha G - Af' - \tfrac{1}{2}Fa'$

at eſt $f' = \dfrac{2\kappa F + \mathfrak{F}}{nn}$ et $g' = \dfrac{2\kappa G + \mathfrak{G}}{nn}$.

Hic igitur primum litterarum, quae ſunt cognitae, valores in numeris ſubſtituantur, eritque

 F′ =

$$F' = 0,93905\,F + 0,00753\,\mathfrak{F} + 0,01443\,G - 0,00753\,\mathfrak{G}$$
$$G' = 2,80121\,G + 0,02505\,F + 0,00753\,\mathfrak{F}$$
$$0 = 0,92847\,\mathfrak{F} + 0,01776\,F - 0,01776\,G - 0,02501\,\mathfrak{G} + 0,37500$$
$$0 = 2,80121\,\mathfrak{G} + 0,01447\,\mathfrak{F} + 0,01776\,F + 1,87500.$$

hincque

$$\mathfrak{F} = -0,01930\,F + 0,01913\,G - 0,42145$$
$$\mathfrak{G} = -0,00623\,F - 0,00010\,G - 0,66717.$$

§. 180. Inde porro colligemus

$$F' = 0,93895\,F + 0,01457\,G + 0,00185$$
$$G' = 0,02491\,F + 2,80135\,G - 0,00317$$
$$f' = 0,01138\,F + 0,00011\,G - 0,00240.$$
$$g' = -0,00004\,F + 0,01148\,G - 0,00380$$

quibus valoribus ſubſtitutis peruenimus ad has aequationes:

$$0,09337\,F = +0,05421\,G + 1,96867$$
$$6,83135\,G = -0,08830\,F - 3,20944$$

vnde fit:

$$F = 20,65700 \;\ldots\; l\,F = 1,315067$$
$$G = -0,73681 \;\ldots\; l{-}G = 9,867356$$
$$\mathfrak{F} = -0,83418 \;\ldots\; l{-}\mathfrak{F} = 9,921260$$
$$\mathfrak{G} = -0,79580 \;\ldots\; l{-}\mathfrak{G} = 9,900604$$
$$F' = +19,38712 \;\ldots\; l\,F' = 1,287512$$
$$G' = -1,55266 \;\ldots\; l{-}G' = 0,191075$$
$$f' = +0,23259 \;\ldots\; l\,f' = 9,366501$$
$$g' = -0,01309 \;\ldots\; l{-}g' = 8,116940$$

§. 181.

§. 181. His valoribus, qui ad inaequalitates abſolutas pertinent, expeditis, progrediamur ad eos, qui ab excentricitate orbitae lunaris pendent, ac his aequationibus continentur:

$$(\alpha-1)\mathfrak{H} - \mathfrak{A}i' + \tfrac{1}{2}\mathfrak{J}a' - \mathfrak{D}f' + \mathfrak{E}g' - \tfrac{1}{2}\mathfrak{F}(c'-d') - \tfrac{3}{2}\mathfrak{G}e'$$

$$+ \frac{1}{2}\frac{k}{b} + \frac{3J}{2nn} + \frac{3(F-G)}{nn} = 0$$

$$(\alpha+1)\mathfrak{J} - \mathfrak{A}b' + \tfrac{1}{2}\mathfrak{H}a' + \mathfrak{D}g' - \mathfrak{E}f' - \tfrac{1}{2}\mathfrak{F}(c'-e') - \tfrac{3}{2}\mathfrak{G}d'$$

$$+ \frac{1}{2}\frac{k}{b} + \frac{3H}{2nn} + \frac{3(F-G)}{nn} = 0$$

$$H' = (\alpha-1)H - Ai' + \tfrac{1}{2}Ja' - Df' + Eg' - \tfrac{1}{2}F(c'-d') - \tfrac{3}{2}Ge'$$

$$J' = (\alpha+1)J - Ab' + \tfrac{1}{2}Ha' + Dg' - Ef' - \tfrac{1}{2}F(c'-e') - \tfrac{3}{2}Gd'$$

$$(\alpha-1)H' - A'i' - \tfrac{1}{2}J'a' - D'f' - E'g' - \tfrac{1}{2}F'(c'+d') - \tfrac{3}{2}G'e'$$

$$+ \frac{1}{2}\frac{k}{b} - 6H - 2n\mathfrak{H} + \tfrac{1}{2}bF + \frac{3J}{4nn} + \frac{3(F+G)}{2nn} + \frac{\mathfrak{C}\mathfrak{F}}{nn} + \frac{3\mathfrak{C}F}{nn}$$

$$+ \frac{(\mathfrak{A}+3A)}{nn}\mathfrak{J} + \frac{(3\mathfrak{A}+3A)}{nn}J + \frac{(\mathfrak{D}+3D)}{nn}\mathfrak{F} + \frac{(3\mathfrak{D}+3D)}{nn}F$$

$$+ \frac{(\mathfrak{E}+3E)}{nn}\mathfrak{G} + \frac{(3\mathfrak{E}+3E)}{nn}G - \frac{3A(F+G)}{2nn} + \frac{9D+15E}{8nn} = 0$$

$$(\alpha+1)J' - A'b' - \tfrac{1}{2}H'a' - E'f' - D'g' - \tfrac{1}{2}F'(c'+e') - \tfrac{3}{2}G'd'$$

$$+ \frac{1}{2}\frac{k}{b} - 6J - 2n\mathfrak{J} + \tfrac{1}{2}bF + \frac{3H}{4nn} + \frac{3(F+G)}{2nn} + \frac{\mathfrak{C}\mathfrak{F}}{nn} + \frac{3\mathfrak{C}F}{nn}$$

$$+ \frac{(\mathfrak{A}+3A)}{nn}\mathfrak{H} + \frac{(3\mathfrak{A}+3A)}{nn}H + \frac{(\mathfrak{D}+3D)}{nn}\mathfrak{G} + \frac{(3\mathfrak{D}+3D)}{nn}G$$

$$+ \frac{(\mathfrak{E}+3E)}{nn}\mathfrak{F} + \frac{(3\mathfrak{E}+3E)}{nn}F - \frac{3A(F+G)}{2nn} + \frac{9E+15D}{8nn} = 0$$

§. 182. In his aequationibus ſubſtituantur valores iam cogniti, atque obtinebimus,

$$-0{,}06626\,\mathfrak{H} + 0{,}81033\,i' - 0{,}00987\,\mathfrak{J} + 0{,}00854\,J \pm 2{,}04620 = 0$$

$$1{,}83374\,\mathfrak{J} + 0{,}81033\,b' - 0{,}00987\,\mathfrak{H} + 0{,}00854\,H + 2{,}10646 = 0$$

$$H' = -0{,}06626\,H + 1{,}31773\,i' - 0{,}00937\,J - 5{,}57458$$

$$J' = 1{,}83374\,J + 1{,}31773\,b' - 0{,}00987\,H - 1{,}55427$$

$$\left.\begin{array}{l} -0{,}06626\,H' + 2{,}47576\,i' + 0{,}00987\,J' - 11{,}86106 \\ -1{,}01591\,H + 0{,}00427\,J - 0{,}02711\,\mathfrak{J} + 2{,}81250 \\ -2{,}01798\,\mathfrak{H} - 0{,}03634\,J \qquad + 31{,}31044 \\ \qquad\qquad + 10{,}60834 \end{array}\right\} = 0$$

$$\left.\begin{array}{l} 1{,}83374\,J' + 2{,}47576\,b' + 0{,}00987\,H' + 0{,}27762 \\ -1{,}01591\,J + 0{,}00427\,H - 0{,}02711\,\mathfrak{H} + 2{,}81250 \\ -2{,}01798\,\mathfrak{J} - 0{,}03634\,H \qquad + 31{,}81380 \\ \qquad\qquad - 0{,}81380 \end{array}\right\} = 0$$

§. 183. Subſtituamus primo loco b' et i' valores, atque noſtrae aequationes reducentur ad formas ſequentes,

$$-0{,}06626\,\mathfrak{H} + 0{,}01785\,J - 0{,}00526\,\mathfrak{J} + 2{,}04620 = 0$$

$$+1{,}83374\,\mathfrak{J} + 0{,}01785\,H - 0{,}00526\,\mathfrak{H} + 2{,}10646 = 0$$

$$H' = -0{,}06626\,H + 0{,}00526\,J + 0{,}00750\,\mathfrak{J} - 5{,}57458$$

$$J' = +1{,}83374\,J + 0{,}00526\,H + 0{,}00750\,\mathfrak{H} - 1{,}55427$$

qui valores in ſequentibus ſubſtituti dant

$$-1{,}01147\,H - 2{,}01791\,\mathfrak{H} + 0{,}01417\,J - 0{,}01352\,\mathfrak{J} + 33{,}22429 = 0$$

$$+2{,}34674\,J - 2{,}01791\,\mathfrak{J} + 0{,}00535\,H + 0{,}00073\,\mathfrak{H} + 30{,}68146 = 0$$

vnde elicimus:

$$\mathfrak{H} = \pm 0{,}00077\,H + 0{,}26935\,J + 30{,}96780$$

$$\mathfrak{J} = -0{,}00974\,H + 0{,}00077\,J - 1{,}05989$$

§. 184.

§. 184. Hi autem valores in poſterioribus aequationibus ſubſtituti producent

$$1,01290\,H + 0,52937\,J + 29,25137 = 0$$

$$2,34539\,J + 0,02500\,H + 32,84282 = 0$$

hincque tandem concluditur:

$H = -21,68110$	. . .	$l{-}H = 1,336081$
$J = -13,77206$	. . .	$l{-}J = 1,138999$
$\mathfrak{H} = +27,24155$	. . .	$l\,\mathfrak{H} = 1,435232$
$\mathfrak{J} = -1,25933$	. . .	$l{-}\mathfrak{J} = 0,100140$
$b' = -0,09396$	. . .	$l{-}b' = 8,972943$
$i' = -0,16533$	. . .	$l{-}i' = 9,218352$

§. 185. Nunc pro excentricitate orbitae ſolaris hae reſtant aequationes,

$$(a-\frac{1}{n})\mathfrak{K}-\mathfrak{A}l'+\frac{1}{2}\mathfrak{L}a'-\mathfrak{Q}f'+\mathfrak{R}g'+\frac{1}{2}(\frac{2}{n}-p')\mathfrak{F}+\frac{1}{2}\mathfrak{F}q'-\frac{3}{2}\mathfrak{G}r'$$
$$-\frac{3}{4}+\frac{3\mathfrak{L}}{2nn}-\frac{9F}{4nn}+\frac{9G}{4nn}=0$$

$$(a+\frac{1}{n})\mathfrak{L}-\mathfrak{A}k'+\frac{1}{2}\mathfrak{K}a'+\mathfrak{Q}g'-\mathfrak{R}f'+\frac{1}{2}(\frac{2}{n}-p')\mathfrak{F}+\frac{1}{2}\mathfrak{F}r'-\frac{3}{2}\mathfrak{G}q'$$
$$-\frac{3}{4}+\frac{3K}{2nn}-\frac{9F}{4nn}+\frac{9G}{4nn}=0$$

$$K'=(a-\frac{1}{n})K-Al'+\frac{1}{2}La'-Qf'+Rg'+\frac{1}{2}(\frac{2}{n}-p')F+\frac{1}{2}Fq'-\frac{3}{2}Gr'$$

$$L'=(a+\frac{1}{n})L-Ak'+\frac{1}{2}Ka'+Qg'-Rf'+\frac{1}{2}(\frac{2}{n}-p')F+\frac{1}{2}Fr'-\frac{3}{2}Gq'$$

$$(a-\frac{1}{n})K'-A'l'-\frac{1}{2}L'a'-Q'f'-R'g'+\frac{1}{2}(\frac{2}{n}-p')F'-\frac{1}{2}F'q'-\frac{3}{2}G'r'$$
$$-\frac{\mathfrak{e}}{4}-6K-2u\mathfrak{K}-\frac{15F}{8nn}-\frac{9G}{8nn}+\frac{3L}{4nn}+\frac{(\mathfrak{A}+3A)}{nn}\mathfrak{L}+\frac{(3\mathfrak{A}+3A)L}{nn}$$

$$+\frac{(\mathfrak{F}+3F)}{nn}(\mathfrak{P}+\mathfrak{Q})+\frac{(3\mathfrak{F}+3F)}{nn}(P+Q)+\frac{(\mathfrak{G}+3G)}{nn}\mathfrak{R}+\frac{(3\mathfrak{G}+3G)}{nn}R$$

$$+\frac{9P}{8nn}+\frac{9Q}{8nn}+\frac{15R}{8nn}=0$$

$$(a+\frac{1}{n})L'l'-A'k'-\tfrac{1}{2}K'a'-R'f'-Q'g'+\tfrac{1}{2}(\frac{2}{n}-p')F'-\tfrac{1}{2}F'r-\tfrac{1}{2}G'g'$$

$$-\tfrac{3}{4}-6L-2n\mathfrak{L}+\frac{3K}{4nn}-\frac{15F}{8nn}-\frac{9G}{8nn}+\frac{(\mathfrak{A}+3A)}{nn}\mathfrak{K}+\frac{(3\mathfrak{A}+3A)}{nn}K$$

$$+\frac{(\mathfrak{F}+3F)}{nn}(\mathfrak{P}+\mathfrak{R})+\frac{(3\mathfrak{F}+3F)}{nn}(P+R)+\frac{(\mathfrak{G}+3G)}{nn}\mathfrak{Q}+\frac{(3\mathfrak{G}+3G)}{nn}Q$$

$$+\frac{9P}{8nn}+\frac{9R}{8nn}+\frac{15Q}{8nn}=0$$

§. 186. Hic autem obſeruo, hanc determinationem maxime eſſe lubricam, cum coefficiens litterae L, quem poſtremo eſt habitura, admodum fiat paruus; vnde is a terminis, quos omiſimus, non mediocrem mutationem perpeti poſſet. Hanc ob cauſam conſultum iudico, in calculum quoque terminos $3\eta-s$ et $3\eta+s$ introducere, quia praeuideo ab iis coefficientes terminorum, quos quaerimus, non leuiter affici. Sequenti ergo modo calculum redintegro.

§. 187. In hunc finem quoque rationem habeamus angulorum $3\eta-s$ et $3\eta+s$, ſitque

$$\int R\,dr = \mathfrak{A}\,\text{coſ}\,2\eta + \mathfrak{P}e\,\text{coſ}\,s + \mathfrak{Q}e\,\text{coſ}(2\eta-s)$$
$$+ \mathfrak{R}e\,\text{coſ}(2\eta+s)$$
$$+ \mathfrak{F}v\,\text{coſ}\,\eta + \mathfrak{K}ve\,\text{coſ}(\eta-s) + \mathfrak{M}ve\,\text{coſ}(3\eta-s)$$
$$+ \mathfrak{G}v\,\text{coſ}\,\eta + \mathfrak{L}ve\,\text{coſ}(\eta+s) + \mathfrak{N}ve\,\text{coſ}(3\eta+s)$$

$$v =$$

$$\Phi = A\cos 2\eta + Pe\cos s + Qe\cos(2\eta - s) + Re\cos(2\eta + s) + Fv\cos\eta + Kve\cos(\eta - s) + Mve\cos(3\eta - s) + Gv\cos 3\eta + Lve\cos(\eta + s) + Nve\cos(3\eta + s)$$

§. 188. Quodſi iam ponamus:

$$\frac{2\varkappa K + \mathfrak{K}}{nn} = k' \quad ; \quad \frac{2\varkappa L + \mathfrak{L}}{nn} = l'$$

$$\frac{2\varkappa M + \mathfrak{M}}{nn} = m' \quad \frac{2\varkappa N + \mathfrak{N}}{nn} = n'$$

erit

$$\frac{d\Phi}{dr} = \text{Praec.} - a'\cos 2\eta - p'e\cos s - q'e\cos(2\eta - s) - r'e\cos(2\eta + s) - f'v\cos\eta - k've\cos(\eta - s) - m've\cos(3\eta - s) - g'v\cos 3\eta - l've\cos(\eta + s) - n've\cos(3\eta + s)$$

atque ob $\frac{ds}{dr} = \frac{1}{n} - \frac{2}{n}e\cos s$ erit

$$\frac{d\eta}{dr} = \alpha - a'\cos 2\eta + \left(\frac{2}{n} - p'\right)e\cos s - q'e\cos(2\eta - s) - r'e\cos(2\eta + s) - f'v\cos\eta - k've\cos(\eta - s) - m've\cos(3\eta - s) - g'v\cos 3\eta - l've\cos(\eta + s) - n've\cos(3\eta + s)$$

§. 189. Formulas nunc aſſumtas differentiemus, ſolosque terminos, quibus opus habemus, in calculo exprimamus ac reperiemus:

$$R = \text{Praec.}$$

$$+ve\sin(\eta - s)\left(+\mathfrak{A}l' - \mathfrak{A}m' + \mathfrak{Q}f' - \mathfrak{R}g' - \tfrac{1}{2}\mathfrak{F}\left(\frac{2}{n} - p'\right) - \tfrac{1}{2}\mathfrak{F}q' + \tfrac{1}{2}\mathfrak{G}r' - \mathfrak{K}\left(\alpha - \frac{1}{n}\right) - \tfrac{1}{2}\mathfrak{L}a' + \tfrac{1}{2}\mathfrak{M}a' + \right.$$

$$+ve\sin(\eta+s)\left(+\mathfrak{A}k'-\mathfrak{A}n'+\mathfrak{R}f'-\mathfrak{Q}g'-\tfrac{1}{2}\mathfrak{F}\left(\tfrac{2}{n}-p'\right)-\tfrac{1}{2}\mathfrak{F}r'+\tfrac{3}{2}\mathfrak{G}q'-\mathfrak{L}\left(\alpha-\tfrac{1}{n}\right)\right.$$
$$\left.-\tfrac{1}{2}\mathfrak{K}a'+\tfrac{3}{2}\mathfrak{N}a'\right)$$
$$+ve\sin(3\eta-s)\left(+\mathfrak{A}k'+\mathfrak{Q}f'+\tfrac{1}{2}\mathfrak{F}q'-\tfrac{3}{2}\mathfrak{G}\left(\tfrac{2}{n}-p'\right)+\tfrac{1}{2}\mathfrak{K}a'-\mathfrak{M}\left(3\alpha-\tfrac{1}{n}\right)\right)$$
$$+ve\sin(3\eta-s)\left(+\mathfrak{A}l'+\mathfrak{R}f'+\tfrac{1}{2}\mathfrak{F}r'-\tfrac{3}{2}\mathfrak{G}\left(\tfrac{2}{n}-p'\right)+\tfrac{1}{2}\mathfrak{L}a'-\mathfrak{N}\left(3\alpha+\tfrac{1}{n}\right)\right)$$

Ac ſi breuitatis gratia ponamus:

$$K'=\left(\alpha-\tfrac{1}{n}\right)K-A(l'-m')-Qf'+Rg'+\tfrac{1}{2}F\left(\tfrac{2}{n}-p'\right)+\tfrac{1}{2}Fq'-\tfrac{3}{2}Gr'$$
$$+\tfrac{1}{2}La'-\tfrac{3}{2}Ma'$$
$$L'=\left(\alpha+\tfrac{1}{n}\right)L-A(k'-n')-Rf'+Qg'+\tfrac{1}{2}F\left(\tfrac{2}{n}-p'\right)+\tfrac{1}{2}Fr'-\tfrac{3}{2}Gq'$$
$$+\tfrac{1}{2}Ka'-\tfrac{3}{2}Na'$$
$$M'=\left(3\alpha-\tfrac{1}{n}\right)M-Ak'-Qf'-\tfrac{1}{2}Fq'+\tfrac{3}{2}G\left(\tfrac{2}{n}-p'\right)-\tfrac{1}{2}Ka'$$
$$N'=\left(3\alpha+\tfrac{1}{n}\right)N-Al'-Rf'-\tfrac{1}{2}Fr'+\tfrac{3}{2}G\left(\tfrac{2}{n}-p'\right)-\tfrac{1}{2}La'$$

erit

$$\frac{dv}{dr}=-A'\sin 2\eta-P'e\sin s-Q'e\sin(2\eta-s)$$
$$-R'e\sin(2\eta+s)$$
$$-F'v\sin\eta-K've\sin(\eta-s)-M've\sin(3\eta-s)$$
$$-G'v\sin 3\eta-L've\sin(\eta+s)-N've\sin(3\eta+s)$$

§. 190. Hinc iam denuo differentiando nanciſcemur

$$\frac{ddv}{dr^2}=\text{Praec.}$$

$$+ve\cos(\eta-s)\begin{cases}A'l'+A'm'+Q'f'+R'g'-\tfrac{1}{2}F'\left(\tfrac{2}{n}-p'\right)+\tfrac{1}{2}F'q'\\ -+\tfrac{3}{2}G'r'+\tfrac{1}{2}L'a'+\tfrac{3}{2}M'a'-\left(\alpha-\tfrac{1}{n}\right)K'\end{cases}$$

$+$

$$+ve\cos(\eta+s)\left\{\begin{array}{r} A'k'+A'n'+R'f'+Q'g'-\frac{1}{2}F'(\frac{2}{n}-p')+\frac{1}{2}F'r' \\ +\frac{1}{2}G'q'+\frac{1}{2}K'a'+\frac{1}{2}N'a'-(\alpha+\frac{1}{n})L' \end{array}\right.$$

$$+ve\cos(3\eta-s)\left\{\begin{array}{r} A'k'+Q'f'+\frac{1}{2}F'q'-\frac{1}{2}G'(\frac{2}{n}-p')+\frac{1}{2}K'a' \\ -(3\alpha-\frac{1}{n})M' \end{array}\right.$$

$$+ve\cos(3\eta+s)\left\{\begin{array}{r} A'l'+R'f'+\frac{1}{2}F'r'-\frac{1}{2}G'(\frac{2}{n}-p')+\frac{1}{2}L'a' \\ -(3\alpha+\frac{1}{n})N' \end{array}\right.$$

§. 191. Quodſi autem valores iam inuenti ſubſtituantur, habebitur

$$R = \text{Praec.}$$

$$+ve\sin(\eta-s)\left\{\begin{array}{r} -0{,}85830\mathfrak{K}+0{,}00526\mathfrak{L}-0{,}02500\mathfrak{M}+0{,}37592 \\ -0{,}00931L+0{,}00931M \end{array}\right]$$

$$+ve\sin(\eta+s)\left\{\begin{array}{r} -1{,}00918\mathfrak{L}+0{,}00526\mathfrak{K}-0{,}02500\mathfrak{N}+0{,}34115 \\ -0{,}00931K+0{,}00931N \end{array}\right]$$

$$+ve\sin(3\eta-s)\left\{\begin{array}{r} -2{,}72578\mathfrak{M}-0{,}01448\mathfrak{K}+0{,}49223 \\ -0{,}00931K \end{array}\right]$$

$$+ve\sin(3\eta+s)\left\{\begin{array}{r} -2{,}87666\mathfrak{N}-0{,}01448\mathfrak{L}+0{,}47116 \\ -0{,}00931L \end{array}\right]$$

 $K' =$

$$ve\,cſ(\eta+s)\begin{cases}-1{,}01923L-0{,}04222K-0{,}12821N-0{,}00014M\\-0{,}00029\mathfrak{L}-0{,}02166\mathfrak{K}-0{,}00642\mathfrak{N}+0{,}00008\mathfrak{M}\\\qquad-2{,}11858\end{cases}$$

$$ve\,cſ(3\eta-s)\begin{cases}-7{,}43058M-0{,}09507K-0{,}00005L+3{,}93257\\+0{,}00008\mathfrak{M}-0{,}03453\mathfrak{K}-0{,}00007\mathfrak{L}\end{cases}$$

$$ve\,cſ(3\eta+s)\begin{cases}-8{,}27529N-0{,}11338L-0{,}00005K+3{,}41830\\+0{,}00008\mathfrak{N}-0{,}03566\mathfrak{L}-0{,}00007\mathfrak{K}\end{cases}$$

§. 193. His expreſſionibus ita euolutis atque ad calculum numericum praeparatis, quaeramus easdem expreſſiones ex formulis ſupra traditis pro R et $\frac{ddv}{dr^2}$, quae continentur in §. 52 et 54. Inde autem omittendis terminis, quos iam tractauimus, conſequemur.

$$R = Pr. + ve\,ſin(\eta-s)\left(-\tfrac{3}{4}+\frac{3L}{2nn}-\frac{3M}{2nn}-\frac{9F}{4nn}+\frac{9G}{4nn}\right)$$

$$+ve\,ſin(\eta+r)\left(-\tfrac{3}{4}+\frac{3K}{2nn}-\frac{3N}{2nn}+\frac{9G}{4nn}-\frac{9F}{4nn}\right)$$

$$+ve\,ſin(3\eta-s)\left(-\tfrac{15}{4}+\frac{3K}{2nn}-\frac{9F}{4nn}\right)$$

$$+ve\,ſin(3\eta+s)\left(-\tfrac{15}{4}+\frac{3L}{2nn}-\frac{9F}{4nn}\right)$$

ddv

$$\frac{ddv}{dr^2} = \text{Praec.}$$

$$+ve\cos(\eta - s)\left\{\begin{aligned}
&-\tfrac{9}{4} + \frac{9P}{8nn} + \frac{9Q}{8nn} + \frac{15R}{8nn} - 6K + \frac{3L}{4nn} + \frac{3M}{4nn} \\
&-\frac{3F}{4nn} - \frac{9F}{8nn} - \frac{9G}{8nn} - 2\kappa\mathfrak{K} + \frac{(\mathfrak{A}+3A)}{nn}(\mathfrak{L}+\mathfrak{M}) \\
&+\frac{(3\mathfrak{A}+3A)}{nn}(L+M) + \frac{(\mathfrak{P}+3P)}{nn}\mathfrak{P} + \frac{(3\mathfrak{P}+3P)}{nn}F \\
&+\frac{(\mathfrak{Q}+3Q)}{nn}\mathfrak{P} + \frac{(3\mathfrak{Q}+3Q)}{nn}F + \frac{(\mathfrak{R}+3R)}{nn}\mathfrak{G} \\
&+\frac{(3\mathfrak{R}+3R)}{nn}G
\end{aligned}\right.$$

$$+ve\cos(\eta + s)\left\{\begin{aligned}
&-\tfrac{9}{4} + \frac{9P}{8nn} + \frac{9R}{8nn} + \frac{15Q}{8nn} - 6L + \frac{3K}{4nn} + \frac{3N}{4nn} \\
&-\frac{3F}{4nn} - \frac{9F}{8nn} - \frac{9G}{8nn} - 2\kappa\mathfrak{L} + \frac{(\mathfrak{A}+3A)}{nn}(\mathfrak{K}+\mathfrak{N}) \\
&+\frac{(3\mathfrak{A}+3A)}{nn}(K+N) + \frac{(\mathfrak{P}+3P)}{nn}\mathfrak{F} + \frac{(3\mathfrak{P}+3P)}{nn}F \\
&+\frac{(\mathfrak{Q}+3Q)}{nn}\mathfrak{G} + \frac{(3\mathfrak{Q}+3Q)}{nn}G + \frac{(\mathfrak{R}+3R)}{nn}\mathfrak{F} \\
&+\frac{(3\mathfrak{R}+3R)}{nn}F
\end{aligned}\right.$$

$$+ve\cos(3\eta - s)\left\{\begin{aligned}
&-\tfrac{15}{4} + \frac{15P}{8nn} + \frac{9Q}{8nn} - 6M + \frac{3K}{4nn} - \frac{3G}{4nn} \\
&-\frac{9F}{8nn} - 2\kappa\mathfrak{M} + \frac{(\mathfrak{A}+3A)}{nn}\mathfrak{K} + \frac{(3\mathfrak{A}+3A)}{nn}K + \frac{(\mathfrak{P}+3P)}{nn}\mathfrak{G} \\
&+\frac{(3\mathfrak{P}+3P)}{nn}G + \frac{(\mathfrak{Q}+3Q)}{nn}\mathfrak{F} + \frac{(3\mathfrak{Q}+3Q)}{nn}F
\end{aligned}\right.$$

$+ve$

$$+ve\cos(3\eta+s)\begin{cases} -\frac{15}{4}+\frac{15P}{8nn}+\frac{9R}{8nn}-6N+\frac{3L}{4nn}-\frac{3G}{4nn}-\frac{9F}{8nn}-2\kappa\mathfrak{N} \\ +\frac{(\mathfrak{A}+3A)}{nn}\mathfrak{L}+\frac{(3\mathfrak{A}+3A)}{nn}L+\frac{(\mathfrak{P}+3P)}{nn}\mathfrak{G}+\frac{(3\mathfrak{P}+3P)}{nn}G \\ +\frac{(\mathfrak{R}+3R)}{nn}\mathfrak{F}+\frac{(3\mathfrak{R}+3R)}{nn}F \end{cases}$$

§. 194. Introducantur hic quoque valores iam cogniti, ac prodibit

$$\begin{aligned} R = Pr. & + ve\sin(\eta-s)[-1{,}02394+0{,}00854L-0{,}00854M] \\ & + ve\sin(\eta+s)[-1{,}02394+0{,}00854K-0{,}00854N] \\ & + ve\sin(3\eta-s)[-4{,}01450+0{,}00854K] \\ & + ve\sin(3\eta+s)[-4{,}01450+0{,}00854L] \end{aligned}$$

$$\frac{ddv}{dr^2} = \text{Praec.}$$

$$+ve\cos(\eta-s)\begin{bmatrix} -1{,}01591K-0{,}03207L-0{,}03207M-1{,}168671 \\ -2{,}01798\mathfrak{K}-0{,}02711\mathfrak{L}-0{,}02711\mathfrak{M} \end{bmatrix}$$

$$+ve\cos(\eta+s)\begin{bmatrix} -1{,}01591L-0{,}03207K-0{,}03207N-1{,}93904 \\ -2{,}01798\mathfrak{L}-0{,}02711\mathfrak{K}-0{,}02711\mathfrak{N} \end{bmatrix}$$

$$+ve\cos(3\eta-s)\begin{bmatrix} -1{,}01591M-0{,}03207K-2{,}49545 \\ -2{,}01798\mathfrak{M}-0{,}02711\mathfrak{K} \end{bmatrix}$$

$$+ve\cos(\eta 3+s)\begin{bmatrix} -1{,}01591N-0{,}03207L-2{,}78450 \\ -2{,}01798\mathfrak{M}-0{,}02711\mathfrak{L} \end{bmatrix}$$

§. 195. Hinc ergo octo ſequentes aequationes reſultabunt

I. $0,85830\,\mathfrak{K} = +\,0,00526\,\mathfrak{L} - 0,02500\,\mathfrak{M} + 1,39986$
$- 0,01785\,L + 0,01785\,M$

II. $1,00918\,\mathfrak{L} = +\,0,00526\,\mathfrak{K} - 0,02500\,\mathfrak{N} + 1,36509$
$- 0,01785\,K + 0,01785\,N$

III. $2,72578\,\mathfrak{M} = -\,0,01448\,\mathfrak{K} + 4,50673$
$- 0,01785\,K$

IV. $2,87666\,\mathfrak{N} = -\,0,01448\,\mathfrak{L} + 4,48566$
$- 0,01785\,L$

V. $+0,27844\,K - 0,01057\,L - 0,08949\,M - 0,00014\,N + 0,10081 = 0$
$+2,01769\,\mathfrak{K} + 0,00658\,\mathfrak{L} - 0,01946\,\mathfrak{M} + 0,00008\,\mathfrak{N}$

VI. $-0,00332\,L - 0,01015\,K - 0,09614\,N - 0,00014\,M - 0,17954 = 0$
$+2,01769\,\mathfrak{L} + 0,00545\,\mathfrak{K} + 0,02069\,\mathfrak{N} + 0,00008\,\mathfrak{M}$

VII. $-6,41487\,M - 0,06300\,K - 0,00005\,L + 6,42802 = 0$
$+2,01806\,\mathfrak{M} - 0,00742\,\mathfrak{K} - 0,00007\,\mathfrak{L}$

VIII. $-7,25938\,N - 0,08131\,L - 0,00005\,K + 6,20280 = 0$
$+2,01806\,\mathfrak{N} - 0,00855\,\mathfrak{L} - 0,00007\,\mathfrak{K}$

§. 196. Ex aequationibus III et IV ſtatim eliciuntur hi valores

$$\mathfrak{M} = -\,0,00531\,\mathfrak{K} - 0,00655\,K + 1,65336$$
$$\mathfrak{N} = -\,0,00503\,\mathfrak{L} - 0,00621\,L + 1,55933$$

qui

qui in I et II ſubſtituti praebent:

$$0,85817\mathfrak{K} = 1,35853 + 0,00526\mathfrak{L} - 0,01785(L-M) + 0,00016K$$
$$1,00905\mathfrak{L} = 1,32611 + 0,00526\mathfrak{K} - 0,01785(K-N) + 0,00015L$$

vnde obtinetur:

$$\mathfrak{K} = +1,59116 - 0,02080(L-M) + 0,00008K \pm 0,00010N$$
$$\mathfrak{L} = +1,32251 - 0,01769(K-N) + 0,00005L + 0,00011M$$
$$\mathfrak{M} = +1,64491 - 0,00655K + 0,00010(L-M)$$
$$\mathfrak{N} = +1,55268 - 0,00621L + 0,00009(K-N)$$

§. 197. His valoribus ſubſtitutis caeterae aequationes abibunt in formas ſequentes:

$$0,27862K - 0,05252L - 0,04754M + 0,00017N + 3,35313 = 0$$
$$-0,00346L - 0,04584K - 0,06045N + 0,00020M + 2,53002 = 0$$
$$-6,41502M - 0,07622K + 0,00030L + 9,73587 = 0$$
$$-7,25971N - 0,09384L + 0,00028K + 9,32499 = 0$$

ex quarum binis poſtremis ſtatim obtinetur:

$$M = -0,01188K + 0,00005L + 1,51765$$
$$N = -0,01293L + 0,00004K + 1,28448$$

vnde colligitur:

$$+0,27918K - 0,05252L + 3,28120 = 0$$
$$-0,00268L - 0,04584K + 2,45267 = 0$$

ac denique

$$K = +40,44710 \quad \ldots \quad lK = 1,606887$$
$$L = +368,40200 \quad \ldots \quad lL = 2,566322$$
$$M = +1,05555 \quad \ldots \quad lM = 0,023478$$
$$N = -3,47730 \quad \ldots \quad l-N = 0,541242$$

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§. 198. Litterarum germanicarum valores hinc erunt:

$\mathfrak{K} = - 5,04677$ . . . $l{-}\mathfrak{K} = 0,703013$

$\mathfrak{L} = + 0,56302$ . . . $l\,\mathfrak{L} = 9,750524$

$\mathfrak{M} = + 1,41672$ . . . $l\,\mathfrak{M} = 0,151283$

$\mathfrak{N} = - 0,73119$ . . . $l\,\mathfrak{N} = 9,864030$

ac litterarum hinc deriuatarum:

$k' = + 0,43578$ . . . $l\,k' = 9,639267$

$l' = + 4,23401$ . . . $l\,l' = 0,626752$

$m' = + 0,02018$ . . . $l\,m' = 8,304921$

$n' = - 0,04409$ . . . $l{-}n' = 8,644340$

§. 199. Nunc igitur intelligimus inaequalitates ab angulis $3\eta - s$ et $3\eta + s$ pendentes tam eſſe paruas, vt ſine vllo errore reiici queant, etiamſi valores K et L aliquantum immutauerint. Diſtantia ergo lunae curtata a terra $x = \frac{(1-kk)\,au}{1-k\,\mathrm{co\text{ſ}}\,r}$ ita ab his inaequalitatibus parallacticis pendebit, vt ſit

	Log. coeff.
$x =$ Praec. $+ 0,11756\; v\; \mathrm{co\text{ſ}}\,\eta$	9,070249
$- 0,00419\; v\; \mathrm{co\text{ſ}}\,3\eta$	7,622540
$- 0,1234\; vk\, \mathrm{co\text{ſ}}\,(\eta - r)$	9,091265
$- 0,0784\; vk\, \mathrm{co\text{ſ}}\,(\eta + r)$	8,894183
$+ 0,2302\; ve\, \mathrm{co\text{ſ}}\,(\eta - s)$	9,362071
$+ 2,0965\; ve\, \mathrm{co\text{ſ}}\,(r + s)$	0,321506

Motus

Motus autem momentaneus ita hinc afficietur, vt ſit

$$\begin{array}{lll|l} \frac{d\Phi}{dr} = \text{Praec.} & - 0,23259 & v \cos\eta & 9,366591 \\ & + 0,01309 & v \cos 3\eta & 8,116940 \\ & + 0,0939 & vk \cos(\eta - r) & 8,972943 \\ & + 0,1653 & vk \cos(\eta + r) & 9,218352 \\ & - 0,4358 & ve \cos(\eta - s) & 9,639267 \\ & - 4,2340 & ve \cos(\eta + s) & 0,626752 \end{array}$$

§. 200. Quodſi iam ipſam longitudinem lunae, quatenus ab his inaequalitatibus parallacticis pendet, ponamus:

$$\Phi = \text{Praec.} + \mathfrak{F}' v \sin\eta + \mathfrak{H}' vk \sin(\eta - r) + \mathfrak{K}' ve \sin(\eta - s)$$
$$+ \mathfrak{G}' v \sin 3\eta + \mathfrak{J}' vk \sin(\eta + r) + \mathfrak{L}' ve \sin(\eta + s)$$

ſequentes obtinebimus aequationes pro horum coefficientium determinatione:

$$-0,23259 = \alpha \mathfrak{F}' - \mathfrak{A}' f' - \mathfrak{A}' g' - \tfrac{1}{2} \mathfrak{F}' a' - \tfrac{1}{2} \mathfrak{G}' a'$$

$$+0,01303 = 3\alpha \mathfrak{G}' - \mathfrak{A}' f' - \tfrac{1}{2} \mathfrak{F}' a'$$

$$+0,0939 = (\alpha - 1) \mathfrak{H}' - \mathfrak{A}' i' - \tfrac{1}{2} \mathfrak{J}' a' - \mathfrak{D}' f' - \mathfrak{E}' g'$$
$$- \tfrac{1}{2} \mathfrak{F}' c' - \tfrac{1}{2} \mathfrak{F}' d' - \tfrac{1}{2} \mathfrak{G}' c'$$

$$+0,1653 = (\alpha + 1) \mathfrak{J}' - \mathfrak{A}' b' - \tfrac{1}{2} \mathfrak{H}' a' - \mathfrak{E}' f' - \mathfrak{D}' g'$$
$$- \tfrac{1}{2} \mathfrak{F}' c' - \tfrac{1}{2} \mathfrak{F}' c' - \tfrac{1}{2} \mathfrak{G}' d'$$

—

$$-0,4358 = \left(a-\frac{1}{n}\right)\mathfrak{K}' - \mathfrak{A}'l' - \tfrac{1}{2}\mathfrak{L}'a' - \mathfrak{Q}'f' - \mathfrak{R}'g'$$
$$+ \tfrac{1}{2}\mathfrak{F}'\left(\frac{2}{n}-p'\right) - \tfrac{1}{2}\mathfrak{F}'q' - \tfrac{1}{2}\mathfrak{G}'r'$$
$$-4,2340 = \left(a+\frac{1}{n}\right)\mathfrak{L}' - \mathfrak{A}'k' - \tfrac{1}{2}\mathfrak{K}'a' - \mathfrak{R}'f' - \mathfrak{Q}'g'$$
$$+ \tfrac{1}{2}\mathfrak{F}'\left(\frac{2}{n}-p'\right) - \tfrac{1}{2}\mathfrak{F}'r' - \tfrac{1}{2}\mathfrak{G}'q'$$

§. 201. Valoribus autem iam cognitis hic ſubſtitutis, aequationes iſtae in ſequentes abibunt formas;

$$-0,23031 = +0,94361\,\mathfrak{F}' + 0,02961\,\mathfrak{G}'$$
$$+0,01550 = +2,80122\,\mathfrak{G}' + 0,00987\,\mathfrak{F}'$$
$$-0,0056 = -0,06626\,\mathfrak{H}' + 0,00987\,\mathfrak{I}' - 0,25665\,\mathfrak{F}' + 0,01926\,\mathfrak{G}'$$
$$+0,1710 = +1,93374\,\mathfrak{I}' + 0,00987\,\mathfrak{H}' - 0,06717\,\mathfrak{F}' - 0,54918\,\mathfrak{G}'$$
$$-0,3968 = +0,85830\,\mathfrak{K}' + 0,00987\,\mathfrak{L}' + 0,06357\,\mathfrak{F}' - 0,04509\,\mathfrak{G}'$$
$$-4,2330 = +1,00918\,\mathfrak{L}' + 0,00987\,\mathfrak{K}' + 0,06729\,\mathfrak{F}' - 0,05625\,\mathfrak{G}'$$

vnde colligitur fore

$\mathfrak{F}' = -0,24427$ . . . $l{-}\mathfrak{F}' = 9,387868$

$\mathfrak{G}' = +0,00639$ . . . $l\,\mathfrak{G}' = 7,805991$

$\mathfrak{H}' = +1,1959$ . . . $l\,\mathfrak{H}' = 0,077694$

$\mathfrak{I}' = +0,0757$ . . . $l\,\mathfrak{I}' = 8,879096$

$\mathfrak{K}' = -0,3959$ . . . $l{-}\mathfrak{K}' = 9,597508$

$\mathfrak{L}' = -4,1738$ . . . $l{-}\mathfrak{L}' = 0,620530$

§. 201.

§. 202. Hinc ergo habebimus ſequentes partes pro longitudine Lunae, quas ſimul ope valorum proxime cognitorum pro ν, k, e ad minuta ſecunda reducamus:

			Log coeff.	Val. coeff. in min ſec.
$\Phi =$ Praec.	$-0,24427$	$\nu \sin \eta$	9,387868	$-175''$
	$+0,00639$	$\nu \sin 3\eta$	7,805991	$+4''$
	$+1,1959$	$\nu k \sin(\eta - r)$	0,077694	$+59''$
	$+0,0757$	$\nu k \sin(\eta + r)$	8,879096	$+4''$
	$-0,3959$	$\nu e \sin(\eta - s)$	9,597508	$-5''$
	$-4,1738$	$\nu e \sin(\eta + s)$	0,620530	$-49''$

Sicque omnes iam adepti ſumus motus lunae inaequalitates, quae quidem ab inclinatione eius orbitae ad eclipticam non pendent. Interim tamen non diffiteor, dari aliquas inſuper inaequalitates, quae alicuius forte ſint momenti, quas in hac inueſtigatione praeterimus, cuiusmodi ſunt eae, quae ab angulis $2\eta - 3r$ et $2\eta - 2r + s$ pendent, quae ad plura minuta ſecunda asſurgere poſſe videntur. Verum earum determinatio tam eſt taedioſa, vt malim eam obſeruationibus relinquere.

§. 203. Quae ergo hactenus inuenimus, in vnum colligamus ac primo pro diſtantia lunae a terra curtata $x = \frac{(1-kk)\,au}{1-k\cos r}$ erit

$u =$

	Log. coeff.	coeff. integri
$u = 1 - 0{,}0074991 \cos 2\eta$	7,875009	$-0{,}007499$
$+0{,}0000532 \cos 4\eta$	5,725912	$+0{,}000053$
$+0{,}191557\, k \cos(2\eta - r)$	9,282297	$+0{,}010430$
$-0{,}003293\, k \cos(2\eta + r)$	7,517525	$-0{,}000179$
$-0{,}003321\, k \cos(4\eta - r)$	7,521296	$-0{,}000181$
$+0{,}000049\, k \cos(4\eta + r)$	5,692584	$+0{,}000005$
$-0{,}00511\, kk \cos 2r$	7,708601	$-0{,}000015$
$-0{,}08022\, kk \cos(2\eta - 2r)$	8,904280	$-0{,}000238$
$-0{,}00237\, kk \cos(2\eta + 2r)$	7,375072	$-0{,}000007$
$+0{,}07892\, kk \cos(4\eta - 2r)$	8,897172	$+0{,}000234$
$+0{,}00001\, kk \cos(4\eta + 2r)$	4,872349	$+0{,}000000$
$-0{,}006400\, e \cos s$	7,806180	$-0{,}000206$
$+0{,}014801\, e \cos(2\eta - s)$	8,170303	$+0{,}000249$
$+0{,}011415\, e \cos(2\eta + s)$	8,057492	$+0{,}000192$
$+0{,}00364\, ee \cos 2s$	7,539670	$+0{,}000001$
$-0{,}01482\, ee \cos(2\eta - 2s)$	8,170799	$-0{,}000004$
$-0{,}00584\, ee \cos(2\eta + 2s)$	7,766856	$-0{,}000001$
$-0{,}37957\, ek \cos(r - s)$	9,579297	$-0{,}000347$
$+0{,}30693\, ek \cos(r + s)$	9,487039	$+0{,}000281$
$-0{,}36337\, ek \cos(2\eta - r + s)$	9,560344	$-0{,}000332$
$+0{,}00993\, ek \cos(2\eta + r - s)$	7,997004	$+0{,}000009$
$-0{,}07752\, ek \cos(2\eta - r - s)$	8,889425	$-0{,}000071$
$+0{,}00305\, ek \cos(2\eta + r + s)$	7,484024	$+0{,}000003$
$+0{,}11756\, v \cos \eta$	9,070249	$+0{,}000408$
$-0{,}00419\, v \cos 3\eta$	7,622540	$-0{,}000015$
$-0{,}1234\, vk \cos(\eta - r)$	9,091265	$-0{,}000024$
$-0{,}0784\, vk \cos(\eta + r)$	8,894183	$-0{,}000015$
$+0{,}2302\, ve \cos(\eta - s)$	9,362071	$+0{,}000013$
$+2{,}0965\, ve \cos(\eta + s)$	0,321506	$+0{,}000122$

Hic ad latus adiunxi valores coefficientium integrorum in numeris abſolutis expreſſos, ponendo $k = 0{,}05445$, $e = 0{,}01680$ et $v = \frac{1}{388}$; quos proinde, ſi hi valores aliter per obſeruationes determinentur, facile erit emendare.

§. 204.

§. 204. Pro motu autem lunae momentaneo, ex quo eius motus horarius definiri poterit, habebimus:

$\frac{d\Phi}{dr} =$	Log. coeff.	Coeff. integri
$1,009176$	0,003967	$+$ 1,009176
$+ 0,0195144\ \mathrm{coſ}\ 2\eta$	8,290355	$+$ 0,019514
$- 0,0000322\ \mathrm{coſ}\ 4\eta$	5,507856	$-$ 0,000032
$- 0,001231\,k\ \mathrm{coſ}\ r$	7,090258	$-$ 0,000067
$- 0,366103\,k\ \mathrm{coſ}\ (2\eta - r)$	9,563604	$-$ 0,019934
$+ 0,012832\,k\ \mathrm{coſ}\ (2\eta + r)$	8,108292	$+$ 0,000699
$+ 0,002829\,k\ \mathrm{coſ}\ (4\eta - r)$	7,451633	$+$ 0,000154
$- 0,000171\,k\ \mathrm{coſ}\ (4\eta + r)$	6,232305	$-$ 0,000009
$+ 0,01182\,kk\ \mathrm{coſ}\ 2r$	8,072618	$+$ 0,000035
$- 0,02057\,kk\ \mathrm{coſ}\ (2\eta - 2r)$	8,313172	$-$ 0,000061
$+ 0,01063\,kk\ \mathrm{coſ}\ (2\eta + 2r)$	8,026598	$+$ 0,000032
$- 0,09883\,kk\ \mathrm{coſ}\ (4\eta - 2r)$	8,994889	$-$ 0,000293
$- 0,00004\,kk\ \mathrm{coſ}\ (4\eta + 2r)$	5,592770	$-$ 0,000000
$+ 0,013760\,e\ \mathrm{coſ}\ s$	8,138618	$+$ 0,000231
$- 0,037487\,e\ \mathrm{coſ}\ (2\eta - s)$	8,573878	$-$ 0,000630
$- 0,030062\,e\ \mathrm{coſ}\ (2\eta + s)$	8,478023	$-$ 0,000505
$- 0,00722\,ee\ \mathrm{coſ}\ 2s$	7,858166	$-$ 0,000002
$+ 0,03470\,ee\ \mathrm{coſ}\ (2\eta - 2s)$	8,540319	$+$ 0,000010
$+ 0,01533\,ee\ \mathrm{coſ}\ (2\eta + 2s)$	8,185614	$+$ 0,000005
$+ 0,75204\,ek\ \mathrm{coſ}\ (r - s)$	9,876241	$+$ 0,000688
$- 0,62626\,ek\ \mathrm{coſ}\ (r + s)$	9,796755	$-$ 0,000573
$+ 0,69420\,ek\ \mathrm{coſ}\ (2\eta - r + s)$	9,841484	$+$ 0,000635
$- 0,02440\,ek\ \mathrm{coſ}\ (2\eta + r - s)$	8,387390	$-$ 0,000022
$+ 0,12160\,ek\ \mathrm{coſ}\ (2\eta - r - s)$	9,084933	$+$ 0,000111
$- 0,02000\,ek\ \mathrm{coſ}\ (2\eta + r + s)$	9,301030	$-$ 0,000018
$- 0,23259\,v\ \mathrm{coſ}\ \eta$	9,366591	$-$ 0,000808
$+ 0,01309\,v\ \mathrm{coſ}\ 3\eta$	8,116940	$+$ 0,000045
$+ 0,0939\,vk\ \mathrm{coſ}\ (\eta - r)$	8,972943	$+$ 0,000018
$+ 0,1653\,vk\ \mathrm{coſ}\ (\eta + r)$	9,218352	$+$ 0,000031
$- 0,4358\,ve\ \mathrm{coſ}\ (\eta - s)$	9,639267	$-$ 0,000025
$- 4,2340\,ve\ \mathrm{coſ}\ (\eta + s)$	0,626753	$-$ 0,000247

§. 205. Si iam longitudo Lunae per ſolam excentricitatem ſecundum regulas Keplerianas determinata ponatur $= \zeta$, ita vt poſita eius anomalia vera $= r$, futurum ſit $\zeta = C + 1{,}0085272\, r$, erit longitudo vera per hactenus inuentas inaequalitates.

	Log. coeff.	Val. coeff. in min. ſec.
$\varphi = \zeta + 0{,}0103597 \sin 2\eta$	8,015347	$+2137''$
$- 0{,}0000382 \sin 4\eta$	5,582063	$-\ 8$
$+ 0{,}010146k \sin r$	8,006295	$+114$
$- 0{,}420226k \sin(2\eta - r)$	9,623483	-4720
$+ 0{,}004992k \sin(2\eta + r)$	7,698261	$+\ 56$
$+ 0{,}005286k \sin(4\eta - r)$	7,723163	$+\ 59$
$- 0{,}000086k \sin(4\eta + r)$	5,935307	$-\ 1$
$+ 0{,}00420kk \sin 2r$	7,623250	$+\ 2\frac{1}{2}$
$+ 0{,}57328kk \sin(2\eta - 2r)$	9,758367	$+351$
$+ 0{,}00318kk \sin(2\eta + 2r)$	7,402427	$+\ 1\frac{1}{2}$
$- 0{,}15083kk \sin(4\eta - 2r)$	9,178488	$-\ 92$
$- 0{,}00002kk \sin(4\eta + 2r)$	5,301030	$-\ 0$
$+ 0{,}201385e \sin s$	9,304026	$+701$
$- 0{,}021889e \sin(2\eta - s)$	8,340237	$-\ 76$
$- 0{,}016368e \sin(2\eta + s)$	8,214002	$-\ 57$
$+ 0{,}06615ee \sin 2s$	8,820508	$+\ 4$
$+ 0{,}02332ee \sin(2\eta - 2s)$	8,367825	$+\ 1$
$+ 0{,}00840ee \sin(2\eta + 2s)$	7,924429	$+\ \frac{1}{2}$
$+ 0{,}74760ek \sin(r - s)$	9,873165	$+141$
$- 0{,}61850ek \sin(r + s)$	9,791317	-118
$+ 0{,}81430ek \sin(2\eta - r + s)$	9,910800	$+154$
$- 0{,}01420ek \sin(2\eta + r - s)$	8,150690	$-\ 3$
$+ 0{,}23960ek \sin(2\eta - r - s)$	9,379550	$+\ 45$
$- 0{,}00610ek \sin(2\eta + r + s)$	7,788910	$-\ 1$
$- 0{,}24427\nu \sin \eta$	9,387868	-175
$+ 0{,}00639\nu \sin 3\eta$	7,805991	$+\ 4$
$+ 1{,}1959\nu k \sin(\eta - r)$	0,077694	$+\ 59$
$+ 0{,}0757\nu k \sin(\eta + r)$	8,879096	$+\ 4$
$- 0{,}3959\nu e \sin(\eta - s)$	9,597508	$-\ 5$
$- 4{,}1738\nu e \sin(\eta + s)$	0,620530	$-\ 49$

CAPUT

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CAPUT XII.

INUESTIGATIO INAEQUALITATUM MOTUM LINEAE NODORUM AFFICIENTIUM.

§. 206.

Antequam reliquas motus Lunae inaequalitates, quae ab inclinatione eius orbitae ad eclipticam pendent, definire licet, cum variationes, quae in motu lineae nodorum Lunae, tum eas, quae in ipſa inclinatione eius orbitae ad eclipticam deprehenduntur, inueſtigari oportet. Reſidua enim pars aequationis noſtrae principalis, qua omnes motus Lunae inaequalitates continentur, litteras π et ρ implicat, quarum illa longitudinem nodi aſcendentis, haec vero ρ inclinationem ad eclipticam deſignat. Niſi igitur vtriusque huius quantitatis incrementa vel decrementa ad differentiale dr reduxerimus, reſiduas motus Lunae inaequalitates determinare non poterimus.

207. Aequatio autem ſupra (55) pro motu lineae nodorum tradita, cum ſit $\frac{2\varkappa v + \int R\,dr}{nn} = \alpha + \frac{1+2ee}{nn} - \frac{d\Phi}{dr}$

ideoque $\frac{2\varkappa v + \int R\,dr}{nn} = a' \operatorname{coſ} 2\eta - (\frac{2}{n} - c')k \operatorname{coſ} r + d'k \operatorname{coſ}(2\eta - r)$

$$+ e'k \operatorname{coſ}(2\eta + r)$$

$$+ p'e \operatorname{coſ} s + q'e \operatorname{coſ}(2\eta - s)$$

$$+ r'e \operatorname{coſ}(2\eta + s)$$

 Si

ſi ponamus breuitatis gratia $\frac{3(1+2kk+\frac{9}{4}ee)}{\varkappa\varkappa\, nn} = i$, vt ſit $i = 0,0168918$, induet formam ſequentem:

$$\frac{d\pi}{dr} = -i\left(\varkappa + \alpha'\cos 2\eta - \left(\frac{2}{n} - c'\right)k\cos r \begin{matrix} + d'k\cos(2\eta - r) + \text{etc.} \\ + e'k\cos(2\eta + r) \end{matrix}\right)$$

$$\left(\begin{matrix} 1 + 4k\cos r + 5kk\cos 2r - 6ek\cos(r-s) \\ -3e\cos s + \frac{3}{2}ee\cos 2s - 6ek\cos(r+s) \end{matrix}\right)\left(\tfrac{1}{4} + \tfrac{1}{4}\cos 2\eta \begin{matrix} -\frac{1}{4}\cos(2\Phi - 2\pi) \\ -\frac{1}{4}\cos(2\theta - 2\pi) \end{matrix}\right)$$

$$-\frac{\varkappa i v}{4}\left(\tfrac{11}{4}\, c\, s\,\eta + \tfrac{5}{4}\, c\, s\, 3\eta - \tfrac{3}{2}\, c\, s(\Phi + \theta - 2\pi) \begin{matrix} -\frac{5}{4}\cos(3\Phi - \theta - 2\pi) \\ -\frac{5}{4}\cos(3\theta - \Phi - 2\pi) \end{matrix}\right)$$

vbi quidem plurimi termini tam fiunt parui, vt facile negligi queant.

§. 208. Productum autem ex duobus prioribus factoribus, quoniam id in formula pro inclinatione recurrit, ſeorſim exhibeamus: fiet id autem reiectis terminis, qui prae reliquis admodum ſunt parui vt ſequitur:

$$\left.\begin{array}{ll} -\varkappa i + 2i\left(\frac{2}{n} - c'\right)kk - \frac{1}{2}ip'ee & \\ \cos 2\eta & (-i\alpha' - 2id'kk \\ k\cos r & \left(i\left(\frac{2}{n} - c'\right) - 4\varkappa i\right. \\ k\cos(2\eta - r) & (-id' - 2i\alpha' \\ k\cos(2\eta + r) & (-ie' - 2i\alpha' \\ kk\cos 2r & \left(-5\varkappa i + 2i\left(\frac{2}{n} - c'\right)\right. \\ e\cos s & (-ip' + 3\varkappa i \\ e\cos(2\eta - s) & (iq' + \frac{3}{2}i\alpha' \\ e\cos(2\eta + s) & (-ir' + \frac{3}{2}i\alpha' \\ ee\cos 2s & (-\frac{3}{2}\varkappa i + \frac{3}{2}ip' \end{array}\right\} = \begin{array}{l} -0,017043 \\ +0,000161\ \cos 2\eta \\ -0,068110\, k\cos r \\ -0,005791\, k\cos(2\eta - r) \\ +0,000828\, k\cos(2\eta + r) \\ -0,085091\, kk\cos 2r \\ +0,051362\, e\cos s \\ -0,000929\, e\cos(2\eta - s) \\ -0,000804\, e\cos(2\eta + s) \\ -0,025914\, ee\cos 2s \end{array}$$

§. 209.

§. 209. His valoribus ſubſtitutis prodibit

$$\frac{d\pi}{dr} = 0{,}004261 + 0{,}000020$$

$$\text{coſ}\,2\eta \quad (+0{,}000040 - 0{,}004261$$

$$k\,\text{coſ}\,r \quad (-0{,}017043 - 0{,}000620$$

$$k\,\text{coſ}\,(2\eta - r)\,(-0{,}001448 - 0{,}008514) - 0{,}010636\,kk\,\text{cſ}(2\eta - 2r)$$

$$k\,\text{coſ}(2\eta + r)\,(+0{,}000207 - 0{,}008514) - 0{,}010636\,kk\,\text{cſ}(2\eta + 2r)$$

$$kk\,\text{coſ}\,2r \quad (-0{,}021273$$

$$\epsilon\,\text{coſ}\,s \quad (+0{,}012840 - 0{,}000217$$

$$\epsilon\,\text{coſ}\,(2\eta - s)\,(-0{,}000232 + 0{,}006420) - 0{,}003239\,\epsilon\epsilon\,\text{cſ}(2\eta - 2s)$$

$$\epsilon\,\text{coſ}\,(2\eta + s)\,(-0{,}000201 + 0{,}006420) - 0{,}003239\,\epsilon\epsilon\,\text{cſ}(2\eta + 2s)$$

$$\epsilon\epsilon\,\text{coſ}\,2s \quad (-0{,}006479)$$

$$\text{coſ}\,2(\Phi - \pi)\,(+0{,}003261 - 0{,}000020) - 0{,}000041\,\text{coſ}\,\eta$$

$$+\,0{,}000019\,\text{coſ}\,(3\Phi - \theta - 2\pi)$$

$$\text{coſ}\,2\,(\theta - \pi)\,(+0{,}004261 - 0{,}000020) - 0{,}000019\,\text{coſ}\,3\eta$$

$$+\,0{,}000019\,\text{coſ}(3\theta - \Phi - 2\pi)$$

$$k\,\text{cſ}(2\Phi - 2\pi - r)(+0{,}008514 + 0{,}000724)$$

$$+\,0{,}000022\,\text{coſ}\,(\Phi + \theta - 2\pi)$$

$$k\,\text{cſ}(2\Phi - 2\pi + r)(+0{,}008514 - 0{,}000103)$$

$$+\,0{,}010636\,kk\,\text{coſ}\,2(\Phi - r - \pi)$$

$$k\,\text{cſ}(2\theta - 2\pi - r)(+0{,}008514 - 0{,}000103)$$

$$k\,\text{cſ}(2\theta - 2\pi + r)\,(+0{,}008514 + 0{,}000724)$$

$$\epsilon\,\text{cſ}(2\Phi - 2\pi - s)(-0{,}006420 + 0{,}000116)$$

$$\epsilon\,\text{cſ}(2\Phi - 2\pi + s)(-0{,}006420 + 0{,}000100)$$

$$\epsilon\,\text{cſ}(2\theta - 2\pi - s)\,(-0{,}006420 + 0{,}000100)$$

$$+\,0{,}003239\,\epsilon\epsilon\,\text{coſ}\,2\,(\theta - s - \pi)$$

$$\epsilon\,\text{cſ}(2 - \theta 2\pi + s)\,(-0{,}006420 + 0{,}000116)$$

vnde oritur:

$$O - 0{,}019744\,\mathfrak{A} + 0{,}004241\,\mathfrak{B} + 0{,}004241\,\mathfrak{C} = 0{,}004241$$

$$1{,}867476\,\mathfrak{A} - 0{,}004241\,\mathfrak{B} - 0{,}004241\,\mathfrak{C} = -0{,}004221$$

$$2{,}026834\,\mathfrak{B} + 0{,}004221\,\mathfrak{C} = 0{,}004241$$

$$0{,}023965\,\mathfrak{B} + 0{,}159358\,\mathfrak{C} = 0{,}004241$$

§. 213. Valores hinc igitur prodibunt ſequentes:

$$O = +\,0{,}004078 \quad \ldots \quad l\,O = 7{,}610447$$

$$\mathfrak{A} = -\,0{,}002196 \quad \ldots \quad l{-}\mathfrak{A} = 7{,}341634$$

$$\mathfrak{B} = +\,0{,}002037 \quad \ldots \quad l\,\mathfrak{B} = 7{,}308991$$

$$\mathfrak{C} = +\,0{,}026307 \quad \ldots \quad l\,\mathfrak{C} = 8{,}420081$$

Vbi primum obſeruo valorem ipſius O iam proxime accedere ad motum medium lunae nodorum, vti per obſeruationes conſtat; inde enim eſſe deberet $O = 0{,}004053$ facile autem intelligitur, hunc exiguum defectum per reliquas inaequalitates ſuppleri poſſe. Quocirca hinc erit

	Valores in min. ſec.
$\pi = \text{Conſt.} - 0{,}004078\,r$	
$-\,0{,}002196\,\text{ſin}\,2\eta$	$-\,453''$
$+\,0{,}002037\,\text{ſin}\,(2\Phi - 2\pi)$	$+\,420$
$+\,0{,}026307\,\text{ſin}\,(2\theta - 2\pi)$	$+\,5426$

quae inaequalitates mirifice conueniunt cum obſeruationibus. His addi poteſt terminus:

$$+\,0{,}000336\ \text{ſin}\,(4\theta - 4\pi)$$

cuius in minutis ſecundis valor eſt $+\,69''$, qui terminus cum poſtremo illo facile coniungi poteſt.

§. 214. Quaeramus iam ſeorſim inaequalitates, quae ab excentricitate orbitae lunaris pendent, ſitque

$\pi =$

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$$\pi = \text{Const.} - Or + \mathfrak{A}\sin 2\eta + \mathfrak{B}\sin(2\varphi - 2\pi) + \mathfrak{C}\sin(2\theta - 2\pi)$$
$$+ \mathfrak{D}k\sin(2\eta - r) + \mathfrak{E}k\sin(2\eta + r) + \mathfrak{F}k\sin r$$
$$+ \mathfrak{G}kk\sin(2\eta - 2r) + \mathfrak{H}kk\sin 2r + \mathfrak{J}k\sin(2\varphi - 2\pi - r)$$
$$+ \mathfrak{K}k\sin(2\varphi - 2\pi + r) + \mathfrak{L}k\sin(2\theta - 2\pi - r) + \mathfrak{M}k\sin(2\theta - 2\pi + r)$$
$$+ \mathfrak{N}kk\sin(2\varphi - 2\pi - 2r)$$

eritque differentiando

$$\frac{d\pi}{dr} = \text{Pr.} + k\cos(2\eta - r)(-\mathfrak{A}c' - 0{,}009238\,\mathfrak{B} - 0{,}009238\,\mathfrak{C} + (2\alpha - 1)\mathfrak{D}$$
$$+ k\cos(2\eta + r)(-\mathfrak{A}c' - 0{,}008411\,\mathfrak{B} - 0{,}008411\,\mathfrak{C} + (2\alpha + 1)\mathfrak{E}$$
$$+ k\cos r(-\mathfrak{A}d' - \mathfrak{A}e' - 0{,}017649\,\mathfrak{B} - 0{,}017649\,\mathfrak{C} + \mathfrak{F} - \mathfrak{D}a' - \mathfrak{E}a'$$
$$+ kk\cos 2r \quad (-0{,}010636\,\mathfrak{B} + 2\mathfrak{H} - \mathfrak{D}e' - \mathfrak{E}d' - \mathfrak{G}a'$$
$$+ kk\cos(2\eta - 2r)(-0{,}010636\,\mathfrak{C} + 2(\alpha - 1)\mathfrak{G} - \mathfrak{D}c'$$
$$+ k\cos(2\theta - 2\pi - r)\left\{\begin{array}{l} -\mathfrak{B}e' + 0{,}008307\,\mathfrak{B} + \frac{2}{n}\mathfrak{C} + 0{,}017663\,\mathfrak{C} \\ \qquad + (\frac{2}{n} - 1)\mathfrak{L} + 0{,}008482\,\mathfrak{L} \end{array}\right.$$
$$+ k\cos(2\theta - 2\pi + r)\left\{\begin{array}{l} -\mathfrak{B}d' + 0{,}009962\,\mathfrak{B} + \frac{2}{n}\mathfrak{C} + 0{,}017663\,\mathfrak{C} \\ \qquad + (\frac{2}{n} + 1)\mathfrak{M} + 0{,}008482\,\mathfrak{M} \end{array}\right.$$
$$+ k\cos(2\varphi - 2\pi - r)\left\{\begin{array}{l} +0{,}017663\,\mathfrak{B} + 0{,}009962\,\mathfrak{C} + 2(\alpha + \frac{1}{n})\mathfrak{J} \\ \qquad - \mathfrak{J} + 0{,}008482\,\mathfrak{J} \end{array}\right.$$
$$+ k\cos(2\varphi - 2\pi + r)\left\{\begin{array}{l} +0{,}017663\,\mathfrak{B} + 0{,}008307\,\mathfrak{C} + 2(\alpha + \frac{1}{n})\mathfrak{K} \\ \qquad + \mathfrak{K} + 0{,}008482\,\mathfrak{K} \end{array}\right.$$
$$+ kk\cos(2\varphi - 2\pi - 2r)\left\{\begin{array}{l} +0{,}021273\,\mathfrak{B} + 0{,}010636\,\mathfrak{C} + 2(\alpha + \frac{1}{n})\mathfrak{N} \\ \qquad - 2\mathfrak{N} + 0{,}008482\,\mathfrak{N} \end{array}\right.$$

§. 195. Hinc ergo octo ſequentes aequationes reſultabunt

$$\text{I.}\quad 0{,}85830\,\mathfrak{K} = +\,0{,}00526\,\mathfrak{L} - 0{,}02500\,\mathfrak{M} + 1{,}39986 - 0{,}01785\,L + 0{,}01785\,M$$

$$\text{II.}\quad 1{,}00918\,\mathfrak{L} = +\,0{,}00526\,\mathfrak{K} - 0{,}02500\,\mathfrak{N} + 1{,}36509 - 0{,}01785\,K + 0{,}01785\,N$$

$$\text{III.}\quad 2{,}72578\,\mathfrak{M} = -\,0{,}01448\,\mathfrak{K} + 4{,}50673 - 0{,}01785\,K$$

$$\text{IV.}\quad 2{,}87666\,\mathfrak{N} = -\,0{,}01448\,\mathfrak{L} + 4{,}48566 - 0{,}01785\,L$$

$$\text{V.}\quad +0{,}27844K - 0{,}01057L - 0{,}08949M - 0{,}00014N + 0{,}10081 = +2{,}01769\mathfrak{K} + 0{,}00658\mathfrak{L} - 0{,}01946\mathfrak{M} + 0{,}00008\mathfrak{N}$$

$$\text{VI.}\quad -0{,}00332L - 0{,}01015K - 0{,}09614N - 0{,}00014M - 0{,}17954 = +2{,}01769\mathfrak{L} + 0{,}00545\mathfrak{K} + 0{,}02069\mathfrak{N} + 0{,}00008\mathfrak{M}$$

$$\text{VII.}\quad -6{,}41487\,M - 0{,}06300\,K - 0{,}00005\,L + 6{,}42802 = +\,2{,}01806\,\mathfrak{M} - 0{,}00742\,\mathfrak{K} - 0{,}00007\,\mathfrak{L}$$

$$\text{VIII.}\quad -7{,}25938\,N - 0{,}08131\,L - 0{,}00005\,K + 6{,}20280 = +\,2{,}01806\,\mathfrak{N} - 0{,}00855\,\mathfrak{L} - 0{,}00007\,\mathfrak{K}$$

§. 196. Ex aequationibus III et IV ſtatim eliciuntur hi valores

$$\mathfrak{M} = -\,0{,}00531\ \mathfrak{K} - 0{,}00655\ K + 1{,}65336$$
$$\mathfrak{N} = -\,0{,}00503\ \mathfrak{L} - 0{,}00621\ L + 1{,}55933$$

qui

qui in I et II ſubſtituti praebent:

$$0{,}85817\mathfrak{K} = 1{,}35853 + 0{,}00526\mathfrak{L} - 0{,}01785(L-M) + 0{,}00016K$$
$$1{,}00905\mathfrak{L} = 1{,}32611 + 0{,}00526\mathfrak{K} - 0{,}01785(K-N) + 0{,}00015L$$

vnde obtinetur:

$$\mathfrak{K} = +1{,}59116 - 0{,}02080(L-M) + 0{,}00008K \pm 0{,}00010N$$
$$\mathfrak{L} = +1{,}32251 - 0{,}01769(K-N) + 0{,}00005L + 0{,}00011M$$
$$\mathfrak{M} = +1{,}64491 - 0{,}00655K + 0{,}00010(L-M)$$
$$\mathfrak{N} = +1{,}55268 - 0{,}00621L + 0{,}00009(K-N)$$

§. 197. His valoribus ſubſtitutis caeterae aequationes abibunt in formas ſequentes:

$$0{,}27862K - 0{,}05252L - 0{,}04754M + 0{,}00017N + 3{,}35313 = 0$$
$$-0{,}00346L - 0{,}04584K - 0{,}06045N + 0{,}00020M + 2{,}53002 = 0$$
$$-6{,}41502M - 0{,}07622K + 0{,}00030L + 9{,}73587 = 0$$
$$-7{,}25971N - 0{,}09384L + 0{,}00028K + 9{,}32499 = 0$$

ex quarum binis poſtremis ſtatim obtinetur:

$$M = -0{,}01188\,K + 0{,}00005\,L + 1{,}51765$$
$$N = -0{,}01293\,L + 0{,}00004\,K + 1{,}28448$$

vnde colligitur:

$$+0{,}27918\,K - 0{,}05252\,L + 3{,}28120 = 0$$
$$-0{,}00268\,L - 0{,}04584\,K + 2{,}45267 = 0$$

ac denique

$$K = +40{,}44710 \;\ldots\; l\,K = 1{,}606887$$
$$L = +368{,}40200 \;\ldots\; l\,L = 2{,}566322$$
$$M = +1{,}05555 \;\ldots\; l\,M = 0{,}023478$$
$$N = -3{,}47730 \;\ldots\; l\,N = 0{,}541242$$

§. 198. Litterarum germanicarum valores hinc erunt:

$\mathfrak{K} = -5{,}04677$ . . . $l{-}\mathfrak{K} = 0{,}703013$
$\mathfrak{L} = +0{,}56302$ . . . $l\,\mathfrak{L} = 9{,}750524$
$\mathfrak{M} = +1{,}41672$ . . . $l\,\mathfrak{M} = 0{,}151283$
$\mathfrak{N} = -0{,}73119$ . . . $l\,\mathfrak{N} = 9{,}864030$

ac litterarum hinc deriuatarum:

$k' = +0{,}43578$ . . . $l\,k' = 9{,}639267$
$l' = +4{,}23401$ . . . $l\,l' = 0{,}626752$
$m' = +0{,}02018$ . . . $l\,m' = 8{,}304921$
$n' = -0{,}04409$ . . . $l{-}n' = 8{,}644340$

§. 199. Nunc igitur intelligimus inaequalitates ab angulis $3\eta - s$ et $3\eta + s$ pendentes tam eſſe paruas, vt ſine vllo errore reiici queant, etiamſi valores K et L aliquantum immutauerint. Diſtantia ergo lunae curtata a terra $x = \frac{(1-kk)\,au}{1-k\,\text{coſ}\,r}$ ita ab his inaequalitatibus parallacticis pendebit, vt ſit

	Log. coeff.
$u =$ Praec. $+ 0{,}11756\; v\; \text{coſ}\,\eta$	9,070249
$- 0{,}00419\; v\; \text{coſ}\,3\eta$	7,622540
$- 0{,}1234\; vk\; \text{coſ}\,(\eta - r)$	9,091265
$- 0{,}0784\; vk\; \text{coſ}\,(\eta + r)$	8,894183
$+ 0{,}2302\; ve\; \text{coſ}\,(\eta - s)$	9,362071
$+ 2{,}0965\; ve\; \text{coſ}\,(r + s)$	0,321506

Motus

Motus autem momentaneus ita hinc afficietur, vt ſit

$\frac{d\Phi}{dt} =$ Praec.	$-\ 0{,}23259\ v \cos\eta$	9,366591
	$+\ 0{,}01309\ v \cos 3\eta$	8,116940
	$+\ 0{,}0939\ vk \cos(\eta - r)$	8,972943
	$+\ 0{,}1653\ vk \cos(\eta + r)$	9,218352
	$-\ 0{,}4358\ ve \cos(\eta - s)$	9,639267
	$-\ 4{,}2340\ ve \cos(\eta + s)$	0,626752

§. 200. Quodſi iam ipſam longitudinem lunae, quatenus ab his inaequalitatibus parallacticis pendet, ponamus:

$$\Phi = \text{Praec.} + \mathfrak{F}' v \sin\eta + \mathfrak{H}' vk \sin(\eta - r) + \mathfrak{K}' ve \sin(\eta - s)$$
$$+ \mathfrak{G}' v \sin 3\eta + \mathfrak{I}' vk \sin(\eta + r) + \mathfrak{L}' ve \sin(\eta + s)$$

ſequentes obtinebimus aequationes pro horum coefficientium determinatione:

$$-0{,}23259 = \alpha\mathfrak{F}' - \mathfrak{A}'f' - \mathfrak{A}'g' - \tfrac{1}{2}\mathfrak{F}'a' - \tfrac{1}{2}\mathfrak{G}'a'$$

$$+0{,}01303 = 3\alpha\mathfrak{G}' - \mathfrak{A}'f' - \tfrac{1}{2}\mathfrak{F}'a'$$

$$+0{,}0939 = (\alpha - 1)\mathfrak{H}' - \mathfrak{A}'i' - \tfrac{1}{2}\mathfrak{I}'a' - \mathfrak{D}'f' - \mathfrak{E}'g'$$
$$- \tfrac{1}{2}\mathfrak{F}'c' - \tfrac{1}{2}\mathfrak{F}'d' - \tfrac{1}{2}\mathfrak{G}'c'$$

$$+0{,}1653 = (\alpha + 1)\mathfrak{I}' - \mathfrak{A}'b' - \tfrac{1}{2}\mathfrak{H}'a' - \mathfrak{E}'f' - \mathfrak{D}'g'$$
$$- \tfrac{1}{2}\mathfrak{F}'c' - \tfrac{1}{2}\mathfrak{F}'c' - \tfrac{1}{2}\mathfrak{G}'d'$$

$$-0{,}4358 = \left(\alpha - \frac{1}{n}\right)\mathfrak{K}' - \mathfrak{A}' l' - \tfrac{1}{2}\mathfrak{L}' a' - \mathfrak{Q}' f' - \mathfrak{M}' g'$$
$$+ \tfrac{1}{2}\mathfrak{F}'\left(\frac{2}{n} - p'\right) - \tfrac{1}{2}\mathfrak{F}' q' - \tfrac{1}{2}\mathfrak{G}' r'$$
$$-4{,}2340 = \left(\alpha + \frac{1}{n}\right)\mathfrak{L}' - \mathfrak{A}' k' - \tfrac{1}{2}\mathfrak{K}' a' - \mathfrak{M}' f' - \mathfrak{Q}' g'$$
$$+ \tfrac{1}{2}\mathfrak{F}'\left(\frac{2}{n} - p'\right) - \tfrac{1}{2}\mathfrak{F}' r' - \tfrac{1}{2}\mathfrak{G}' q'$$

§. 201. Valoribus autem iam cognitis hic ſubſtitutis, aequationes iſtae in ſequentes abibunt formas;

$$-0{,}23031 = +0{,}94361\,\mathfrak{F}' + 0{,}02961\,\mathfrak{G}'$$
$$+0{,}01550 = +2{,}80122\,\mathfrak{G}' + 0{,}00987\,\mathfrak{F}'$$
$$-0{,}0056 = -0{,}06626\,\mathfrak{H}' + 0{,}00987\,\mathfrak{J}' - 0{,}25665\,\mathfrak{F}' + 0{,}01926\,\mathfrak{G}'$$
$$+0{,}1710 = +1{,}93374\,\mathfrak{J}' + 0{,}00987\,\mathfrak{H}' - 0{,}06717\,\mathfrak{F}' - 0{,}54918\,\mathfrak{G}'$$
$$-0{,}3968 = +0{,}85830\,\mathfrak{K}' + 0{,}00987\,\mathfrak{L}' + 0{,}06357\,\mathfrak{F}' - 0{,}04509\,\mathfrak{G}'$$
$$-4{,}2330 = +1{,}00918\,\mathfrak{L}' + 0{,}00987\,\mathfrak{K}' + 0{,}06729\,\mathfrak{F}' - 0{,}05625\,\mathfrak{G}'$$

vnde colligitur fore

$\mathfrak{F}' = -0{,}24427$	. . .	$l{-}\mathfrak{F}' = 9{,}387868$
$\mathfrak{G}' = +0{,}00639$	. . .	$l\,\mathfrak{G}' = 7{,}805991$
$\mathfrak{H}' = +1{,}1959$	. . .	$l\,\mathfrak{H}' = 0{,}077694$
$\mathfrak{J}' = +0{,}0757$	. . .	$l\,\mathfrak{J}' = 8{,}879096$
$\mathfrak{K}' = -0{,}3959$	. . .	$l{-}\mathfrak{K}' = 9{,}597508$
$\mathfrak{L}' = -4{,}1738$	. . .	$l{-}\mathfrak{L}' = 0{,}620530$

§. 201.

§. 202. Hinc ergo habebimus ſequentes partes pro longitudine Lunae, quas ſimul ope valorum proxime cognitorum pro ν, k, e ad minuta ſecunda reducamus:

			Log coeff.	Val. coeff. in min ſec.
φ = Praec.	$-0,24427$	$\nu \sin \eta$	9,387868	$-175''$
	$+0,00639$	$\nu \sin 3\eta$	7,805991	$+\ \ 4''$
	$+1,1959$	$\nu k \sin(\eta-r)$	0,077694	$+\ 59''$
	$+0,0757$	$\nu k \sin(\eta+r)$	8,879096	$+\ \ 4''$
	$-0,3959$	$\nu e \sin(\eta-s)$	9,597508	$-\ \ 5''$
	$-4,1738$	$\nu e \sin(\eta+s)$	0,620530	$-\ 49''$

Sicque omnes iam adepti ſumus motus lunae inaequalitates, quae quidem ab inclinatione eius orbitae ad eclipticam non pendent. Interim tamen non diffiteor, dari aliquas inſuper inaequalitates, quae alicuius forte ſint momenti, quas in hac inueſtigatione praeterimus, cuiusmodi ſunt eae, quae ab angulis $2\eta-3r$ et $2\eta-2r+s$ pendent, quae ad plura minuta ſecunda aſſurgere poſſe videntur. Verum earum determinatio tam eſt taedioſa, vt malim eam obſeruationibus relinquere.

§. 203. Quae ergo hactenus inuenimus, in vnum colligamus ac primo pro diſtantia lunae a terra curtata $x = \frac{(1-kk)\, au}{1-k\cos r}$ erit

$u =$

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	Log. coeff.	coeff. integri
$u = 1 - 0{,}0074991 \cos 2\eta$	7,875009	$-0{,}007499$
$+0{,}0000532 \cos 4\eta$	5,725912	$+0{,}000053$
$+0{,}191557 k \cos(2\eta - r)$	9,282297	$+0{,}010430$
$-0{,}003293 k \cos(2\eta + r)$	7,517525	$-0{,}000179$
$-0{,}003321 k \cos(4\eta - r)$	7,521296	$-0{,}000181$
$+0{,}000049 k \cos(4\eta + r)$	5,692584	$+0{,}000005$
$-0{,}00511 kk \cos 2r$	7,708601	$-0{,}000015$
$-0{,}08022 kk \cos(2\eta - 2r)$	8,904280	$-0{,}000238$
$-0{,}00237 kk \cos(2\eta + 2r)$	7,375072	$-0{,}000007$
$+0{,}07892 kk \cos(4\eta - 2r)$	8,897172	$+0{,}000234$
$+0{,}00001 kk \cos(4\eta + 2r)$	4,872349	$+0{,}000000$
$-0{,}006400 e \cos s$	7,806180	$-0{,}000206$
$+0{,}014801 e \cos(2\eta - s)$	8,170303	$+0{,}000249$
$+0{,}011415 e \cos(2\eta + s)$	8,057492	$+0{,}000192$
$+0{,}00364 ee \cos 2s$	7,539670	$+0{,}000001$
$-0{,}01482 ee \cos(2\eta - 2s)$	8,170799	$-0{,}000004$
$-0{,}00584 ee \cos(2\eta + 2s)$	7,766856	$-0{,}000001$
$-0{,}37957 ek \cos(r - s)$	9,579297	$-0{,}000347$
$+0{,}30693 ek \cos(r + s)$	9,487039	$+0{,}000281$
$-0{,}36337 ek \cos(2\eta - r + s)$	9,560344	$-0{,}000332$
$+0{,}00993 ek \cos(2\eta + r - s)$	7,997004	$+0{,}000009$
$-0{,}07752 ek \cos(2\eta - r - s)$	8,889425	$-0{,}000071$
$+0{,}00305 ek \cos(2\eta + r + s)$	7,484024	$+0{,}000003$
$+0{,}11756 \nu \cos \eta$	9,070249	$+0{,}000408$
$-0{,}00419 \nu \cos 3\eta$	7,622540	$-0{,}000015$
$-0{,}1234 \nu k \cos(\eta - r)$	9,091265	$-0{,}000024$
$-0{,}0784 \nu k \cos(\eta + r)$	8,894183	$-0{,}000015$
$+0{,}2302 \nu e \cos(\eta - s)$	9,362071	$+0{,}000013$
$+2{,}0965 \nu e \cos(\eta + s)$	0,321506	$+0{,}000122$

Hic ad latus adiunxi valores coefficientium integrorum in numeris abſolutis expreſſos, ponendo $k = 0{,}05445$, $e = 0{,}01680$ et $\nu = \frac{1}{268}$; quos proinde, ſi hi valores aliter per obſeruationes determinentur, facile erit emendare.

§. 204.

§. 204. Pro motu autem lunae momentaneo, ex quo eius motus horarius definiri poterit, habebimus:

	Log. coeff.	Coeff. integri
$\frac{d\Phi}{dr} =$ 1,009176	0,003967	+ 1,009176
+ 0,0195144 coſ 2*η*	8,290355	+ 0,019514
— 0,0000322 coſ 4*η*	5,507856	— 0,000032
— 0,001231 *k* coſ *r*	7,090258	— 0,000067
— 0,366103 *k* coſ (2*η* − *r*)	9,563604	— 0,019934
+ 0,012832 *k* coſ (2*η* + *r*)	8,108292	+ 0,000699
+ 0,002829 *k* coſ (4*η* − *r*)	7,451633	+ 0,000154
— 0,000171 *k* coſ (4*η* + *r*)	6,232305	— 0,000009
+ 0,01182 *kk* coſ 2*r*	8,072618	+ 0,000035
— 0,02057 *kk* coſ (2*η* − 2*r*)	8,313172	— 0,000061
+ 0,01063 *kk* coſ (2*η* + 2*r*)	8,026598	+ 0,000032
— 0,09883 *kk* coſ (4*η* − 2*r*)	8,994889	— 0,000293
— 0,00004 *kk* coſ (4*η* + 2*r*)	5,592770	— 0,000000
+ 0,013760 *e* coſ *s*	8,138618	+ 0,000231
— 0,037487 *e* coſ (2*η* − *s*)	8,573878	— 0,000630
— 0,030062 *e* coſ (2*η* + *s*)	8,478023	— 0,000505
— 0,00722 *ee* coſ 2 *s*	7,858166	— 0,000002
+ 0,03470 *ee* coſ (2*η* − 2*s*)	8,540319	+ 0,000010
+ 0,01533 *ee* coſ (2*η* + 2*s*)	8,185614	+ 0,000005
+ 0,75204 *ek* coſ (*r* − *s*)	9,876241	+ 0,000688
— 0,62626 *ek* coſ (*r* + *s*)	9,796755	— 0,000573
+ 0,69420 *ek* coſ (2*η* − *r* + *s*)	9,841484	+ 0,000635
— 0,02440 *ek* coſ (2*η* + *r* − *s*)	8,387390	— 0,000022
+ 0,12160 *ek* coſ (2*η* − *r* − *s*)	9,084933	+ 0,000111
— 0,02000 *ek* coſ (2*η* + *r* + *s*)	9,301030	— 0,000018
— 0,23259 *v* coſ *η*	9,366591	— 0,000808
+ 0,01309 *v* coſ 3 *η*	8,116940	+ 0,000045
+ 0,0939 *vk* coſ (*η* − *r*)	8,972943	+ 0,000018
+ 0,1653 *vk* coſ (*η* + *r*)	9,218352	+ 0,000031
— 0,4358 *ve* coſ (*η* − *s*)	9,639267	— 0,000025
— 4,2340 *ve* coſ (*η* + *s*)	0,626753	— 0,000247

§. 205. Si iam longitudo Lunae per ſolam excentricitatem ſecundum regulas Keplerianas determinata ponatur $= \zeta$, ita vt poſita eius anomalia vera $= r$, futurum ſit $\zeta = C + 1{,}0085272\, r$, erit longitudo vera per hactenus inuentas inaequalitates.

	Log. coeff.	Val. coeff. in min. ſec.
$\varphi = \zeta + 0{,}0103597 \sin 2\eta$	8,015347	$+2137''$
$- 0{,}0000382 \sin 4\eta$	5,582063	$-\ 8$
$+ 0{,}010146k \sin r$	8,006295	$+114$
$- 0{,}420226k \sin(2\eta - r)$	9,623483	-4720
$+ 0{,}004992k \sin(2\eta + r)$	7,698261	$+\ 56$
$+ 0{,}005286k \sin(4\eta - r)$	7,723163	$+\ 59$
$- 0{,}000086k \sin(4\eta + r)$	5,935307	$-\ 1$
$+ 0{,}00420kk \sin 2r$	7,623250	$+\ 2\frac{1}{2}$
$+ 0{,}57328kk \sin(2\eta - 2r)$	9,758367	$+351$
$+ 0{,}00318kk \sin(2\eta + 2r)$	7,402427	$+\ 1\frac{1}{2}$
$- 0{,}15083kk \sin(4\eta - 2r)$	9,178488	$-\ 92$
$- 0{,}00002kk \sin(4\eta + 2r)$	5,301030	$-\ 0$
$+ 0{,}201385e \sin s$	9,304026	$+701$
$- 0{,}021889e \sin(2\eta - s)$	8,340237	$-\ 76$
$- 0{,}016368e \sin(2\eta + s)$	8,214002	$-\ 57$
$+ 0{,}06615ee \sin 2s$	8,820508	$+\ 4$
$+ 0{,}02332ee \sin(2\eta - 2s)$	8,367825	$+\ 1$
$+ 0{,}00840ee \sin(2\eta + 2s)$	7,924429	$+\ \frac{1}{2}$
$+ 0{,}74760ek \sin(r - s)$	9,873165	$+141$
$- 0{,}61850ek \sin(r + s)$	9,791317	-118
$+ 0{,}81430ek \sin(2\eta - r + s)$	9,910800	$+154$
$- 0{,}01420ek \sin(2\eta + r - s)$	8,150690	$-\ 3$
$+ 0{,}23960ek \sin(2\eta - r - s)$	9,379550	$+\ 45$
$- 0{,}00610ek \sin(2\eta + r + s)$	7,788910	$-\ 1$
$- 0{,}24427\nu \sin \eta$	9,387868	-175
$+ 0{,}00639\nu \sin 3\eta$	7,805991	$+\ 4$
$+ 1{,}1959\nu k \sin(\eta - r)$	0,077694	$+\ 59$
$+ 0{,}0757\nu k \sin(\eta + r)$	8,879096	$+\ 4$
$- 0{,}3959\nu e \sin(\eta - s)$	9,597508	$-\ 5$
$- 4{,}1738\nu e \sin(\eta + s)$	0,620530	$-\ 49$

CAPUT

CAPUT XII.

INUESTIGATIO INAEQUALITATUM MOTUM LINEAE NODORUM AFFICIENTIUM.

§. 206.

Antequam reliquas motus Lunae inaequalitates, quae ab inclinatione eius orbitae ad eclipticam pendent, definire licet, cum variationes, quae in motu lineae nodorum Lunae, tum eas, quae in ipſa inclinatione eius orbitae ad eclipticam deprehenduntur, inueſtigari oportet. Reſidua enim pars aequationis noſtrae principalis, qua omnes motus Lunae inaequalitates continentur, litteras π et ϱ implicat, quarum illa longitudinem nodi aſcendentis, haec vero ϱ inclinationem ad eclipticam deſignat. Niſi igitur vtriusque huius quantitatis incrementa vel decrementa ad differentiale dr reduxerimus, reſiduas motus Lunae inaequalitates determinare non poterimus.

207. Aequatio autem ſupra (55) pro motu lineae nodorum tradita, cum ſit $\frac{2\kappa v + \int R\,dr}{nn} = \alpha + \frac{1+2ee}{nn} - \frac{d\Phi}{dr}$ ideoque

$$\frac{2\kappa v + \int R\,dr}{nn} = a'\cos 2\eta - \left(\frac{2}{n} - c'\right)k\cos r + d'k\cos(2\eta - r)$$
$$+ e'k\cos(2\eta + r)$$
$$+ p'e\cos s + q'e\cos(2\eta - s)$$
$$+ r'e\cos(2\eta + s)$$

 Si

ſi ponamus breuitatis gratia $\frac{3(1+2kk+\frac{9}{8}ee)}{\varkappa\varkappa\, nn} = i$, vt ſit $i = 0,0168918$, induet formam ſequentem:

$$\frac{d\pi}{dr} = -i\left(\varkappa + \alpha'\cos 2\eta - \left(\frac{2}{n} - c'\right)k\cos r \begin{matrix} +d'k\cos(2\eta-r) \\ +e'k\cos(2\eta+r) \end{matrix} + \text{etc.}\right)$$

$$\left(\begin{matrix} 1+4k\cos r+5kk\cos 2r-6ek\cos(r-s) \\ -3e\cos s+\frac{3}{2}ee\cos 2s-6ek\cos(r+s) \end{matrix}\right)\left(\frac{1}{4}+\frac{1}{4}\cos 2\eta \begin{matrix} -\frac{1}{4}\cos(2\Phi-2\pi) \\ -\frac{1}{4}\cos(2\theta-2\pi) \end{matrix}\right)$$

$$-\frac{\varkappa i v}{4}\left(\frac{1}{4}\cos\eta + \frac{5}{4}\cos 3\eta - \frac{3}{2}\cos(\Phi+\theta-2\pi) \begin{matrix} -\frac{5}{4}\cos(3\Phi-\theta-2\pi) \\ -\frac{5}{4}\cos(3\theta-\Phi-2\pi) \end{matrix}\right)$$

vbi quidem plurimi termini tam fiunt parui, vt facile negligi queant.

§. 208. Productum autem ex duobus prioribus factoribus, quoniam id in formula pro inclinatione recurrit, ſeorſim exhibeamus: fiet id autem reiectis terminis, qui prae reliquis admodum ſunt parui vt ſequitur:

$$\left.\begin{array}{ll} -\varkappa i + 2i\left(\frac{2}{n}-c'\right)kk - \frac{3}{2}ip'ee & \\ \cos 2\eta & (-i\alpha' - 2id'kk \\ k\cos r & (i(\frac{2}{n}-c') - 4\varkappa i \\ k\cos(2\eta-r) & (-id' - 2i\alpha' \\ k\cos(2\eta+r) & (-ie' - 2i\alpha' \\ kk\cos 2r & (-5\varkappa i + 2i(\frac{2}{n}-c') \\ e\cos s & (-ip' + 3\varkappa i \\ e\cos(2\eta-s) & (iq' + \frac{3}{2}i\alpha' \\ e\cos(2\eta+s) & (-ir' + \frac{3}{2}i\alpha' \\ ee\cos 2s & (-\frac{3}{2}\varkappa i + \frac{3}{2}ip' \end{array}\right\} = \begin{array}{l} -0,017043 \\ +0,000161\cos 2\eta \\ -0,068110\,k\cos r \\ -0,005791\,k\cos(2\eta-r) \\ +0,000828\,k\cos(2\eta+r) \\ -0,085091\,kk\cos 2r \\ +0,051362\,e\cos s \\ -0,000929\,e\cos(2\eta-s) \\ -0,000804\,e\cos(2\eta+s) \\ -0,025914\,ee\cos 2s \end{array}$$

§. 209.

§. 209. His valoribus ſubſtitutis prodibit

$\frac{d\pi}{dr} = $ $0{,}004261 + 0{,}000020$

coſ 2η $(+0{,}000040 - 0{,}004261$

k coſ r $(-0{,}017043 - 0{,}000620$

k coſ $(2\eta - r)$ $(-0{,}001448 - 0{,}008514) - 0{,}010636\,kk$ cſ $(2\eta - 2r)$

k coſ $(2\eta + r)$ $(+0{,}000207 - 0{,}008514) - 0{,}010636\,kk$ cſ $(2\eta + 2r)$

kk coſ $2r$ $(-0{,}021273$

e coſ s $(+0{,}012840 - 0{,}000217$

e coſ $(2\eta - s)$ $(-0{,}000232 + 0{,}006420) - 0{,}003239\,ee$ cſ $(2\eta - 2s)$

e coſ $(2\eta + s)$ $(-0{,}000201 + 0{,}006420) - 0{,}003239\,ee$ cſ $(2\eta + 2s)$

ee coſ $2s$ $(-0{,}006479)$

coſ $2(\Phi - \pi)$ $(+0{,}003261 - 0{,}000020) - 0{,}000041$ coſ η

$+ 0{,}000019$ coſ $(3\Phi - \theta - 2\pi)$

coſ $2(\theta - \pi)$ $(+0{,}004261 - 0{,}000020) - 0{,}000019$ coſ 3η

$+ 0{,}000019$ coſ $(3\theta - \Phi - 2\pi)$

k cſ $(2\Phi - 2\pi - r)$ $(+0{,}008514 + 0{,}000724)$

$+ 0{,}000022$ coſ $(\Phi + \theta - 2\pi)$

k cſ $(2\Phi - 2\pi + r)$ $(+0{,}008514 - 0{,}000103)$

$+ 0{,}010636\,kk$ coſ $2(\Phi - \nu - \pi)$

k cſ $(2\theta - 2\pi - r)$ $(+0{,}008514 - 0{,}000103)$

k cſ $(2\theta - 2\pi + r)$ $(+0{,}008514 + 0{,}000724)$

e cſ $(2\Phi - 2\pi - s)$ $(-0{,}006420 + 0{,}000116)$

e cſ $(2\Phi - 2\pi + s)$ $(-0{,}006420 + 0{,}000100)$

e cſ $(2\theta - 2\pi - s)$ $(-0{,}006420 + 0{,}000100)$

$+ 0{,}003239\,ee$ coſ $2(\theta - s - \pi)$

e cſ $(2 - \theta 2\pi + s)$ $(-0{,}006420 + 0{,}000116)$

 §. 110.

§. 210. Habebimus ergo

$$
\begin{array}{ll}
\frac{d\pi}{dr} = -0,004241 & -0,000041 \cos \eta \\
-0,004221 \cos 2\eta & -0,000019 \cos 3\eta \\
-0,017663\, k \cos r & +0,000020 \cos 4\eta \\
-0,009962\, k \cos (2\eta - r) & +0,004241 \cos (2\Phi - 2\pi) \\
-0,008307\, k \cos (2\eta + r) & +0,004241 \cos (2\theta - 2\pi) \\
-0,010636\, kk \cos (2\eta - 2r) & +0,000022 \cos (\Phi + \theta - 2\pi) \\
-0,010636\, kk \cos (2\eta + 2r) & +0,000019 \cos (3\Phi - \theta - 2\pi) \\
-0,021273\, kk \cos r & +0,000019 \cos (3\theta - \Phi - 2\pi) \\
+0,012623\, e \cos s & \\
+0,006188\, e \cos (2\eta - s) & \\
+0,006219\, e \cos (2\eta + s) & \\
-0,006479\, ee \cos 2s & \\
-0,003239\, ee \cos (2\eta - 2s) & \\
-0,003239\, ee \cos (2\eta + 2s) & \\
+0,009238\, k \cos (2\Phi - 2\pi - r) & -0,006304\, e \cos (2\Phi - 2\pi - s) \\
+0,008411\, k \cos (2\Phi - 2\pi + r) & -0,006320\, e \cos (2\Phi - 2\pi + s) \\
+0,008411\, k \cos (2\theta - 2\pi - r) & -0,006320\, e \cos (2\theta - 2\pi - s) \\
+0,009238\, k \cos (2\theta - 2\pi + r) & -0,006304\, e \cos (2\theta - 2\pi + s) \\
+0,010636\, kk \cos (2\Phi - 2\pi - 2r) & +0,003239\, ee \cos (2\theta - 2\pi - 2s)
\end{array}
$$

§. 211. Quanquam plurimi horum terminorum tam ſunt parui, vt in ſe ſpectati tuto relici poſſent, tamen quidam per integrationem ad magnitudinem ſatis notabilem excreſcere poſſunt. Huius autem indolis ſunt illi termini, qui eiusmodi complectuntur angulos, quorum

rum differentialia ad dr admodum paruam tenent rationem, cuiusmodi ſunt anguli s, $2s$, $2\theta-2\pi$, $2\theta-2\pi-s$, $2\theta-2\pi+s$, $2\Phi-2\pi-2r$ et $2\theta-2\pi-2s$; quorum natura differentialium ex ſequentibus formulis colligi poteſt:

$$\frac{d\eta}{dr} = \alpha - a'\operatorname{cof} 2\eta - c'k\operatorname{cof} r - d'k\operatorname{cof}(2\eta-r) - e'k\operatorname{cof}(2\eta+r)$$
$$+ (\frac{2}{n}-p')e\operatorname{cof} s - q'e\operatorname{cof}(2\eta-s) - r'e\operatorname{cof}(2\eta+s)$$

$$\frac{d\Phi}{dr} = \alpha + \frac{1+2ee}{n} - a'\operatorname{cof} 2\eta + (\frac{2}{n}-c')k\operatorname{cof} r$$
$$- d'k\operatorname{cof}(2\eta-r) - p'e\operatorname{cof} s - q'e\operatorname{cof}(2\eta-s)$$
$$- e'k\operatorname{cof}(2\eta+r) \qquad - r'e\operatorname{cof}(2\eta+s)$$

$$\frac{ds}{dr} = \frac{d\theta}{dr} = \frac{1+2ee}{n} + \frac{2}{n}k\operatorname{cof} r - \frac{2}{n}e\operatorname{cof} s$$

§. 212. Quaeramus primo inaequalitates motus nodorum, quae neque ab excentricitate orbitae lunaris neque ſolaris pendent, ſitque:

$$\pi = \text{Conſt.} - \mathfrak{O}r + \mathfrak{A}\operatorname{fin} 2\eta + \mathfrak{B}\operatorname{fin}(2\Phi-2\pi) + \mathfrak{C}\operatorname{fin}(2\theta-2\pi)$$

reiectis reliquis terminis, quos praeuidemus fore minimos, ac differentiando obtinebimus:

$$\frac{d\pi}{dr} = -\mathfrak{O} - \mathfrak{A}a' - 0{,}004241\,\mathfrak{B} - 0{,}004241\,\mathfrak{C}$$
$$+ \operatorname{cof} 2\eta\ (2\alpha\mathfrak{A} - 0{,}004241\,\mathfrak{B} - 0{,}004241\,\mathfrak{C}$$
$$+ \operatorname{cf}(2\Phi-2\pi)\ (2(\alpha+\frac{1}{n})\mathfrak{B} + 0{,}008482\,\mathfrak{B} + 0{,}004221\,\mathfrak{C}$$
$$+ \operatorname{cf}(2\theta-2\pi)\ (-\mathfrak{B}a' + 0{,}004221\,\mathfrak{B} + \frac{2\mathfrak{C}}{n} + 0{,}008482\,\mathfrak{C}$$

vnde

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vnde oritur:

$$O - 0{,}019744\,\mathfrak{A} + 0{,}004241\,\mathfrak{B} + 0{,}004241\,\mathfrak{C} = 0{,}004241$$
$$1{,}867476\,\mathfrak{A} - 0{,}004241\,\mathfrak{B} - 0{,}004241\,\mathfrak{C} = -0{,}004221$$
$$2{,}026834\,\mathfrak{B} + 0{,}004221\,\mathfrak{C} = 0{,}004241$$
$$0{,}023965\,\mathfrak{B} + 0{,}159358\,\mathfrak{C} = 0{,}004241$$

§. 213. Valores hinc igitur prodibunt ſequentes:

$$O = +\,0{,}004078 \quad \ldots \quad l\,O = 7{,}610447$$
$$\mathfrak{A} = -\,0{,}002196 \quad \ldots \quad l{-}\mathfrak{A} = 7{,}341634$$
$$\mathfrak{B} = +\,0{,}002037 \quad \ldots \quad l\,\mathfrak{B} = 7{,}308991$$
$$\mathfrak{C} = +\,0{,}026307 \quad \ldots \quad l\,\mathfrak{C} = 8{,}420081$$

Vbi primum obſeruo valorem ipſius O iam proxime accedere ad motum medium lunae nodorum, vti per obſeruationes conſtat; inde enim eſſe deberet $O = 0{,}004053$ facile autem intelligitur, hunc exiguum defectum per reliquas inaequalitates ſuppleri poſſe. Quocirca hinc erit

	Valores in min. ſec.
$\pi = \text{Conſt.} - 0{,}004078\, r$	
$-\,0{,}002196 \,\text{ſin}\, 2\eta$	$-\;453''$
$+\,0{,}002037 \,\text{ſin}\, (2\Phi - 2\pi)$	$+\;420$
$+\,0{,}026307 \,\text{ſin}\, (2\theta - 2\pi)$	$+\;5426$

quae inaequalitates mirifice conueniunt cum obſeruationibus. His addi poteſt terminus:

$$+\,0{,}000336 \,\text{ſin}\, (4\theta - 4\pi)$$

cuius in minutis ſecundis valor eſt $+\,69''$, qui terminus cum poſtremo illo facile coniungi poteſt.

§. 214. Quaeramus iam ſeorſim inaequalitates, quae ab excentricitate orbitae lunaris pendent, ſitque $\pi =$

$$\begin{aligned}\pi = \text{Conſt.} - Or &+ \mathfrak{A}\sin 2\eta + \mathfrak{B}\sin(2\varphi-2\pi) + \mathfrak{C}\sin(2\theta-2\pi)\\ &+ \mathfrak{D}k\sin(2\eta-r) + \mathfrak{E}k\sin(2\eta+r) + \mathfrak{F}k\sin r\\ &+ \mathfrak{G}kk\sin(2\eta-2r) + \mathfrak{H}kk\sin 2r + \mathfrak{J}k\sin(2\varphi-2\pi-r)\\ &+ \mathfrak{K}k\sin(2\varphi-2\pi+r) + \mathfrak{L}k\sin(2\theta-2\pi-r) + \mathfrak{M}k\sin(2\theta-2\pi+r)\\ &+ \mathfrak{N}kk\sin(2\varphi-2\pi-2r)\end{aligned}$$

eritque differentiando

$$\begin{aligned}\frac{d\pi}{dr} = \text{Pr.} &+ k\cos(2\eta-r)(-\mathfrak{A}c'-0{,}009238\,\mathfrak{B}-0{,}009238\,\mathfrak{C}+(2\alpha-1)\mathfrak{D}\\ &+ k\cos(2\eta+r)(-\mathfrak{A}c'-0{,}008411\,\mathfrak{B}-0{,}008411\,\mathfrak{C}+(2\alpha+1)\mathfrak{E}\\ &+ k\cos r(-\mathfrak{A}d'-\mathfrak{A}e'-0{,}017649\,\mathfrak{B}-0{,}017649\,\mathfrak{C}+\mathfrak{F}-\mathfrak{D}a'-\mathfrak{E}a'\\ &+ kk\cos 2r\quad(-0{,}010636\,\mathfrak{B}+2\mathfrak{H}-\mathfrak{D}c'-\mathfrak{E}d'-\mathfrak{G}a'\\ &+ kk\cos(2\eta-2r)(-0{,}010636\,\mathfrak{C}+2(\alpha-1)\mathfrak{G}-\mathfrak{D}c'\end{aligned}$$

$$+ k\cos(2\theta-2\pi-r)\begin{cases}-\mathfrak{B}e'+0{,}008307\,\mathfrak{B}+\frac{2}{n}\mathfrak{C}+0{,}017663\,\mathfrak{C}\\ \qquad +(\frac{2}{n}-1)\,\mathfrak{L}+0{,}008482\,\mathfrak{L}\end{cases}$$

$$+ k\cos(2\theta-2\pi+r)\begin{cases}-\mathfrak{B}d'+0{,}009962\,\mathfrak{B}+\frac{2}{n}\mathfrak{C}+0{,}017663\,\mathfrak{C}\\ \qquad +(\frac{2}{n}+1)\,\mathfrak{M}+0{,}008482\,\mathfrak{M}\end{cases}$$

$$+ k\cos(2\varphi-2\pi-r)\begin{cases}+0{,}017663\,\mathfrak{B}+0{,}009962\,\mathfrak{C}+2(\alpha+\frac{1}{n})\mathfrak{J}\\ \qquad -\mathfrak{J}+0{,}008482\,\mathfrak{J}\end{cases}$$

$$+ k\cos(2\varphi-2\pi+r)\begin{cases}+0{,}017663\,\mathfrak{B}+0{,}008307\,\mathfrak{C}+2(\alpha+\frac{1}{n})\mathfrak{K}\\ \qquad +\mathfrak{K}+0{,}008482\,\mathfrak{K}\end{cases}$$

$$+ kk\cos(2\varphi-2\pi-2r)\begin{cases}+0{,}021273\,\mathfrak{B}+0{,}010636\,\mathfrak{C}+2(\alpha+\frac{1}{n})\mathfrak{N}\\ \qquad -2\mathfrak{N}+0{,}008482\,\mathfrak{N}\end{cases}$$

§. 215. Superfluum foret maiorem curam in his differentialibus adhibere, quia vero proxime tantum rem determinare ſufficit; erit ergo:

$$0,867476\,\mathfrak{D} = -0,009962$$
$$2,867476\,\mathfrak{E} = -0,008307$$

Hincque et in min ſec.

$$\mathfrak{D} = -0,011480 \quad | \quad \mathfrak{D}k = -129''$$
$$\mathfrak{E} = -0,002900 \quad | \quad \mathfrak{E}k = -33''$$

quae inaequalitates in loco nodi vix alicuius ſunt momenti, vnde eas exactius determinare non eſt opus.

§. 216. Calculo autem euoluto erit

$\pi =$ Pr.	$-0,011480k\ \sin(2\eta - r)$	8,059940	$-129''$
	$-0,002900k\ \sin(2\eta + r)$	7,462400	-33
	$-0,017663k\ \sin r$	8,247064	-198
	$+0,090497kk\ \sin(2\eta - 2r)$	8,956634	$+55$
	$-0,011978kk\ \sin 2r$	8,078384	-7
	$+0,008707k\ \sin(2\Phi - 2\pi - r)$	7,939851	$+98$
	$+0,002701k\ \sin(2\Phi - 2\pi + r)$	7,431516	$+30$
	$-0,004680k\ \sin(2\theta - 2\pi - r)$	7,670224	-53
	$+0,004685k\ \sin(2\theta - 2\pi + r)$	7,670680	$+53$
	$+0,384848kk\ \sin(2\Phi - 2\pi - 2r)$	9,585289	$+235$

§. 217. Simili modo inueſtigemus inaequalitates motus nodorum, quae pendent ab excentricitate orbitae ſolaris ſitque:

$\pi =$

$$\pi = \text{Conſt.} - O r + \mathfrak{A}\,\text{ſin}\,2\eta + \mathfrak{B}\,\text{ſin}\,(2\phi-2\pi) + \mathfrak{C}\,\text{ſin}(2\theta-2\pi)$$
$$+ \mathfrak{D} e\,\text{ſin}\,s + \mathfrak{E} e\,\text{ſin}(2\eta-s) + \mathfrak{F} e\,\text{ſin}(2\eta+s)$$
$$+ \mathfrak{G} ee\,\text{ſin}\,2s + \mathfrak{H} e\,\text{ſin}(2\phi-2\pi-s) + \mathfrak{K} e\,\text{ſin}(2\theta-2\pi-s)$$
$$+ \mathfrak{I} e\,\text{ſin}(2\phi-2\pi+s) + \mathfrak{L} e\,\text{ſin}(2\theta-2\pi+s)$$
$$+ \mathfrak{M} ee\,\text{ſin}(2\theta-2\pi-2s)$$

vnde differentiandó pro terminis quaeſitis erit:

$$\frac{d\pi}{dr} = \text{Praec.}$$

$$+ e\,\text{coſ}\,s \left\{ \begin{array}{l} -\mathfrak{A}q' - \mathfrak{A}r' + 0,006304\,\mathfrak{B} + 0,006320\,\mathfrak{C} + \frac{1}{n}\mathfrak{D} - \mathfrak{E}s' - \mathfrak{F}s' \\ \quad + 0,006320\,\mathfrak{B} + 0,006304\,\mathfrak{C} \end{array} \right.$$

$$+ e\,\text{coſ}(2\eta-s) \left\{ \mathfrak{A}\left(\frac{2}{n}-p'\right) + 0,006304\,\mathfrak{B} + 0,006304\,\mathfrak{C} + \left(2\alpha - \frac{1}{n}\right)\mathfrak{E} \right.$$

$$+ e\,\text{coſ}(2\eta+s) \left\{ \mathfrak{A}\left(\frac{2}{n}-p'\right) + 0,006320\,\mathfrak{B} + 0,006320\,\mathfrak{C} + \left(2\alpha + \frac{1}{n}\right)\mathfrak{F} \right.$$

$$+ ee\,\text{coſ}\,2s \left\{ -0,003239\,\mathfrak{C} - \frac{1}{n}\mathfrak{D} - \mathfrak{E}r' - \mathfrak{F}q' + \frac{2}{n}\mathfrak{G} \right.$$

$$+ e\,\text{coſ}\,(2\phi-2\pi-s) \left\{ \begin{array}{l} -\mathfrak{B}p' - 0,012623\,\mathfrak{B} - 0,006188\,\mathfrak{C} \\ + \left(2\alpha + \frac{1}{n} + 0,008482\right)\mathfrak{H} \end{array} \right.$$

$$+ e\,\text{coſ}(2\phi-2\pi+s) \left\{ \begin{array}{l} -\mathfrak{B}p' - 0,012623\,\mathfrak{B} - 0,006219\,\mathfrak{C} \\ + \left(2\alpha + \frac{3}{n} + 0,008482\right)\mathfrak{I} \end{array} \right.$$

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$$+\, e\cos(2\theta-2\pi-s)\left\{\begin{array}{l}-\ \mathfrak{B}r'-0{,}006219\,\mathfrak{B}-\frac{2}{n}\mathfrak{E}-0{,}012623\,\mathfrak{E}\\ +\left(\frac{1}{n}+0{,}008482\right)\mathfrak{K}\end{array}\right.$$

$$+\, e\cos(2\theta-2\pi+s)\left\{\begin{array}{l}-\ \mathfrak{B}q'-0{,}006188\,\mathfrak{B}-\frac{2}{n}\mathfrak{E}-0{,}012623\,\mathfrak{E}\\ +\left(\frac{3}{n}+0{,}008482\right)\mathfrak{L}\end{array}\right.$$

$$+\, ee\cos(2\theta-2\pi-2s)\left\{\begin{array}{l}+0{,}003239\,\mathfrak{B}+\frac{1}{2n}\mathfrak{E}+0{,}006479\,\mathfrak{E}\\ +0{,}008482\,\mathfrak{M}-\frac{1}{n}\mathfrak{K}-0{,}012623\,\mathfrak{K}\end{array}\right.$$

§. 218. Hinc reperiuntur ſequentes valores

$\mathfrak{D}=0{,}159070$.	$l\,\mathfrak{D}=9{,}201585$;	$\mathfrak{D}e=\ 551''$
$\mathfrak{E}=0{,}003562$.	$l\,\mathfrak{E}=7{,}551680$;	$\mathfrak{E}e=\ 12\frac{1}{4}$
$\mathfrak{F}=0{,}003301$.	$l\,\mathfrak{F}=7{,}518677$;	$\mathfrak{F}e=\ 11\frac{1}{4}$
$\mathfrak{G}=0{,}031650$.	$l\,\mathfrak{G}=8{,}587191$;	$\mathfrak{G}ee=\ 2''$
$\mathfrak{H}=-0{,}003153$.	$l-\mathfrak{H}=7{,}498692$;	$\mathfrak{H}e=-\ 11''$
$\mathfrak{I}=-0{,}002932$.	$l-\mathfrak{I}=7{,}467118$;	$\mathfrak{I}e=-\ 10''$
$\mathfrak{K}=-0{,}025750$.	$l-\mathfrak{K}=8{,}410784$;	$\mathfrak{K}e=-\ 90$
$\mathfrak{L}=-0{,}009076$.	$l-\mathfrak{L}=7{,}957885$;	$\mathfrak{L}e=-\ 32$

At valor ipſius $\mathfrak{M}$ tam ſit paruus, vt merito pro nihilo haberi poſſit.

§. 219.

§. 219. Colligamus ergo has inaequalitates in vnam ſummam, atque obtinebimus longitudinem veram nodi aſcendentis

	Valor. in minut. ſec.
$\pi =$ Conſt. $-0{,}004053\ r$	
$-0{,}002196$ ſin 2η	$-\ 453''$
$+0{,}002037$ ſin $(2\varphi-2\pi)$	$+\ 420$
$+0{,}026307$ ſin $(2\theta-2\pi)$	$+\ 5426$
$+0{,}000370$ ſin $(4\theta-4\pi)$	$+\ 75$
$-0{,}01766k$ ſin r	$-\ 198$
$-0{,}01148k$ ſin $(2\eta-r)$	$-\ 129$
$-0{,}00290k$ ſin $(2\eta+r)$	$-\ 33$
$+0{,}0905kk$ ſin $(2\eta-2r)$	$+\ 55$
$-0{,}0120kk$ ſin $2r$	$-\ 7$
$+0{,}00871k$ ſin $(2\varphi-2\pi-r)$	$+\ 98$
$+0{,}00270k$ ſin $(2\varphi-2\pi+r)$	$+\ 30$
$-0{,}00468k$ ſin $(2\theta-2\pi-r)$	$-\ 53$
$+0{,}00468k$ ſin $(2\theta-2\pi+r)$	$+\ 53$
$+0{,}3848kk$ ſin $(2\varphi-2\pi-2r)$	$+\ 235$
$+0{,}15907e$ ſin s	$+\ 551$
$-0{,}02575e$ ſin $(2\theta-2\pi-s)$	$-\ 90$
$-0{,}00907e$ ſin $(2\theta-2\pi+s)$	$-\ 32$

omiſſis ſcilicet iis inaequalitatibus, quae non ſupra $30''$ exſurgunt.

CAPUT XIII.

INUESTIGATIO INCLINATIONIS ORBITAE LUNARIS AD ECLIPTICAM.

§. 220.

Pro inclinatione orbitae lunaris ad eclipticam inuenienda, forma §. 208. euoluta multiplicari debet per $-\frac{1}{4}\,\text{ſin}\,2\eta + \frac{1}{4}\,\text{ſin}\,2(\Phi-\pi) + \frac{1}{4}\,\text{ſin}\,2(\theta-\pi)$, ac productum erit $= \frac{d.\,l\,\text{tang}\,\rho}{dr}$: Hinc ergo habebitur:

$$
\begin{array}{lll}
\frac{d.\,l\,\text{tang}\,\rho}{dr} = & +0{,}004261\ \text{ſin}\,2\eta & -0{,}000020\,\text{ſin}\,4\eta \\
 & +0{,}008514k\ \text{ſin}(2\eta-r) & \\
 & +0{,}008514k\ \text{ſin}(2\eta+r) & \\
 & +0{,}000827k\ \text{ſin}\,r & \\
 & -0{,}004261\ \text{ſin}(2\Phi-2\pi) & \text{adiice} \\
 & -0{,}004261\ \text{ſin}(2\theta-2\pi) & \text{ad coeff.} \\
 & -0{,}008514k\ \text{ſin}(2\Phi-2\pi-r) & -0{,}000724 \\
 & -0{,}008514k\ \text{ſin}(2\Phi-2\pi+r) & +0{,}000103 \\
 & -0{,}008514k\ \text{ſin}(2\theta-2\pi-r) & +0{,}000103 \\
 & -0{,}008514k\ \text{ſin}(2\theta-2\pi+r) & -0{,}000724 \\
 & -0{,}010636kk\,\text{ſin}(2\Phi-2\pi-2r) & \\
 & +0{,}006420e\ \text{ſin}(2\theta-2\pi-s) & -0{,}000100 \\
 & +0{,}006420e\ \text{ſin}(2\theta-2\pi+s) & -0{,}000116
\end{array}
$$

§. 221. Quaeramus primo terminos, qui a neutra excentricitate pendent, ſitque

$$l\,\frac{\text{tang}\,\varrho}{\text{tang}\,\epsilon} =$$

$$\mathfrak{A}\,\text{coſ}\,2\eta + \mathfrak{a}\,\text{coſ}\,4\eta + \mathfrak{B}\,\text{coſ}(2\Phi - 2\pi) + \mathfrak{C}\,\text{coſ}(2\theta - 2\pi) + \mathfrak{c}\,\text{coſ}(4\theta - 4\pi)$$

eritque differentiando:

$$\frac{d\,l\,\text{tang}\,\varrho}{d\,r} = \begin{cases} \text{ſin}\,2\ [-2\alpha\mathfrak{A} + 0{,}004241\,\mathfrak{B} - 0{,}004241\,\mathfrak{C} \\ \text{ſin}\,4\eta\ [-4\alpha\mathfrak{a} + \mathfrak{A}\alpha' \\ \text{ſin}(2\Phi - 2\pi)\left\{-2(\alpha + \frac{1}{n})\mathfrak{B} - 0{,}008482\,\mathfrak{B} - 0{,}004227\,\mathfrak{C}\right. \\ \text{ſin}(2\theta - 2\pi)\left\{-\frac{2}{n}\mathfrak{C} + \mathfrak{B}\alpha' - 0{,}004221\,\mathfrak{B} - 0{,}008482\,\mathfrak{C}\right. \\ \text{ſin}(4\theta - 4\pi)\left\{+ 0{,}004241\,\mathfrak{C} - \frac{4}{n}\,\mathfrak{c}\right. \end{cases}$$

§. 222. Ex his iam reperitur:

$\mathfrak{A} = -0{,}002630 \quad . \quad . \quad . \quad l{-}\mathfrak{A} = 7{,}419914$
$\mathfrak{B} = +0{,}002037 \quad . \quad . \quad . \quad l\,\mathfrak{B} = 7{,}308991$
$\mathfrak{C} = +0{,}026307 \quad . \quad . \quad . \quad l\,\mathfrak{C} = 8{,}420081$
$\mathfrak{a} = +0{,}000019 \quad . \quad . \quad . \quad l\,\mathfrak{a} = 5{,}278753$
$\mathfrak{c} = +0{,}000370 \quad . \quad . \quad . \quad l\,\mathfrak{c} = 6{,}567931$

ita vt hinc ſit:

$$l\,\frac{\text{tang}\,\varrho}{\text{tang}\,\epsilon} = -0{,}002630\ \text{coſ}\,2\eta$$
$$+ 0{,}000019\ \text{coſ}\,4\eta$$
$$+ 0{,}002037\ \text{coſ}(2\Phi - 2\pi)$$
$$+ 0{,}026307\ \text{coſ}(2\theta - 2\pi)$$
$$+ 0{,}000370\ \text{coſ}(4\theta - 4\pi)$$

§. 223.

§. 223. Quaeramus iam ſeorſim terminos ab excentricitate Lunae pendentes: ſitque

$$l\frac{\operatorname{tang}\varrho}{\operatorname{tang}\epsilon} = \mathfrak{A}\cos 2\eta + \mathfrak{B}\cos(2\Phi-2\pi) + \mathfrak{C}\cos(2\theta-2\pi) + \mathfrak{D}k\cos(2\eta-r) + \mathfrak{E}k\cos(2\eta+r) + \mathfrak{F}k\cos r + \mathfrak{G}k\cos(2\Phi-2\pi-r) + \mathfrak{I}k\cos(2\theta-2\pi-r) + \mathfrak{H}k\cos(2\Phi-2\pi+r) + \mathfrak{K}k\cos(2\theta-2\pi+r) + \mathfrak{L}k^2\cos(2\Phi-2\pi-2r)$$

vnde differentialibus ſumendis habebitur: $\frac{d.\, l\operatorname{tang}\varrho}{dr} =$

$$k\sin(2\eta-r)\left\{\begin{array}{l} +\mathfrak{A}c' + 0{,}09238\,\mathfrak{B} - 0{,}009238\,\mathfrak{C} + 0{,}004241\,\mathfrak{G} \\ +(2\alpha-1)\mathfrak{D} - 0{,}004241\,\mathfrak{K} \end{array}\right.$$

$$k\sin(2\eta+r)\left\{\begin{array}{l} +\mathfrak{A}c' + 0{,}008411\,\mathfrak{B} - 0{,}008411\,\mathfrak{C} + (2\alpha+1)\mathfrak{E} \\ +0{,}004241\,\mathfrak{H} - 0{,}004241\,\mathfrak{I} \end{array}\right.$$

$$k\sin r\left\{\begin{array}{l} +\mathfrak{A}d' - \mathfrak{A}c' + 0{,}000827\,\mathfrak{B} - 0{,}000827\,\mathfrak{C} - \mathfrak{F} - 0{,}004241\,\mathfrak{G} \\ +0{,}004241\,\mathfrak{H} - \mathfrak{D}a' + \mathfrak{E}a' - 0{,}004241\,\mathfrak{I} + 0{,}004241\,\mathfrak{K} \end{array}\right.$$

$$k\sin(2\Phi-2\pi-r)\left\{\begin{array}{l} -0{,}017663\,\mathfrak{B} - 0{,}009962\,\mathfrak{C} - 2(\alpha+\frac{1}{n})\,\mathfrak{G} + \mathfrak{G} \\ -0{,}008482\,\mathfrak{G} - 0{,}004221\,\mathfrak{I} \end{array}\right.$$

$$k\sin(2\Phi-2\pi+r)\left\{\begin{array}{l} -0{,}017663\,\mathfrak{B} - 0{,}008307\,\mathfrak{C} - 2(\alpha+\frac{1}{n})\mathfrak{H} - \mathfrak{H} \\ -0{,}008482\,\mathfrak{H} - 0{,}004221\,\mathfrak{K} \end{array}\right.$$

$$k\sin(2\theta-2\pi-r)\left\{\begin{array}{l} +\mathfrak{B}c' - 0{,}008307\,\mathfrak{B} - \frac{2}{n}\mathfrak{C} - 0{,}017663\,\mathfrak{C} \\ +\mathfrak{G}a' - 0{,}004221\,\mathfrak{G} - \frac{2}{n}\mathfrak{I} + \mathfrak{I} - 0{,}008482\,\mathfrak{I} \end{array}\right.$$

k ſin

$$k\,\text{ſin}(2\theta-2\pi+r)\left\{\begin{array}{l}+\mathfrak{B}d'-0{,}009962\,\mathfrak{B}-\frac{2}{n}\mathfrak{C}-0{,}017663\,\mathfrak{C}\\ +\mathfrak{H}a'-0{,}004221\,\mathfrak{H}-\frac{2}{n}\mathfrak{K}-\mathfrak{K}-0{,}008482\,\mathfrak{K}\end{array}\right.$$

$$kk\,\text{ſin}(2\Phi-2\pi-2r)\left\{\begin{array}{l}-0{,}021273\,\mathfrak{B}-0{,}010636\,\mathfrak{C}-2(\alpha+\frac{1}{n})\,\mathfrak{L}+2\,\mathfrak{L}\\ \qquad -0{,}008482\,\mathfrak{L}\end{array}\right.$$

hincque reperitur:

$$\begin{array}{llll}
\mathfrak{D}= & 0{,}010487 & \ldots & l\,\mathfrak{D}=8{,}020638\\
\mathfrak{E}= & 0{,}003166 & \ldots & l\,\mathfrak{E}=7{,}500439\\
\mathfrak{F}= & -0{,}001600 & \ldots & l{-}\mathfrak{F}=7{,}204120\\
\mathfrak{G}= & +0{,}008719 & \ldots & l\,\mathfrak{G}=7{,}940484\\
\mathfrak{H}= & +0{,}002699 & \ldots & l\,\mathfrak{H}=7{,}431136\\
\mathfrak{J}= & -0{,}004460 & \ldots & l{-}\mathfrak{J}=7{,}649305\\
\mathfrak{K}= & +0{,}004717 & \ldots & l\,\mathfrak{K}=7{,}623628\\
\mathfrak{L}= & +0{,}384890 & \ldots & l\,\mathfrak{L}=9{,}585335
\end{array}$$

§. 224. Nunc denique pro inaequalitatibus ab excentricitate orbitae ſolaris pendentibus ponatur.

$$l\,\frac{\text{tang}\,\varrho}{\text{tang}\,\epsilon}=\mathfrak{A}\,\text{coſ}\,2\eta+\mathfrak{B}\,\text{coſ}(2\Phi-2\pi)+\mathfrak{M}e\,\text{coſ}(2\theta-2\pi-s)$$
$$+\mathfrak{C}\,\text{coſ}(2\theta-2\pi)+\mathfrak{N}e\,\text{coſ}(2\theta-2\pi+s)$$

ac differentiando prodibit: $\frac{d.\,l\,\text{tang}\,\varrho}{dr}=$

$$e\,\text{ſin}(2\theta-2\pi-s)\left\{\begin{array}{l}+\mathfrak{B}r'+0{,}006219\,\mathfrak{B}+\frac{2}{n}\mathfrak{C}+0{,}012623\,\mathfrak{C}\\ \qquad -\frac{1}{n}\mathfrak{M}-0{,}008482\,\mathfrak{M}\end{array}\right.$$

$$\epsilon \sin(2\theta-2\pi+s)\left\{\begin{array}{l} +\mathfrak{B}q' + 0{,}06188\,\mathfrak{B} + \frac{2}{n}\mathfrak{C} + 0{,}012623\,\mathfrak{C} \\ \quad -\frac{3}{n}\mathfrak{N} - 0{,}008482\,\mathfrak{N} \end{array}\right.$$

vnde reperitur

$$\mathfrak{M} = -\ 0{,}024034 \quad . \ . \quad l\text{-}\mathfrak{M} = 8{,}380835$$

$$\mathfrak{N} = -\ 0{,}008519 \quad . \ . \quad l\text{-}\mathfrak{N} = 7{,}930332$$

§. 225. Si ergo ϵ denotet inclinationem mediam orbitae lunaris ad eclipticam, et ϱ inclinationem veram, erit

			log. coeff.
$l\frac{\tang\varrho}{\tang\epsilon} =$	$-\ 0{,}002630$	$\cos 2\eta$	7,419915
	$+\ 0{,}000019$	$\cos 4\eta$	5,278753
	$+\ 0{,}002037$	$\cos(2\Phi-2\pi)$	7,308991
	$+\ 0{,}026307$	$\cos(2\theta-2\pi)$	8,420081
	$+\ 0{,}000370$	$\cos(4\theta-4\pi)$	6,567931
	$+\ 0{,}01049k$	$\cos(2\eta-r)$	8,020638
	$+\ 0{,}00317k$	$\cos(2\eta+r)$	7,500439
	$-\ 0{,}00160k$	$\cos r$	7,204120
	$+\ 0{,}00872k$	$\cos(2\Phi-2\pi-r)$	7,940484
	$+\ 0{,}00270k$	$\cos(2\Phi-2\pi+r)$	7,431136
	$-\ 0{,}00446k$	$\cos(2\theta-2\pi-r)$	7,649305
	$+\ 0{,}00472k$	$\cos(2\theta-2\pi+r)$	7,673628
	$+\ 0{,}3849kk$	$\cos(2\Phi-2\pi-2r)$	9,585335
	$-\ 0{,}02403\epsilon$	$\cos(2\theta-2\pi-s)$	8,380835
	$-\ 0{,}00852\epsilon$	$\cos(2\theta-2\pi+s)$	7,930332

§. 226.

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§. 226. Quodſi iam ponatur $l\frac{\text{tang}\,\varrho}{\text{tang}\,\epsilon} = \text{S}$, erit ad numeros ipſos procedendo $\frac{\text{tang}\,\varrho}{\text{tang}\,\epsilon} = 1 + \text{S} + \frac{1}{2}\,\text{SS}$ Hinc igitur negligendo terminos minimos, conſequemur:

$$
\begin{aligned}
\frac{\text{tang}\,\varrho}{\text{tang}\,\epsilon} = \; 1 & - 0{,}002604 \quad \text{coſ}\,2\,\eta \\
& + 0{,}000020 \quad \text{coſ}\,4\,\eta \\
& + 0{,}002003 \quad \text{coſ}\,(2\,\Phi - 2\,\pi) \\
& + 0{,}026307 \quad \text{coſ}\,(2\,\theta - 2\,\pi) \\
& + 0{,}000490 \quad \text{coſ}\,(4\,\theta - 4\,\pi) \\
& - 0{,}00160k \quad \text{coſ}\,r \\
& + 0{,}01049k \quad \text{coſ}\,(2\,\eta - r) \\
& + 0{,}00317k \quad \text{coſ}\,(2\,\eta + r) \\
& + 0{,}00885k \quad \text{coſ}\,(2\Phi - 2\pi - r) \\
& + 0{,}00274k \quad \text{coſ}\,(2\Phi - 2\pi + r) \\
& - 0{,}00448k \quad \text{coſ}\,(2\theta - 2\pi - r) \\
& + 0{,}00470k \quad \text{coſ}\,(2\theta - 2\pi + r) \\
& + 0{,}3849kk \quad \text{coſ}\,(2\Phi - 2\pi - 2r) \\
& - 0{,}02403e \quad \text{coſ}\,(2\,\theta - 2\pi - s) \\
& - 0{,}00852e \quad \text{coſ}\,(2\,\theta - 2\pi + s)
\end{aligned}
$$

§. 227. Cum in aequatione noſtra principali, quae motum Lunae continet, inſit terminus $\frac{\operatorname{tang} \rho^2}{\operatorname{tang} \varepsilon^2}$, huiusquoque valorem euolui conueniet: erit ergo

$$
\begin{aligned}
\frac{\operatorname{tang} \rho^2}{\operatorname{tang} \varepsilon^2} = {} & 1 \\
& - 0{,}005156 \cos 2\eta \quad + 0{,}000040 \cos 4\eta \\
& + 0{,}003938 \cos(2\varphi - 2\pi) \\
& + 0{,}052614 \cos(2\theta - 2\pi) + 0{,}001320 \cos(4\theta - 4\pi) \\
& - 0{,}00290k \cos r \\
& + 0{,}02098k \cos(2\eta - r) \\
& + 0{,}00634k \cos(2\eta + r) \\
& + 0{,}01796k \cos(2\varphi - 2\pi - r) \\
& + 0{,}00556k \cos(2\varphi - 2\pi + r) \\
& - 0{,}00896k \cos(2\theta - 2\pi - r) \\
& + 0{,}00940k \cos(2\theta - 2\pi + r) \\
& + 0{,}7698kk \cos(2\varphi - 2\pi - 2r) \\
& + 0{,}04806e \cos(2\theta - 2\pi - s) \\
& + 0{,}01704e \cos(2\theta - 2\pi + s)
\end{aligned}
$$

Hicque ergo valor in ſuperiori illa aequatione ſubſtitui poterit.

§. 228.

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§. 228. Celeb. autem Clairaut concluſit inclinationem mediam ε ex obſeruationibus exquiſitiſſimis $5^\circ\, 8'\, 9''$, ex qua igitur ad quoduis tempus inclinationem veram elicere licebit. Sit enim $\varrho = \varepsilon + \omega$, erit tang $\varrho = \frac{\text{tg}\,\varepsilon + \omega}{1 - \omega\,\text{tg}\,\varepsilon}$ $= \text{tang}\,\varepsilon + \frac{\omega}{\cos \varepsilon^2} = V\,\text{tang}\,\varepsilon$, ponendo V pro expreſſione ipſius $\frac{\text{tang}\,\varrho}{\text{tang}\,\varepsilon}$. Hinc erit $\omega = (V-1) \sin \varepsilon \cos \varepsilon = \frac{1}{2}(V-1) \sin 2\varepsilon = 0{,}08915(V-1)$: vnde reperitur in minutis ſecundis

$$
\begin{aligned}
\varrho = \varepsilon &- 48'' \cos 2\eta \\
&+ 36 \cos(2\varphi - 2\pi) \\
&+ 484 \cos(2\theta - 2\pi) \\
&+ 9 \cos(4\theta - 4\pi) \\
&- 2 \cos r \\
&+ 11 \cos(2\eta - r) \\
&+ 3 \cos(2\eta + r) \\
&+ 9 \cos(2\varphi - 2\pi - r) \\
&+ 3 \cos(2\varphi - 2\pi + r) \\
&- 5 \cos(2\theta - 2\pi - r) \\
&+ 5 \cos(2\theta - 2\pi + r) \\
&+ 23 \cos(2\varphi - 2\pi - 2r) \\
&- 7 \cos(2\theta - 2\pi - s) \\
&- 3 \cos(2\theta - 2\pi + s)
\end{aligned}
$$

§. 229. Hic notandum eſt, etiamſi valor inclinationis mediae *ι* aliquantillum immutetur, aequationes has tamen inde vix alterari, ita vt eae ſemper eaedem ſint manſurae. Perſpicuum quoque eſt in calculo aſtronomico ſufficere tres inaequalitates primores, et reliquas omnes ſine errore ſenſibili praetermitti poſſe; niſi forte aequatio 23 coſ $(2\varphi - 2\pi - 2r)$ retinenda cenſeatur, quae inter reliquas eſt maxima. Exprimit autem angulus $2\varphi - 2\pi - 2r$ duplam diſtantiam apogei Lunae ab eius nodo, a quo angulo quoque locum nodi non mediocriter affici vidimus, cum correctio hinc oriunda pro loco nodi vsque ad 235″ aſſurgere poſſit.

CAPUT

CAPUT XIV.

INUESTIGATIO INAEQUALITATUM MOTUS LUNAE AB EIUS INCLINATIONE AD ECLIPTICAM ORIUNDARUM.

§. 230.

Ponamus more adhuc vſitato:

$$\begin{aligned}\int R\,dr = {} & \mathfrak{A}\cos 2\eta + \mathfrak{C}k\cos r + \mathfrak{D}k\cos(2\eta - r) + \mathfrak{P}e\cos s + \mathfrak{Q}e\cos(2\eta - s)\\ & + \mathfrak{E}k\cos(2\eta + r) \qquad + \mathfrak{R}e\cos(2\eta + s)\\ & + \mathfrak{F}f\cos 2\eta + \mathfrak{G}f\cos(2\varphi - 2\pi) + \mathfrak{J}fk\cos r + \mathfrak{K}fk\cos(2\eta - r)\\ & + \mathfrak{H}f\cos(2\theta - 2\pi) \qquad + \mathfrak{L}fk\cos(2\eta + r)\\ & + \mathfrak{M}fk\cos(2\varphi - 2\pi - r) + \mathfrak{S}fk\cos(2\theta - 2\pi - r)\\ & + \mathfrak{N}fk\cos(2\varphi - 2\pi + r) + \mathfrak{T}fk\cos(2\theta - 2\pi + r)\\ & + \mathfrak{O}fkk\cos(2\varphi - 2\pi - 2r) + \mathfrak{U}fe\cos(2\theta - 2\pi - s)\\ & + \mathfrak{V}fe\cos(2\theta - 2\pi + s)\end{aligned}$$

$$\begin{aligned}\text{et } v = {} & A\cos 2\eta\ \ldots + Dk\cos(2\eta - r) + Pe\cos s + Qe\cos(2\eta - s)\\ & + Ek\cos(2\eta + r) \qquad + Re\cos(2\eta + s)\\ & + Ff\cos 2\eta + Gf\cos(2\varphi - 2\pi) + Jfk\cos r + Kfk\cos(2\eta - r)\\ & + Hf\cos(2\theta - 2\pi) \qquad + Lfk\cos(2\eta + r)\\ & + Mfk\cos(2\varphi - 2\pi - r) + Sfk\cos(2\theta - 2\pi - r)\\ & + Nfk\cos(2\varphi - 2\pi + r) + Tfk\cos(2\theta - 2\pi + r)\\ & + Ofkk\cos(2\varphi - 2\pi - 2r) + Ufe\cos(2\theta - 2\pi - s)\\ & + Vfe\cos(2\theta - 2\pi + s)\end{aligned}$$

§. 231.

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§. 231. His valoribus ſubſtitutis in formula §. 52. orietur

$$R = f \sin 2\eta \Big(\quad \ldots \quad fk \sin(2\theta - 2\pi - r)\Big(-\frac{3M}{2nn} - \frac{3G}{nn}$$

$$f \sin(2\phi - 2\pi)\Big(+\frac{3H}{2nn} \quad \ldots \quad fk \sin(2\theta - 2\pi + r)\Big(-\frac{3N}{2nn} - \frac{3G}{nn}$$

$$f \sin(2\theta - 2\pi)\Big(-\frac{3G}{2nn} \quad \ldots \quad fe \sin(2\theta - 2\pi - s)\Big(+\frac{9G}{4nn}$$

$$fk \sin r\Big(+\frac{3K}{2nn} - \frac{3L}{2nn} \quad \ldots \quad f\epsilon \sin(2\theta - 2\pi + s)\Big(+\frac{9G}{4nn}$$

$$fk \sin(2\eta - r)\Big(+\frac{3I}{2nn}$$

$$fk \sin(2\eta - r)\Big(+\frac{3I}{2nn}$$

$$fk \sin(2\phi - 2\pi - r)\Big(+\frac{3S}{2nn} + \frac{3H}{nn}$$

$$fk \sin(2\phi - 2\pi + r)\Big(+\frac{3T}{2nn} + \frac{3H}{nn}$$

$$fk \sin(2\phi - 2\pi - 2r)\Big(+\frac{3S}{nn}$$

§. 232. Altera vero aequatio fundamentalis induet formam ſequentem:

$$\frac{ddv}{dr^2} = \text{Praec.} + f\cos 2\eta \;[-\mathfrak{E}F - 2\varkappa\,\mathfrak{F} - 0{,}005156 - 0{,}026307$$
$$+ f\cos(2\phi - 2\pi) \;[-\mathfrak{E}G - 2\varkappa\,\mathfrak{G} + 0{,}003938 - 1$$
$$+ f\cos(2\theta - 2\pi) \;[-\mathfrak{E}H - 2\varkappa\,\mathfrak{H} + 0{,}052614 + 0{,}002578$$

$$+fk\cos r \quad \left[-\beta J - 2\varkappa\mathfrak{J} - 0{,}00290 - 0{,}00893 \right.$$
$$\left. - 0{,}00098 - 0{,}00278 - 0{,}00098 \right.$$
$$+fk\cos(2\eta - r) \quad \left[-\beta K + \tfrac{1}{2}bF - 2\varkappa\mathfrak{K} + 0{,}02098 - 0{,}00470 \right.$$
$$\left. - 0{,}00258 - 0{,}01315 \right.$$
$$+fk\cos(2\eta + r) \quad \left[-\beta L + \tfrac{1}{2}bF - 2\varkappa\mathfrak{L} + 0{,}00634 + 0{,}00448 \right.$$
$$\left. - 0{,}00258 - 0{,}01315 \right.$$
$$+fk\cos(2\varphi - 2\pi - r) \quad \left[-\beta M + \tfrac{1}{2}bG - 2\varkappa\mathfrak{M} + 0{,}01796 + 0{,}00145 \right.$$
$$\left. + 0{,}00197 - \tfrac{1}{2} \right.$$
$$+fk\cos(2\varphi - 2\pi + r) \quad \left[-\beta N + \tfrac{1}{2}bG - 2\varkappa\mathfrak{N} + 0{,}00556 + 0{,}00145 \right.$$
$$\left. + 0{,}00197 - \tfrac{1}{2} \right.$$
$$+fkk\cos(2\varphi - 2\pi - 2r) \quad \left[-\beta O + \tfrac{1}{2}hM + \tfrac{1}{4}G - 2\varkappa\mathfrak{O} + 0{,}7698 + 0{,}0089 \right.$$
$$\left. + 0{,}0010 + 0{,}0007 - \tfrac{1}{4} \right.$$
$$+fk\cos(2\theta - 2\pi - r) \quad \left[-\beta S + \tfrac{1}{2}bH - 2\varkappa\mathfrak{S} - 0{,}00896 - 0{,}00317 \right.$$
$$\left. + 0{,}02631 + 0{,}00129 \right.$$
$$+fk\cos(2\theta - 2\pi + r) \quad \left[-\beta T + \tfrac{1}{2}bH - 2\varkappa\mathfrak{T} + 0{,}00940 - 0{,}01049 \right.$$
$$\left. + 0{,}02631 + 0{,}00129 \right.$$
$$+fe\cos(2\theta - 2\pi - s) \quad \left[-\beta U - 2\varkappa\mathfrak{U} - 0{,}04806 \right.$$
$$+fe\cos(2\theta - 2\pi \mp s) \quad \left[-\beta V - 2\varkappa\mathfrak{V} - 0{,}01704 \right.$$

§. 233. Quoniam manifeſtum eſt, coefficientes F, G, H etc. admodum fore paruos; cum maxima huius generis inaequalitas aliquot minuta prima non excedat, hi iidem coefficientes per nn diuiſi tam euadent parvi, vt ſine errore reiici queant. Hoc autem facto quoque litterae germanicae $\mathfrak{F}$, $\mathfrak{G}$, $\mathfrak{H}$ etc. pro nihilo erunt habendae, ex quo ſola poſterior aequatio differentio-differentialis reſoluenda ſupererit; in qua ob eandem rationem terminos ex diuiſione coefficientium per nn oriundos omiſimus, cum in tam operoſo calculo ſuffi-

ciat correctiones inde reſultantes proxime ſaltem determinaſſe; praeſertim cum haec praetermiſſio vix ad aliquot minuta ſecunda ſit aſcenſura.

§. 234. Ob eandem rationem licebit in valoribus differentialium $\frac{d\Phi}{dr}$ et $\frac{d\eta}{dr}$ particulas ab inclinatione pendentes negligere, vnde erit: $\frac{dv}{dr} =$

$$f\,\mathrm{ſin}\,2\eta\ [-2\alpha F = -F']$$

$$f\,\mathrm{ſin}\,(2\varphi-2\pi)\ [-2(\alpha+\frac{1}{n})G-0{,}008482\,G=-G']$$

$$f\,\mathrm{ſin}\,(2\theta-2\pi)\ [+G\alpha'-\frac{2}{n})H-0{,}008482\,H=-H']$$

$$fk\,\mathrm{ſin}\,r\ [Fd'-Fc'-J = -J']$$

$$fk\,\mathrm{ſin}\,(2\eta-r)\ [+Fc'-(2\alpha-1)K=-K']$$

$$fk\,\mathrm{ſin}\,(2\eta+r)\ [+Fc'-(2\alpha+1)L=-L']$$

$$fk\,\mathrm{ſin}(2\varphi-2\pi-r)[-2(\alpha+\frac{1}{n})M+M-0{,}008482\,M=-M']$$

$$fk\,\mathrm{ſin}(2\varphi-2\pi+r)[-2(\alpha+\frac{1}{n})N-N-0{,}008482\,N=-N']$$

$$fkk\,\mathrm{ſin}(2\varphi\ 2\pi-2r)[-2(\alpha+\frac{1}{n})O+2O-0{,}008482\,O=-O']$$

$$fk\,\mathrm{ſin}(2\theta-2\pi-r)\ [+Ge'-\frac{2}{n}H-\frac{2}{n}S+S-0{,}008482\,S=-S']$$

$$fk\,\mathrm{ſin}(2\theta-2\pi+r)[+Gd'-\frac{2}{n}H-\frac{2}{n}T-T-0{,}008482\,T=-T']$$

$$fe\,\mathrm{ſin}(2\theta-2\pi-s)[+Gr'+\frac{2}{n}H-\frac{1}{n}U-0{,}008482\,U=-U']$$

$$fe\,\mathrm{ſin}(2\theta-2\pi+s)[+Gq'+\frac{2}{n}H-\frac{3}{n}V-0{,}008482\,V=-V']$$

§. 235.

§. 235. Si nunc ſimili modo denuo differentiemus, prodibit: $\frac{ddv}{dr^2} =$

$f \operatorname{coſ} 2\eta \quad [-2\alpha F']$

$f \operatorname{coſ}(2\varphi - 2\pi) \quad [-2(\alpha + \frac{1}{n}) G' - 0{,}008482\, G']$

$f \operatorname{coſ}(2\theta - 2\pi) \quad [+ G'a' - \frac{2}{n} H' - 0{,}008482\, H']$

$fk \operatorname{coſ} r \quad [F'd' + Fe' - J']$

$fk \operatorname{coſ}(2\eta - r) \quad [F'e' - (2\alpha - 1) K']$

$fk \operatorname{coſ}(2\eta + r) \quad [F'c' - (2\alpha + 1) L']$

$fk \operatorname{coſ}(2\varphi - 2\pi - r) [-2(\alpha + \frac{1}{n}) M' + M' - 0{,}008482\, M']$

$fk \operatorname{coſ}(2\varphi - 2\pi + r) [-2(\alpha + \frac{1}{n}) N' - N' - 0{,}008482\, N']$

$fkk \operatorname{coſ}(2\varphi - 2\pi - 2r) [-2(\alpha + \frac{1}{n}) O' + 2O' - 0{,}008482\, O']$

$fk \operatorname{coſ}(2\theta - 2\pi - r) [G'e' - \frac{2}{n} H' - \frac{2}{n} S' + S' - 0{,}008482\, S']$

$fk \operatorname{coſ}(2\theta - 2\pi + r) [G'd' - \frac{2}{n} H' - \frac{2}{n} T' - T' - 0{,}008482\, T']$

$fe \operatorname{coſ}(2\theta - 2\pi - s) [G'r' + \frac{2}{n} H' - \frac{1}{n} U' - 0{,}008482\, U']$

$fe \operatorname{coſ}(2\theta - 2\pi + s) [G'q' + \frac{2}{n} H' - \frac{3}{n} V' - 0{,}008482\, V']$

§. 236. Hinc autem ſequentes eliciuntur valores

$F =$	0,01273	. . .	$l\,F = 8{,}104833$
$G =$	0,32213	. . .	$l\,G = 9{,}508032$
$H =$	0,06976	. . .	$l\,H = 8{,}843590$
$J =$	$-1{,}87800$	. . .	$l{-}J = 0{,}273710$

 K =

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$K = +0{,}05615$	. . .	$l\text{-}K = 8{,}749352$
$L = -0{,}00077$	. . .	$l\text{-}L = 6{,}888904$
$M = -0{,}29638$	. . .	$l\text{-}M = 9{,}471854$
$N = +0{,}00012$	. . .	$l\,N = 6{,}089109$
$O = +0{,}32287$	. . .	$l\,O = 9{,}509034$
$S = +0{,}39091$	. . .	$l\,S = 9{,}592073$
$T = +0{,}69579$	. . .	$l\,T = 9{,}842475$
$U = -0{,}07922$	. . .	$l\text{-}U = 8{,}898830$
$V = -0{,}05141$	. . .	$l\text{-}V = 8{,}711093$

§. 237. Pro diſtantia ergo lunae a ſole curtata $x = \frac{(1-kk)au}{1-k\,\mathrm{coſ}\,r}$ erit

$$u = \text{Praec.}$$

		Log. coeff.	Valores coeff. integri
$+\,0{,}000072f$	$\mathrm{coſ}\,2\eta$	5,860017	$+\,0{,}000079$
$+\,0{,}001833f$	$\mathrm{coſ}(2\Phi-2\pi)$	7,263216	$+\,0{,}002005$
$+\,0{,}000397f$	$\mathrm{coſ}(2\theta-2\pi)$	6,598774	$+\,0{,}000434$
$-\,0{,}01069fk$	$\mathrm{coſ}\,r$	8,028894	$-\,0{,}000634$
$+\,0{,}00032fk$	$\mathrm{coſ}(2\eta-r)$	6,504536	$+\,0{,}000019$
$-\,0{,}00000fk$	$\mathrm{coſ}(2\eta+r)$	4,644088	$-\,0{,}000000$
$-\,0{,}00169fk$	$\mathrm{coſ}(2\Phi-2\pi-r)$	7,227038	$-\,0{,}000100$
$+\,0{,}00000fk$	$\mathrm{coſ}(2\Phi-2\pi+r)$	3,834293	$+\,0{,}000000$
$+\,0{,}00184fk^2$	$\mathrm{coſ}(2\Phi-2\pi-2r)$	7,264218	$+\,0{,}000006$
$+\,0{,}00223fk$	$\mathrm{coſ}(2\theta-2\pi-r)$	7,347257	$+\,0{,}000132$
$+\,0{,}00396fk$	$\mathrm{coſ}(2\theta-2\pi+r)$	7,597659	$+\,0{,}000235$
$-\,0{,}00045fe$	$\mathrm{coſ}(2\theta-2\pi-s)$	6,654014	$-\,0{,}000000$
$-\,0{,}00029fe$	$\mathrm{coſ}(2\theta-2\pi+s)$	6,466277	$-\,0{,}000000$

vbi notandum eſt eſſe $f = 1{,}093756$, et $l\,f = 0{,}038921$.

§. 238. Deinde pro motu momentaneo habebitur

$$\frac{d\Phi}{dr} = \text{Praec.}$$

	Log. coeff.	Valores coeff. in numeris.
$-0{,}000146f \operatorname{coſ} 2\eta$	6,164934	$-0{,}000160$
$-0{,}003700f \operatorname{coſ}(2\Phi-2\pi)$	7,568133	$-0{,}004046$
$-0{,}000801f \operatorname{coſ}(2\theta-2\pi)$	6,903691	$-0{,}000876$
$+0{,}02157fk \operatorname{coſ} r$	8,333811	$+0{,}001286$
$-0{,}00065fk \operatorname{coſ}(2\eta-r)$	6,809453	$-0{,}000038$
$+0{,}00000fk \operatorname{coſ}(2\eta+r)$	4,949005	$+0{,}000001$
$+0{,}00340fk \operatorname{coſ}(2\Phi-2\pi-r)$	7,531955	$+0{,}000203$
$-0{,}00000fk \operatorname{coſ}(2\Phi-2\pi+r)$	4,139210	$-0{,}000000$
$-0{,}00371fk^2 \operatorname{coſ}(2\Phi-2\pi-2r)$	7,569135	$-0{,}000012$
$-0{,}00449fk \operatorname{coſ}(2\theta-2\pi-r)$	7,652174	$-0{,}000267$
$-0{,}00799fk \operatorname{coſ}2\theta-2\pi+r)$	7,902576	$-0{,}000476$
$+0{,}00091fe \operatorname{coſ}2\theta-2\pi-s)$	6,958931	$+0{,}000000$
$+0{,}00059fe \operatorname{coſ}(2\theta-2\pi+s)$	6,771194	$+0{,}000000$

§. 239. Pro correctione longitudinis verae hinc oriunda ponatur,

$$\Phi = \text{Praec.}$$

$$+ \mathfrak{A}'f \operatorname{ſin} 2\eta \quad + \mathfrak{J}'fk \operatorname{ſin} r \quad + \mathfrak{M}'fk \operatorname{ſin}(2\Phi-2\pi-r)$$

$$+ \mathfrak{G}'f \operatorname{ſin}(2\Phi-2\pi) + \mathfrak{K}'fk \operatorname{ſin}(2\eta-r) + \mathfrak{N}'fk \operatorname{ſin}(2\Phi-2\pi+r)$$

$$+ \mathfrak{H}'f \operatorname{ſin}(2\theta-2\pi) + \mathfrak{L}'fk \operatorname{ſin}(2\eta+r) + \mathfrak{O}'fkk \operatorname{ſin}(2\Phi-2\pi-2r)$$

$$+ \mathfrak{S}'fk \operatorname{ſin}(2\theta-2\pi-r) \; + \mathfrak{U}'fe \operatorname{ſin}(2\theta-2\pi-s)$$

$$+ \mathfrak{T}'fk \operatorname{ſin}(2\theta-2\pi+r) \; + \mathfrak{W}'fe \operatorname{ſin}(2\theta-2\pi+s)$$

 erit-

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eritque :

$$2\alpha\,\mathfrak{F}' = -\,0{,}000146$$
$$0{,}026834\,\mathfrak{G}' = -\,0{,}003700$$
$$0{,}159358\,\mathfrak{H}' - \mathfrak{G}'a' = -\,0{,}000801$$
$$\mathfrak{I}' - \mathfrak{F}'d' - \mathfrak{F}'e' = +\,0{,}02157$$
$$0{,}867476\,\mathfrak{K}' - \mathfrak{F}'c' = -\,0{,}00065$$
$$2{,}867476\,\mathfrak{L}' - \mathfrak{F}'c' = -\,0{,}00000$$
$$1{,}026834\,\mathfrak{M}' = +\,0{,}00340$$
$$3{,}026834\,\mathfrak{N}' = -\,0{,}00000$$
$$0{,}026834\,\mathfrak{O}' = -\,0{,}00371$$
$$-\,0{,}840642\,\mathfrak{S}' - \mathfrak{G}'e' + \frac{2}{n}\,\mathfrak{H}' = -\,0{,}00449$$
$$+\,1{,}159358\,\mathfrak{T}' - \mathfrak{G}'d' + \frac{2}{n}\,\mathfrak{H}' = -\,0{,}00799$$
$$+\,0{,}083920\,\mathfrak{U}' - \mathfrak{G}'r' - \frac{2}{n}\,\mathfrak{H}' = +\,0{,}00091$$
$$+\,0{,}234796\,\mathfrak{V}' - \mathfrak{G}'q' - \frac{2}{n}\,\mathfrak{H}' = +\,0{,}00059$$

§. 240. Expeditis igitur his formulis orietur :

	Log. coeff.	coeff. tot. in ſec.
$\Phi = \mathrm{Pr.} - 0{,}000078 f$ ſin 2η	5,893680	$-\ 18''$
$-\,0{,}001825 f$ ſin $(2\Phi - 2\pi)$	7,261316	$-\ 422$
$-\,0{,}004800 f$ ſin $(2\theta - 2\pi)$	7,681286	$-\ 1083$
$+\,0{,}02154 fk$ ſin r	8,333246	$+\ 264$
$-\,0{,}00074 fk$ ſin $(2\eta - r)$	6,867925	$-\ 9$
$+\,0{,}00332 fk$ ſin $(2\Phi - 2\pi - r)$	7,520465	$+\ 41$
$-\,0{,}13818 fk^2$ ſin $(2\Phi - 2\pi - 2r)$	9,140450	$-\ 92$
$+\,0{,}00446 fk$ ſin $(2\theta - 2\pi - r)$	7,649421	$+\ 55$
$-\,0{,}00685 fk$ ſin $(2\theta - 2\pi + r)$	7,835624	$-\ 84$
$+\,0{,}00310 fe$ ſin $(2\theta - 2\pi - s)$	7,491107	$+\ 11$
$-\,0{,}00030 fe$ ſin $(2\theta - 2\pi + s)$	6,474418	$-\ 1$

§. 242.

§. 241. Haec omnia ſatis conueniunt cum notis inaequalitatibus motus lunae, niſi quod inaequalitas ab angulo $2\theta - 2\pi$ pendens plane aduerſari videatur, cum nullum eius veſtigium in tabulis aſtronomicis occurrat; quod quidem eo magis eſt mirandum, cum correctio inde oriunda ad $18', 3''$ exſurgat. Lubens equidem agnoſco, in hoc calculo non omnem curam eſſe adhibitam, vt hanc aequationem tanquam omnibus numeris abſolutam ſpectare liceat, quoniam ad plurimos terminos, quos formulae noſtrae ſuppeditant, non reſpexi. Interim tamen calculum repetenti mox patebit, non admodum enormiter eſſe aberratum, praeſertim cum aequatio ab angulo $2\Phi - 2\pi$ pendens, quae pari paſſu procedit, veritati perquam conſentanea prodierit, cum ea reductio lunae ad eclipticam contineatur. Ac ſi quidem haec inaequalitas ad ſemiſſem vsque diminuatur, tamen tanta remanet, vt merito dubitare debeamus, eius effectum ab Aſtronomis non eſſe animaduerſum; cum eius omiſſio vix per aliam aequationem compenſari queat. Hancobrem, ſiue omiſſio terminorum neglectorum ſit in cauſa, ſive etiam in calculo numerico error fuerit admiſſus, quod facile euenire potuit, iſtam inueſtigationem in capite ſequenti accuratius ſuſcipiamus.

CAPUT XV.

ACCURATIOR INUESTIGATIO INAEQUALITATUM LUNAE AB INCLINATIONE EJUS ORBITAE PENDENTIUM.

§. 242.

Quoniam praecipuum dubium circa inaequalitatem ab angulo $2\theta-2\pi$ pendentem verſatur, noſtram inueſtigationem ab iis inaequalitatibus, quae ſimul ab alterutra excentricitate pendent, abſtrahamus. Ponamus ergo:

$$\int R\,dr = \mathfrak{A}\cos 2\eta + \mathfrak{F}f\cos 2\eta + \mathfrak{G}f\cos(2\Phi-2\pi) + \mathfrak{H}f\cos(2\theta-2\pi) + \mathfrak{J}f\cos(4\theta-4\pi)$$

$$\text{et } v = A\cos 2\eta + Ff\cos 2\eta + Gf\cos(2\Phi-2\pi) + Hf\cos(2\theta-2\pi) + Jf\cos(4\theta-4\pi)$$

Poſitoque $\frac{2\kappa F+\mathfrak{F}}{nn} = f'$; $\frac{2\kappa G+\mathfrak{G}}{nn} = g'$; $\frac{2\kappa H+\mathfrak{H}}{nn} = h'$; $\frac{2\kappa J+\mathfrak{J}}{nn} = i'$ erit:

$$\frac{d\Phi}{dr} = \alpha + \frac{1}{n} - a'\cos 2\eta - f'f\cos 2\eta - g'f\cos(2\Phi-2\pi) - h'f\cos(2\theta-2\pi) - i'f\cos(4\theta-4\pi)$$

$$\frac{d\eta}{dr} = \alpha - a'\cos 2\eta - f'f\cos 2\eta - g'f\cos(2\Phi-2\pi) - h'f\cos(2\theta-2\pi) - i'f\cos(4\theta-4\pi)$$

$$\frac{d\theta}{dr} = \frac{1}{n} \text{ et } \frac{d\pi}{dr} = -0{,}004241 - 0{,}004221\cos 2\eta + 0{,}004241\cos(2\Phi-2\pi) + 0{,}004241\cos(2\theta-2\pi)$$

§. 243.

§. 243. His valoribus in formulis principalibus ſubſtitutis habebimus has aequationes:

$$R = -\frac{3G}{2nn} f\,\text{ſin}\,(2\theta - 2\pi) + \frac{3H}{2nn} f\,\text{ſin}\,(2\Phi - 2\pi)$$

$$\frac{ddv}{dr^2} = f\,\text{coſ}\,2\eta \left\{ -\beta F - 2\varkappa\mathfrak{F} \right.$$

$$f\,\text{coſ}(2\Phi - 2\pi) \left\{ \begin{array}{l} -\beta G + \frac{3H}{4nn} - 2\varkappa\mathfrak{G} + \frac{\mathfrak{A}\mathfrak{H}}{nn} + \frac{3A\mathfrak{H}}{nn} \\ + \frac{3\mathfrak{A}H}{nn} + \frac{3AH}{nn} \end{array} \right.$$

$$f\,\text{coſ}\,(2\theta 2\pi) \left\{ \begin{array}{l} -\beta H \pm \frac{3G}{4nn} - 2\varkappa\mathfrak{H} + \frac{\mathfrak{A}\mathfrak{G}}{nn} + \frac{3A\mathfrak{G}}{nn} \\ + \frac{3\mathfrak{A}G}{nn} + \frac{3AG}{nn} \end{array} \right.$$

$$f\,\text{coſ}\,(4\theta - 4\pi)[-\beta J - 2\varkappa\mathfrak{J}$$

$$f\,\text{coſ}\,2\eta\left(-0{,}005156 - 0{,}026307 + 0{,}001969\frac{A}{nn}\right)$$

$$f\,\text{coſ}(2\Phi - 2\pi)\left(+0{,}003938 - 1 - 0{,}052614\frac{A}{nn} - 0{,}002578\frac{A}{nn}\right)$$

$$f\,\text{coſ}(2\theta - 2\pi)\left(+0{,}052614 + 0{,}002578 - 0{,}001969\frac{2A}{nn} + \frac{A}{nn}\right)$$

$$f\,\text{coſ}(4\theta - 4\pi)\left(+0{,}001320 + 0{,}026307\frac{A}{nn}\right)$$

§. 244. Vel in numeris erit

$$R = 0{,}008540\,H f\,\text{ſin}(2\Phi - 2\pi) - 0{,}008540\,G f\,\text{ſin}(2\theta - 2\pi)$$

$$\frac{ddv}{dr^2} =$$

$$f \cos 2\eta \; [-1{,}01591\,F - 2\varkappa\,\mathfrak{F} - 0{,}031478$$

$$f\,\mathrm{c}ſ(2\Phi-2\pi)\left\{\begin{array}{l} -1{,}01591\,G - 2\varkappa\,\mathfrak{G} + 0{,}000636\,H - 0{,}845648 \\ \qquad\qquad -0{,}027110\,\mathfrak{H} \end{array}\right.$$

$$f\,\mathrm{c}ſ(2\theta-2\pi)\left\{\begin{array}{l} -1{,}01591\,H - 2\varkappa\,\mathfrak{H} + 0{,}000636\,G + 0{,}047722 \\ \qquad\qquad -0{,}027110\,\mathfrak{G} \end{array}\right.$$

$$f\,\mathrm{c}ſ(4\theta-4\pi)\,[-1{,}01591\,J - 2\varkappa\,\mathfrak{J} + 0{,}001123$$

Nunc autem ex formulis aſſumtis erit

$$R = f \sin 2\eta \; [-2\alpha\,\mathfrak{F} + 0{,}004241\,\mathfrak{G} - 0{,}004241\,\mathfrak{H}]$$
$$f \sin(2\Phi-2\pi)\,[+\mathfrak{A}\,b' - 2{,}026834\,\mathfrak{G} + 0{,}004221\,\mathfrak{H}]$$
$$f \sin(2\theta-2\pi)\,[-\mathfrak{A}\,g' - 0{,}023965\,\mathfrak{G} - 0{,}159358\,\mathfrak{H}]$$
$$f \sin(4\theta-4\pi)\,[+0{,}004241\,\mathfrak{H} - 0{,}318716\,\mathfrak{J}]$$

at eſt

$$\mathfrak{A}b' = -0{,}00931\,H - 0{,}00461\,\mathfrak{H}; \text{ et } \mathfrak{A}g' = -0{,}00931\,G - 0{,}00461\,\mathfrak{G}$$

vnde fit:

$$1{,}867476\,\mathfrak{F} = 0{,}004241\,(\mathfrak{G} - \mathfrak{H})$$
$$2{,}026874\,\mathfrak{G} = -0{,}01785\,H - 0{,}00039\,\mathfrak{H}$$
$$0{,}159358\,\mathfrak{H} = +0{,}01785\,G - 0{,}01935\,\mathfrak{G}$$
$$0{,}318716\,\mathfrak{J} = +0{,}004241\,\mathfrak{H}$$

§. 245. Tum ſimili modo differentiando valorem ipſius $\dot{v}$ ponatur

$$F' = 1{,}867476\,F - 0{,}004241\,G + 0{,}004241\,H$$
$$G' = 2{,}026834\,G + 0{,}01091\,H + 0{,}00750\,\mathfrak{H}$$
$$H' = 0{,}159358\,H + 0{,}03909\,G - 0{,}00750\,\mathfrak{G}$$
$$J' = 0{,}318716\,F - 0{,}004241\,H$$

vt ſit $\frac{dv}{dr} =$

$$A'\,\text{ſin}\,2\eta - F'f\,\text{ſin}\,2\eta - G'f\,\text{ſin}(2\Phi-2\pi) - H'f\,\text{ſin}(2\theta-2\pi) - J'f\,\text{ſin}(4\theta-4\pi)$$

eritque

$$\frac{ddv}{dr^2} = f\,\text{coſ}\,2\,\eta\ [-2aF' + 0{,}004241\,G' + 0{,}004241\,H']$$
$$f\,\text{coſ}(2\Phi-2\pi)[+A'b' - 2{,}026834\,G' - 0{,}004221\,H']$$
$$f\,\text{coſ}(2\Phi-2\pi)[+A'g' - 0{,}159358\,H' - 0{,}023965\,G']$$
$$f\,\text{coſ}(4\theta-4\pi)[+0{,}004241\,H' - 0{,}318716\,J']$$

ſeu $\frac{ddv}{dr^2} =$

$$f\,\text{coſ}\,2\eta\ \begin{cases} -3{,}48745\,F + 0{,}01668\,G - 0{,}00721\,H - 0{,}00003\,\mathfrak{G} \\ \qquad + 0{,}00007\,\mathfrak{H} \end{cases}$$
$$f\,\text{cſ}(2\Phi-2\pi)\ [-4{,}10820\,G - 0{,}05121\,H - 0{,}02929\,\mathfrak{H} + 0{,}00003\,\mathfrak{G}$$
$$f\,\text{cſ}(2\theta-2\pi)\ [-0{,}02565\,H - 0{,}08323\,G - 0{,}01290\,\mathfrak{G} - 0{,}00018\,\mathfrak{H}$$
$$f\,\text{cſ}(4\theta-4\pi)\ [-0{,}10158\,J + 0{,}00016\,G + 0{,}00202\,H - 0{,}00003\,\mathfrak{G}$$

§. 246. Hinc pro $\mathfrak{F}$ et $\mathfrak{J}$ ſubſtitutis valoribus

$$\mathfrak{F} = 0{,}00227\,(\mathfrak{G}-\mathfrak{H}) \quad \text{et} \quad \mathfrak{J} = 0{,}01331\,\mathfrak{H}$$

habebimus has aequationes

$$+2{,}47154\,F - 0{,}01668\,G + 0{,}00721\,H - 0{,}00456\,\mathfrak{G} + 0{,}00456\,\mathfrak{H} = 0{,}031478$$
$$+3{,}09229\,G + 0{,}05185\,H + 0{,}00218\,\mathfrak{H} - 2{,}01799\,\mathfrak{G} = 0{,}995648$$
$$-0{,}99026\,H + 0{,}08387\,G - 0{,}01421\,\mathfrak{G} - 2{,}01780\,\mathfrak{H} = -0{,}047722$$
$$-0{,}91433\,J - 0{,}00016\,G - 0{,}00202\,H + 0{,}00003\,\mathfrak{G} - 0{,}02685\,\mathfrak{H} = -0{,}001123$$

§. 247. Deinde reperitur

$$\mathfrak{G} = -0,00881\ H - 0,00002\ G$$

$$\mathfrak{H} = +0,11201\ G + 0,00107\ H$$

qui valores ſubſtituti praebent:

$$+2,47154\ F - 0,01618\ G + 0,00725\ H = 0,031478$$

$$+3,09256\ G + 0,06964\ H = 0,995648$$

$$+0,99229\ H + 0,14239\ G = 0,047722$$

$$-0,91430\ J - 0,00297\ G - 0,00205\ H = -0,001123$$

vnde tandem reperitur

$F = +0,014830$	. . .	$l\ F = 8,171166$
$G = +0,321910$	. . .	$l\ G = 9,507734$
$H = +0,001901$	. . .	$l\ H = 7,278927$
$J = +0,000180$	. . .	$l\ J = 6,256381$
	atque	
$\mathfrak{F} = -0,000080$	. . .	$l{-}\mathfrak{F} = 5,903090$
$\mathfrak{G} = -0,000023$	. . .	$l{-}\mathfrak{G} = 5,361728$
$\mathfrak{H} = +0,036056$	. . .	$l\ \mathfrak{H} = 8,556977$
$\mathfrak{J} = +0,000480$	. . .	$l\ \mathfrak{J} = 6,681241$

§. 248. Hinc ergo pro diſtantia $x = \frac{(1-kk)\,au}{1-k\,\mathrm{cof}\,r}$ reperitur

	log.coeff.	val.coeff.
$x = \mathrm{Pr.} + 0,000084 f\ \mathrm{cof}\,2\eta$	5,926350	$+0,000092$
$+ 0,001832 f\ \mathrm{cof}(2\Phi - 2\pi)$	7,262918	$+0,002004$
$+ 0,000011 f\ \mathrm{cof}(2\theta - 2\pi)$	5,034111	$+0,000012$
$+ 0,000001 f\ \mathrm{cof}(4\theta - 4\pi)$	4,011565	$+0,000001$

At

At pro motu momentaneo erit

	log.coeff.	val.coeff.
$\frac{d\Phi}{dr} = \text{Pr.} - 0{,}000169 f \cos 2\eta$	6,227887	$-0{,}000185$
$- 0{,}003697 f \cos(2\Phi - 2\pi)$	7,567849	$-0{,}004043$
$- 0{,}000227 f \cos(2\theta - 2\pi)$	6,356026	$-0{,}000248$
$- 0{,}000005 f \cos(4\theta - 4\pi)$	4,698970	$-0{,}000005$

vnde quidem iam patet inaequalitatem ab angulo $2\theta - 2\pi$ pendentm multo fore minorem, quam ſupra inueneramus, in quo non paruum veritatis criterium cernitur.

§. 249. Pro ipſa iam longitudine lunae ponamus:

$$\Phi = \text{Pr.} + \mathfrak{A}' \sin 2\eta + \mathfrak{F}' f \sin 2\eta + \mathfrak{G}' f \sin(2\Phi - 2\pi)$$
$$+ \mathfrak{H}' f \sin(2\theta - 2\pi) + \mathfrak{J}' f \sin(4\theta - 4\pi)$$

atque obtinebimus has aequationes:

$$2\alpha \mathfrak{F}' - 0{,}004241 (\mathfrak{G}' + \mathfrak{H}') = -0{,}000169$$
$$2{,}026834 \mathfrak{G}' + 0{,}004221 \mathfrak{H}' - 0{,}000227 \mathfrak{A}' = -0{,}003697$$
$$0{,}159358 \mathfrak{H}' + 0{,}023965 \mathfrak{G}' - 0{,}003697 \mathfrak{A}' = -0{,}000227$$
$$0{,}318716 \mathfrak{J}' - 0{,}004241 \mathfrak{H}' = -0{,}000005$$

vnde erit

	log. coeff.	vol.tot.coeff. in min. ſec.
$\Phi = \text{Pr.} - 0{,}000096 f \sin 2\eta$	5,984018	$-22''$
$- 0{,}001823 f \sin(2\Phi - 2\pi)$	7,260310	-411
$- 0{,}000910 f \sin(2\theta - 2\pi)$	6,959131	-205
$- 0{,}000028 f \sin(4\theta - 4\pi)$	5,450835	-6

Patet ergo reuera aequationem ab angulo $2\theta - 2\pi$ ortam multo eſſe minorem, quam capite praecedente inueneramus, atque nunc quidem non vltra $205''$ ſeu $3', 25''$ aſcendere. Nullum igitur eſt dubium, quin haec aequatio tabulas lunares ad multo maiorem perfectionem ſit euectura.

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§. 250. Cum igitur neglectus terminorum, minimorum tantum errorem pepererit in aequatione ab angulo $2\theta - 2\pi$ pendente, operae erit pretium, etiam aequationes inſuper ab excentricitate orbitae lunaris pendentes curatius inueſtigare, quae quidem alicuius videntur eſſe momenti. In hunc finem ponamus.

$$\begin{aligned} \int R\,dr = {} & \mathfrak{A} \operatorname{cſ} 2\eta + \mathfrak{C} k \operatorname{cſ} r + \mathfrak{D} k \operatorname{cſ}(2\eta - r) + \mathfrak{E} k \operatorname{cſ}(2\eta + r) + 32 kk \operatorname{cſ}(2\eta - 2r) \\ & + \mathfrak{G} f \operatorname{coſ}(2\theta - 2\pi) + \mathfrak{H} f \operatorname{coſ}(2\theta - 2\pi) \\ & + \mathfrak{J} fk \operatorname{coſ} r + \mathfrak{M} fk \operatorname{coſ}(2\Phi - 2\pi - r) + \mathfrak{S} fk \operatorname{coſ}(2\theta - 2\pi - r) \\ & + \mathfrak{O} fkk \operatorname{cſ}(2\Phi - 2\pi - 2r) + \mathfrak{T} fk \operatorname{coſ}(2\theta + 2\pi + r) \end{aligned}$$

$$\begin{aligned} \text{et } v = {} & A \operatorname{cſ} 2\eta + D k \operatorname{cſ}(2\eta - r) + E k \operatorname{coſ}(2\eta + r) - 14 kk \operatorname{coſ}(2\eta - 2r) \\ & + G f \operatorname{coſ}(2\Phi - 2\pi) + H f \operatorname{coſ}(2\theta - 2\pi) \\ & + J fk \operatorname{coſ} r + M fk \operatorname{coſ}(2\Phi - 2\pi - r) + S fk \operatorname{coſ}(2\theta - 2\pi - r) \\ & + O fkk \operatorname{coſ}(2\Phi - 2\pi - 2r) + T fk \operatorname{coſ}(2\theta - 2\pi + r) \end{aligned}$$

§. 251. His valoribus in aequationibus noſtris principalibus ſubſtitutis habebimus:

$$\begin{aligned} R = {} & fk \sin r \quad (0) \\ & fk \sin(2\Phi - 2\pi - r) \quad (0{,}00854\,S + 0{,}01708\,H) \\ & fkk \sin(2\Phi - 2\pi - 2r) \quad (0{,}01708\,S) \\ & fk \sin(2\theta - 2\pi - r) \quad (-0{,}00854\,M - 0{,}01708\,G) \\ & fk \sin(2\theta - 2\pi + r) \quad (-0{,}01708\,G) \end{aligned}$$

et:

$$\frac{ddv}{dr^2} = fk \operatorname{coſ} r \begin{cases} -6J - 2k\mathfrak{J} - 0{,}00290 - 0{,}00893 - 0{,}00278 \\ -0{,}00098 - 0{,}00098 - 0{,}00448 \frac{A}{nn} + 0{,}00470 \frac{A}{nn} \\ + 0{,}01315 \frac{A}{nn} + 0{,}01315 \frac{A}{nn} + 0{,}0263 \frac{D}{nn} \\ + 0{,}0263 \frac{E}{nn} \end{cases}$$

fk

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$$fk\cos(2\Phi-2\pi-r)\left\{\begin{aligned}
&-\epsilon M-2\kappa\mathfrak{M}+\tfrac{1}{2}bG+0{,}00427S-0{,}00854H\\
&-0{,}02711\mathfrak{S}-0{,}03634S+\frac{\mathfrak{C}}{nn}\mathfrak{G}+\frac{3\mathfrak{C}}{nn}G\\
&+0{,}55422\mathfrak{H}+0{,}51332H-\frac{3AH}{2nn}+0{,}01796\\
&+0{,}00145+0{,}001969-\tfrac{1}{2}+0{,}00896\frac{A}{nn}\\
&+0{,}01049\frac{A}{nn}-0{,}02631\frac{A}{nn}-0{,}00197\frac{A}{nn}\\
&-0{,}00129\frac{A}{nn}-0{,}05261\frac{D}{nn}-0{,}00258\frac{E}{nn}
\end{aligned}\right.$$

$$fkk\cos(2\Phi-2\pi-2r)\left\{\begin{aligned}
&-\epsilon O-2\kappa\mathfrak{D}+\tfrac{1}{2}bM+\tfrac{3}{4}\mathfrak{G}-\frac{10}{nn}\mathfrak{H}+\frac{54}{nn}H\\
&-\frac{3DH}{2nn}+0{,}7698+0{,}00898+0{,}00098\\
&+0{,}00072-\tfrac{1}{4}+0{,}00448\frac{A}{nn}+0{,}0052\frac{A}{nn}\\
&-0{,}00064\frac{A}{nn}+0{,}00896\frac{D}{nn}-0{,}01315\frac{A}{nn}\\
&-0{,}02630\frac{D}{nn}+0{,}01049\frac{E}{nn}-0{,}00129\frac{E}{nn}\\
&-0{,}00129\frac{E}{nn}-0{,}00042
\end{aligned}\right.$$

$$fk\cos(2\theta-2\pi-r)\left\{\begin{aligned}
&-\epsilon S-2\kappa\mathfrak{S}+\tfrac{1}{2}bH+0{,}00427M-0{,}00854G\\
&-0{,}02711\mathfrak{M}-0{,}03634M+\frac{\mathfrak{C}}{nn}\mathfrak{H}+\frac{3\mathfrak{C}}{nn}H\\
&-0{,}01606\mathfrak{G}-0{,}02844G-\frac{3AG}{2nn}-0{,}00896\\
&-0{,}00327+0{,}026307+0{,}00129-0{,}00179\frac{A}{nn}\\
&-0{,}00145\frac{A}{nn}+\frac{A}{2nn}-0{,}00394\frac{E}{nn}+\frac{E}{nn}
\end{aligned}\right.$$

$$f k\,\text{coſ}(2\theta-2\pi+r)\begin{cases} -\beta\mathrm{T}-2\varkappa\mathfrak{T}+\tfrac{1}{2}b\mathrm{H}+0{,}00854\,\mathrm{G}+\dfrac{\mathfrak{C}}{nn}\mathfrak{H}+\dfrac{3\mathfrak{C}}{nn}\mathrm{H} \\ +0{,}55422\mathfrak{G}+0{,}51332\mathrm{G}-\dfrac{3\mathrm{AG}}{2nn}+0{,}00940 \\ -0{,}01049+0{,}026307+0{,}00129-0{,}00556\dfrac{\mathrm{A}}{nn} \\ -0{,}00145\dfrac{\mathrm{A}}{nn}-0{,}003938\dfrac{\mathrm{A}}{nn}+\dfrac{\mathrm{A}}{2nn} \\ -0{,}0394\dfrac{\mathrm{D}}{nn}+\dfrac{\mathrm{D}}{nn} \end{cases}$$

§. 252. Hae autem formulae intricatae reducuntur ad ſequentes

$$\frac{ddv}{dr}=\text{Pr.}+fk\,\text{coſ}\,r\,(-\beta\mathrm{J}-2\varkappa\mathfrak{J}-0{,}01182)$$

$$fk\,\text{coſ}(2\Phi-2\pi-r)\begin{pmatrix}-\beta\mathrm{M}-2\varkappa\mathfrak{M}-0{,}03207\mathrm{S}+0{,}53619\\ -0{,}02711\mathfrak{S}\end{pmatrix}$$

$$fkk\,\text{coſ}(2\Phi-2\pi-2r)\,(-\beta\mathrm{O}-2\varkappa\mathfrak{O}+1{,}52115\mathrm{M}+0{,}76615)$$

$$fk\,\text{coſ}(2\theta-2\pi-r)\begin{pmatrix}-\beta\mathrm{S}-2\varkappa\mathfrak{S}-0{,}03207\mathrm{M}+0{,}00286\\ -0{,}02711\mathfrak{M}\end{pmatrix}$$

$$fk\,\text{coſ}(2\theta-2\pi+r)\,(-\beta\mathrm{T}-2\varkappa\mathfrak{T}+0{,}38147)$$

§. 253. Nunc eosdem valores ex formulis aſſumtis eruamus, ac poſito more adhuc recepto $\frac{2\varkappa\mathrm{J}+\mathfrak{J}}{nn}=i'$, $\frac{2\varkappa\mathrm{M}+\mathfrak{M}}{nn}=m'$ etc. erit

$$\frac{d\Phi}{dr}=\alpha+\frac{1}{n}-a'\,\text{coſ}\,2\eta-d'k\,\text{coſ}(2\eta-r)\;-g'f\,\text{coſ}(2\Phi-2\pi)$$
$$-e'k\,\text{coſ}(2\eta+r)\;-b'f\,\text{coſ}(2\theta-2\pi)$$
$$-i'fk\,\text{coſ}\,r\;-m'fk\,\text{coſ}(2\Phi-2\pi-r)\;-s'fk\,\text{coſ}(2\theta-2\pi-r)$$
$$-n'fkk\,\text{coſ}(2\Phi-2\pi-2r)\;-t'fk\,\text{coſ}(2\theta-2\pi+r)$$

$d\eta$

$$\frac{d\eta}{dr} = \alpha - a'\cos 2\eta - c'k\cos r - d'k\cos(2\eta - r) - g'k\cos(2\Phi - 2\pi)$$
$$- e'k\cos(2\eta + r) - b'k\cos(2\theta - 2\pi)$$
$$- i'fk\cos r - m'fk\cos(2\Phi - 2\pi - r) - s'fk\cos(2\theta - 2\pi - r)$$
$$- n'fkk\cos(2\Phi - 2\pi - 2r) - t'fk\cos(2\theta - 2\pi + r)$$

$$\frac{d\theta}{dr} = \frac{1}{n} + \frac{2}{n}k\cos r + \frac{3}{2n}kk\cos 2r$$ atque

$$\frac{d\pi}{dr} =$$
$$-0{,}004241 \qquad +0{,}004241\cos(2\Phi - 2\pi)$$
$$-0{,}004221\cos 2\eta + 0{,}004241\cos(2\theta - 2\pi)$$
$$-0{,}01766k\cos r - 0{,}00996k\cos(2\eta - r) - 0{,}010636kk\cos(2\eta - 2r)$$
$$-0{,}00831k\cos(2\eta + r) - 0{,}021273kk\cos 2r$$
$$+0{,}00924k\cos(2\Phi - 2\pi - r) + 0{,}00841k\cos(2\theta - 2\pi - r)$$
$$+0{,}00841k\cos(2\Phi - 2\pi + r) + 0{,}00924k\cos(2\theta - 2\pi + r)$$
$$+0{,}01064kk\cos(2\Phi - 2\pi - 2r)$$

§. 254. His iam valoribus introducendis differentiemus formulas noſtras aſſumtas pro $\int R\,dr$ et v; atque obtinebimus primo:

$$R = \text{Praec.}$$
$$+ fk\sin r \begin{Bmatrix} +0{,}00924\mathfrak{G} + 0{,}00841\mathfrak{H} - 0{,}004241\mathfrak{M} \\ -0{,}00841\mathfrak{G} - 0{,}00924\mathfrak{H} - 0{,}004241\mathfrak{S} \\ +0{,}004241\mathfrak{T} \end{Bmatrix}$$
$$+ fk\sin(2\Phi - 2\pi - r) \begin{Bmatrix} +\mathfrak{A}s' - 0{,}01766\mathfrak{G} - 0{,}00996\mathfrak{H} \\ -1{,}026834\mathfrak{M} - 0{,}004221\mathfrak{S} \end{Bmatrix}$$
$$+ fkk\sin(2\Phi - 2\pi - 2r) \begin{Bmatrix} +\mathfrak{D}s' + 32h' - 0{,}02127\mathfrak{G} - 0{,}01064\mathfrak{H} \\ -0{,}026834\mathfrak{O} - 0{,}01766\mathfrak{M} - 0{,}00996\mathfrak{S} \end{Bmatrix}$$

$$+fk\sin(2\theta-2\pi-r)\left\{\begin{array}{l}-\mathfrak{A}m'+\mathfrak{D}b'+\mathfrak{G}e'-\frac{2}{n}\mathfrak{H}+0{,}840642\mathfrak{S}+\mathfrak{M}a' \\ -\mathfrak{E}g'-0{,}00831\mathfrak{G}-0{,}01766\mathfrak{H}-0{,}004221\mathfrak{M}\end{array}\right\}$$

$$+fk\sin(2\theta-2\pi+r)\left\{\begin{array}{l}-\mathfrak{D}g'+\mathfrak{E}b'+\mathfrak{G}d'-\frac{2}{n}\mathfrak{H}-1{,}159358\mathfrak{T} \\ +0{,}00996\mathfrak{G}-0{,}01766\mathfrak{H}\end{array}\right\}$$

§. 255. Hinc igitur conſequimur iſtas aequationes

$$0{,}00083\,(\mathfrak{G}-\mathfrak{H})-\mathfrak{J}-0{,}004241\,(\mathfrak{M}+\mathfrak{S}-\mathfrak{T})=0$$

$$\mathfrak{A}s'-0{,}01766\mathfrak{G}-0{,}00996\mathfrak{H}-0{,}004221\mathfrak{S}-1{,}026834\mathfrak{M}= 0{,}01708\,H+0{,}00854\,S$$

$$\mathfrak{D}s'+32b'-0{,}002127\mathfrak{G}-0{,}01064\mathfrak{H}-0{,}026834\mathfrak{O}-0{,}01766\mathfrak{M} -0{,}00996\mathfrak{S}=0{,}01708\,S$$

$$-\mathfrak{A}m'+\mathfrak{D}b'-\mathfrak{E}g'+\mathfrak{G}e'-0{,}00831\mathfrak{G}-\frac{2}{n}\mathfrak{H}-0{,}01766\mathfrak{H}+\mathfrak{M}a' -0{,}004221\mathfrak{M}+0{,}840642\mathfrak{S}=-0{,}01708G-0{,}00854M$$

$$-\mathfrak{D}g'+\mathfrak{E}b'+\mathfrak{G}d'-0{,}00996\mathfrak{G}-\frac{2}{n}\mathfrak{H}-0{,}01766\mathfrak{H}-1{,}159358\mathfrak{T}= -0{,}01708G$$

§. 256. Ponatur nunc vlterius:

$$J'=\mathfrak{J}-0{,}00083\,(G-H)+0{,}004241\,(M+S-T)$$

$$M'=1{,}026834M-As'+0{,}01766G+0{,}00996H+0{,}004221S$$

$$O'=0{,}026834\,O-Ds'+14b'+0{,}02127G+0{,}01064H +0{,}01766M+0{,}00996\,S$$

$$S'=-0{,}840642\,S+Am'-Db'+Eg'-Ge'+0{,}00831G+\frac{2}{n}H +0{,}01766H-Ma'+0{,}004221M$$

$$T'=1{,}159358\,T+Dg'-Eb'-Gd'+0{,}00996\,G+\frac{2}{n}H +0{,}01766\,H$$

eritque

eritque $$\frac{ddv}{dr} = \text{Praec.}$$

$$+fk\,\mathrm{coſ}\,r\,[+0{,}01766\,(G'+H') - J' + 0{,}004241\,(M'+S'+T')]$$

$$+fk\,\mathrm{coſ}\,(2\varphi-2\pi-r)\left\{\begin{matrix} A's' - 0{,}01766\,G' - 0{,}00996\,H' \\ - 0{,}004221\,S' - 1{,}026834\,M' \end{matrix}\right\}$$

$$+fkk\,\mathrm{coſ}\,(2\varphi-2\pi-2r)\left\{\begin{matrix} D's' - 2{,}6b' - 0{,}02127\,G' - 0{,}01064\,H' \\ - 0{,}01766\,M' - 0{,}00996\,S' \\ - 0{,}026834\,O' \end{matrix}\right\}$$

$$+fk\,\mathrm{coſ}\,(2\theta-2\pi-r)\left\{\begin{matrix} A'm' + D'b' + E'g' + G'c' + 0{,}840642\,S' \\ - 0{,}00831\,G' - \frac{2}{n}H' - 0{,}01766\,H' \\ + M'a' - 0{,}004221\,M' \end{matrix}\right\}$$

$$+fk\,\mathrm{coſ}\,(2\theta-2\pi+r)\left\{\begin{matrix} D'g' + E'b' + G'd' - 0{,}00996\,G - \frac{2}{n}H' \\ - 0{,}01766\,H' - 1{,}159358\,T' \end{matrix}\right\}$$

§. 257. Prioris ordinis aequationes huc reducuntur:

$$\mathfrak{J} = -\,0{,}004241\,(\mathfrak{M}+\mathfrak{S}+\mathfrak{T})$$
$$1{,}026834\,\mathfrak{M} = -\,0{,}01785\,S - 0{,}00883\,\mathfrak{S} - 0{,}00039$$
$$0{,}026834\,\mathfrak{O} = -\,0{,}05835\,S - 0{,}03041\,\mathfrak{S} - 0{,}01766\,\mathfrak{M} + 0{,}00690$$
$$-\,0{,}840642\,\mathfrak{S} = +\,0{,}01785\,M - 0{,}01935\,\mathfrak{M} + 0{,}00263$$
$$1{,}159358\,\mathfrak{T} = +\,0{,}01247$$

Hinc fit

$$\mathfrak{J} = +\,0{,}00007\,S + 0{,}00008\,M + 0{,}00006 \quad \text{ſeu } \mathfrak{J} = 0$$
$$\mathfrak{M} = -\,0{,}01738\,S + 0{,}00018\,M - 0{,}00035$$
$$\mathfrak{O} = -\,2{,}16266\,S + 0{,}02394\,M + 0{,}26092$$
$$\mathfrak{S} = -\,0{,}00040\,S - 0{,}02123\,M - 0{,}00313$$
$$\mathfrak{T} = -\,0{,}01076$$

§. 258. Porro reperietur

$$J' = J + 0{,}004241\,(M+S-T) - 0{,}00026$$
$$M' = 1{,}026834\,M + 0{,}01935\,S - 0{,}00016\,M + 0{,}00568$$
$$O' = 0{,}026834\,O - 0{,}37652\,S + 0{,}01805\,M + 0{,}01063$$
$$S' = 0{,}840642\,S + 0{,}00013\,S + 0{,}00883\,M - 0{,}00167$$
$$T' = 1{,}159358\,T + 0{,}01023$$

ac ſuccinctius habebitur $\frac{ddv}{dr^2} =$ Praec.

$$+ fk \cos r\, [-J' + 0{,}004241(M'+S'+T') + 0{,}01175]$$
$$+ fk \cos(2\Phi - 2\pi - r) \begin{cases} -1{,}026834\,M' - 0{,}00422\,S' - 0{,}02842\,S \\ + 0{,}00030\,M - 0{,}01160 \end{cases}$$
$$+ fkk \cos(2\Phi - 2\pi - 2r) \begin{cases} -0{,}026834\,O' - 0{,}01766\,M' - 0{,}00354\,M \\ -0{,}01511 - 0{,}00996\,S' + 0{,}33746\,S \end{cases}$$
$$+ fk \cos(2\theta - 2\pi - r) \begin{cases} +0{,}840642\,S' - 0{,}02396\,M' - 0{,}02843\,M \\ -0{,}01472 + 0{,}00025\,S \end{cases}$$
$$+ fk \cos(2\theta - 2\pi + r)\, [-1{,}159358\,T' + 0{,}33856]$$

§. 259. Hinc tandem valores quaeſiti eliciuntur

$J = -0{,}81144$ . . . $l{-}J = 9{,}909256$
$M = -1{,}25325$ . . . $l{-}M = 0{,}098046$
$O = -2{,}12630$ . . . $l{-}O = 0{,}327624$
$S = -0{,}13490$ . . . $l{-}S = 9{,}130012$
$T = -0{,}10080$ . . . $l{-}T = 9{,}003441$

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ac $\mathfrak{J} = -0{,}00005 \quad \ldots \quad l{-}\mathfrak{J} = 5{,}698970$
$\mathfrak{M} = +0{,}00177 \quad \ldots \quad l\,\mathfrak{M} = 7{,}247973$
$\mathfrak{O} = +0{,}52266 \quad \ldots \quad l\,\mathfrak{O} = 9{,}718219$
$\mathfrak{S} = +0{,}02355 \quad \ldots \quad l\,\mathfrak{S} = 8{,}371991$
$\mathfrak{T} = +0{,}01076 \quad \ldots \quad l\,\mathfrak{T} = 8{,}031812$

§. 260. Ex his iam pro diſtantia Lunae a terra erit

$$u = \text{Praec.}$$

	Log. coeff.	Valor. coeff. totius
$-0{,}00462\,fk\ \operatorname{coſ} 2\eta$	7,664440	$-0{,}000275$
$-0{,}00773\,fk\ \operatorname{coſ}(2\Phi-2\pi-r)$	7,853230	$-0{,}000424$
$-0{,}01210\,fkk\ \operatorname{coſ}(2\Phi-2\pi-2r)$	8,082808	$-0{,}000039$
$-0{,}00077\,fk\ \operatorname{coſ}(2\theta-2\pi-r)$	6,885196	$-0{,}000046$
$-0{,}00057\,fk\ \operatorname{coſ}(2\theta-2\pi+r)$	6,758625	$-0{,}000034$

et pro motu momentaneo

$$\frac{d\Phi}{dr} = \text{Praec.}$$

	Log. coeff.	Val. coeff.
$+0{,}00932\,fk\ \operatorname{coſ} r$	7,969416	$+0{,}000555$
$+0{,}01558\,fk\ \operatorname{coſ}(2\Phi-2\pi-r)$	8,192568	$+0{,}000928$
$+0{,}02142\,fkk\ \operatorname{coſ}(2\Phi-2\pi-2r)$	8,330819	$+0{,}000069$
$+0{,}00142\,fk\ \operatorname{coſ}(2\theta-2\pi-r)$	7,152288	$+0{,}000085$
$+0{,}00109\,fk\ \operatorname{coſ}(2\theta-2\pi+r)$	7,037426	$+0{,}000064$

§. 261. Pro longitudine autem lunae ſequentes reſolui debent aequationes:

$$+0{,}00932 = \mathfrak{J}' - 0{,}01766\,(\mathfrak{G}' + \mathfrak{H}') - 0{,}004241\,(\mathfrak{M}' + \mathfrak{S}' + \mathfrak{T}')$$

$$+0{,}01558 = 1{,}026834\,\mathfrak{M}' - \mathfrak{A}'s' + 0{,}01766\,\mathfrak{G}' + 0{,}00996\,\mathfrak{H}' + 0{,}004221\,\mathfrak{S}'$$

$$+0{,}02142 = 0{,}026834\,\mathfrak{O}' - \mathfrak{D}'r' + 2{,}6\,b' + 0{,}02127\,\mathfrak{G}'$$
$$+0{,}01064\,\mathfrak{H}' + 0{,}01766\,\mathfrak{M}' + 0{,}00996\,\mathfrak{S}'$$
$$+0{,}00142 = -0{,}840642\,\mathfrak{S}' - \mathfrak{A}'m' - \mathfrak{D}'b' - \mathfrak{E}'g' + 0{,}02114\,\mathfrak{G}'$$
$$+0{,}16853\,\mathfrak{H}' + 0{,}02396\,\mathfrak{M}'$$
$$+0{,}00109 = 1{,}159358\,\mathfrak{T}' - \mathfrak{D}'g' - \mathfrak{E}'b' - 0{,}35615\,\mathfrak{G}'$$
$$+0{,}16850\,\mathfrak{H}'$$

hincque prodit pro longitudine vera:

		Log. coeff.	Val. coeff. in min. ſec.
$\Phi =$ Pr.	$+0{,}00932\,fk$ ſin r	7,969416	+115
	$+0{,}01521\,fk$ ſin $(2\Phi-2\pi-r)$	8,182130	+187
	$+0{,}79079\,fkk$ ſin $(2\Phi-2\pi-2r)$	9,898060	+529
	$-0{,}00121\,fk$ ſin $(2\theta-2\pi-r)$	7,083939	− 15
	$-0{,}00082\,fk$ ſin $(2\theta-2\pi+r)$	6,913527	− 10

§. 262. Ob inclinationem ergo orbitae lunaris ad eclipticam omnes correctiones huc redeunt, vt ſit

I. Pro diſtantia lunae a terra:

$$u = \text{Praec.}$$

		Log. coeff.	Val. coeff.
$+0{,}000084\,f$	coſ 2η	5,926350	+0,000092
$+0{,}001832\,f$	coſ $(2\Phi-2\pi)$	7,262918	+0,002004
$+0{,}000011\,f$	coſ $(2\theta-2\pi)$	5,034111	+0,000012
$+0{,}000001\,f$	coſ $(4\theta-4\pi)$	4,011565	+0,000001
$-0{,}00462\,fk$	coſ r	7,664440	−0,000275
$-0{,}00773\,fk$	coſ $(2\Phi-2\pi-r)$	7,853230	−0,000424
$-0{,}01210\,fkk$	coſ $(2\Phi-2\pi-2r)$	8,082808	−0,000039
$-0{,}00077\,fk$	coſ $(2\theta-2\pi-r)$	6,885196	−0,000046
$-0{,}00057\,fk$	coſ $(2\theta-2\pi+r)$	1,758625	−0,000034

II. Pro

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II. Pro motu momentaneo:

$$\frac{d\Phi}{dr} = \text{Praec.}$$

	Log. coeff.	Val. coeff.
$-0{,}000169f \cos 2\eta$	6,227887	$-0{,}000185$
$-0{,}003697f \cos(2\Phi-2\pi)$	7,567849	$-0{,}004043$
$-0{,}000227f \cos(2\theta-2\pi)$	6,356026	$-0{,}000248$
$-0{,}000005f \cos(4\theta-4\pi)$	4,698970	$-0{,}000005$
$+0{,}00932fk \cos r$	7,969416	$+0{,}000555$
$+0{,}01558fk \cos(2\Phi-2\pi-r)$	8,192568	$+0{,}000928$
$+0{,}02142fkk \cos(2\Phi-2\pi-2r)$	8,330819	$+0{,}000069$
$+0{,}00142fk \cos(2\theta-2\pi-r)$	7,152288	$+0{,}000085$
$+0{,}00109fk \cos(2\theta-2\pi+r)$	7,037426	$+0{,}000064$

III. Pro longitudine lunae vera

	Log. coeff.	Val. coeff. in min. sec.
$\Phi = \text{Pr.} -0{,}000096f \sin 2\eta$	5,984018	$-22''$
$-0{,}001823f \sin(2\Phi-2\pi)$	7,260310	-411
$-0{,}000910f \sin(2\theta-2\pi)$	6,959131	-205
$-0{,}000028f \sin(4\theta-4\pi-r)$	5,450835	-6
$+0{,}00932fk \sin r$	7,969416	$+115$
$+0{,}01521fk \sin(2\theta-2\pi-r)$	8,182130	$+187$
$+0{,}79079fkk \sin(2\Phi-2\pi-2r)$	9,898060	$+529$
$-0{,}00121fk \sin(2\theta-2\pi-r)$	7,083939	-15
$-0{,}00082fk \sin(2\theta-2\pi+r)$	6,913527	-10

CAPUT

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CAPUT XVI.

EXPOSITIO INAEQUALITATUM LUNAE HACTENUS INUENTARUM.

§. 263.

Quas igitur inuenimus hactenus lunae inaequalitates eae primum, ſi originem earum ſpectemus, ad ſex claſſes reducuntur. Quatenus enim luna in motu ſuo a regulis Keplerianis, in quibus quidem motum apogei compectimur, recedit, eius errores vel primo a ſolo lunae aſpectu, ſeu eius diſtantia a ſole pendent, ſeu quod eodem redit, per angulum η tantum definiuntur, quibus variatio lunae continetur. Ad ſecundam claſſem refero eas lunae inaequalitates, quae inſuper ab excentricitate eius orbitae pendent. Tertia claſſis eas complectitur inaequalitates, quae ab excentricitate orbitae ſolis ortum trahunt. Quarta vero eas, quae per vtramque excentricitatem coniunctim determinantur. Quintae porro claſſi annumeramus eas inaequalitates, quae parallaxin ſolis inuoluunt, atque errores quatuor ante memoratorum generum implicant. Sexta denique claſſis ſuppeditat eas inaequalitates, quae praeterea ab inclinatione orbitae lunaris ad eclipticam pendent.

§. 264. Quodſi vero ad vſum harum inaequalitatum attendamus, prouti eae ad lunam accommodari debent, tum eae in quinque claſſes commodiſſime diſtribuuntur. Primo enim perpendendae ſunt eae inaequali-

qualitates, quarum ope vera diſtantia lunae a terra determinatur, vt inde porro tam lunae diameter apparens, quam eius parallaxis horizontalis aſſignari poſſit. Secundo loco formulae erunt collocandae illae, quae motui momentaneo definiendo inſeruiunt, ex quibus deinceps motus lunae horarius accurate exhiberi poterit. Tertium locum occupabunt eae inaequalitates, quae veram longitudinem lunae ad eclipticam relatam praebent. Quarto vero poſitio lineae nodorum lunae, ſeu longitudo nodi aſcendentis; ac quinto vera inclinatio orbitae lunaris ad eclipticam inueniri debebit; vt deinde vera lunae latitudo concludi poſſit. Manifeſtum autem eſt, has inaequalitates plurimum inter ſe permiſceri, ita vt vix vllum habeatur genus, cuius inaequalitates non a reliquis generibus pendeant; cui tamen incommodo facile medela adhibetur.

§. 265. Quanquam numerus inaequalitatum, quas ſumus conſecuti, tantopere increuit, vt calculus ſine maxima moleſtia expediri nequeat, tamen iam monui, non omnes inaequalitates, quibus motus Lunae perturbatur, eſſe definitas, ſed potius earum numerum omnino eſſe infinitum. Facile quidem intelligitur, pleraſque has praetermiſſas inaequalitates nullius fere eſſe momenti, atque ſine notabili errore iis ſuperſederi poſſe: verum tamen ſunt inter eas nonnullae, quae ad plura minuta ſecunda aſſurgere videntur, quarum argumenta ſupra iam innui; ex quo omnino operae eſſet pretium in eas omni cura inquirere. Sed earum inueſtigatio tam eſt lubrica et incerta, vt leuiſſima omiſſio in calculo facta eas

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maxime afficiat. Cum igitur in calculo plurimos terminos reiicere cogamur, iſtam inueſtigationem fruſtra plane ſuſciperemus, quamdiu ſcilicet rem ſine appropinquatione exequi non licet. Cuius defectus eximium habemus exemplum in inaequalitatibus poſtremo loco inuentis, quae ſtatim atque in negligendo minus largi fueramus, mirum quantum prodierunt immutatae; ac nullum plane eſt dubium, ſi calculum adhuc accuratius proſequi liceret, quin valores inuenti notabilem inſuper mutationem ſint ſubiturae. Imprimis autem aequatio ab angulo $2\Phi-2\pi-2r$ ſeu a dupla diſtantia apogei a nodo pendens, eſt ſuſpecta, ac minime pro certa haberi poteſt, cum leuiſſima circumſtantia eam magnopere perturbare valeat.

§. 266. Si enim in cauſam inquiramus, cur analyſis poſterior tam diuerſos valores pro his inaequalitatibus ſuppeditauerit, primo quidem ſtatim patet, neglectum litterarum germanicarum 𝔊, 𝔐, 𝔒, etc. in calculo priori potiſſimum hoc diſcrimen produxiſſe: ingens enim valor litterae 𝔒 imprimis aequationem ab angulo $2\Phi-2\pi-2r$ pendentem tantopere auxit. Praeterea vero etiam non parum augmenti haec aequatio inde eſt nacta, quod in calculo poſteriori rationem quoque habuimus termini $\cos(2\eta-2r)$, qui tam in valore $\frac{d\Phi}{dr}$ quam $\frac{d\eta}{dr}$ ineſſe eſt deprehenſus; vnde tuto colligere licet, ſi alios quoque terminos ſimiles veluti $2\theta-2\pi-2r$, etſi per ſe ſunt minimi, in calculum introduxiſſemus, coefficien-

efficientes terminorum $2\Phi-2\pi-2r$ non mediocrem inde mutationem ſubituros fuiſſe. Quamobrem plus hinc colligere non poſſumus, niſi inaequalitatem Lunae ab angulo hoc $2\Phi-2\pi-2r$ pendentem minime eſſe contemnendam, etiamſi fortaſſe tanta non ſit, quam inuenimus. Vera autem eius quantitas certius ex obſeruationibus quam ex Theoria colligi poſſe videtur.

§. 267. Quoniam vero hae inaequalitates omnes ad anomaliam Lunae veram referuntur, antequam eas ad vſum adhibere liceat, modum tradi conveniet ad quodvis tempus propoſitum anomaliam Lunae veram determinandi. Cognita autem excentricitate orbitae lunaris k et motu anomaliae mediae, inde ad quodvis tempus facile anomalia media p colligitur. Verum ex anomalia media p et excentricitate k anomalia vera r definiri debet ope huius aequationis $dp = \frac{(1-kk)^{\frac{3}{2}}\, dr}{(1-k\,\mathrm{coſ}\,r)^2}$; vnde quidem non difficulter, ſi nota eſſet anomalia vera r, viciſſim inveniri poſſet anomalia media p. Calculo enim peracto, ſi breuitatis gratia ponatur

$$\delta = \frac{1-\sqrt{(1-kk)}}{k} = \tfrac{1}{2}k + \frac{1.1}{2.4}k^3 + \frac{1.1.3}{2.4.6}k^5 + \text{etc.}$$

reperitur:

$$p = r + 2k\,\mathrm{ſin}\,r + 2\delta(k-\tfrac{1}{2}\delta)\,\mathrm{ſin}\,2s + 2\delta\delta(k-\tfrac{2}{3}\delta)\,\mathrm{ſin}\,3s + 2\delta^3(k-\tfrac{3}{4}\delta)\,\mathrm{ſin}\,4s \text{ etc.}$$

cuius ſeriei progreſſio primo intuitu patet. Cum autem k ac proinde δ ſit valde parvum, erit ſatis exacte:

 $p =$

$$p = r + 2k\,\mathrm{ſin}\,r + (\tfrac{3}{4}kk + \tfrac{1}{8}k^4 + \tfrac{3}{64}k^6)\,\mathrm{ſin}\,2r + (\tfrac{1}{3}k^3 + \tfrac{1}{8}k^5)\,\mathrm{ſin}\,3r$$
$$+ (\tfrac{5}{32}k^4 + \tfrac{3}{32}k^6)\,\mathrm{ſin}\,4r + \tfrac{3}{40}k^5\,\mathrm{ſin}\,5r + \tfrac{7}{192}k^6\,\mathrm{ſin}\,6r$$

§. 268. Data ergo anomalia media Lunae p, eius anomalia vera r elici debebit ex hac aequatione

$$r = p - 2k\,\mathrm{ſin}\,r - (\tfrac{3}{4}kk + \tfrac{1}{3}k^4 + \tfrac{3}{64}k^6)\,\mathrm{ſin}\,2r - (\tfrac{1}{3}k^3 + \tfrac{1}{8}k^5)\,\mathrm{ſin}\,3r$$
$$- (\tfrac{5}{32}k^4 + \tfrac{3}{32}k^6)\,\mathrm{ſin}\,4r - \tfrac{3}{40}k^5\,\mathrm{ſin}\,5r - \tfrac{7}{192}k^6\,\mathrm{ſin}\,6r$$

cuius quidem ope, ſi cognita fuerit excentricitas k, calculus non difficulter expedietur. Quoniam enim termini ſinus inuoluentes ſunt admodum parui, in iis ſtatim poni poterit $r = p$, vnde valor verior pro r eruetur, qui deinde iterum in his terminis adhibitus, iuſtiorem valorem pro r ſuppeditabit. Atque hoc modo poſt aliquot operationes verus tandem valor pro anomalia vera r obtinebitur. Interim tamen, quo iſte calculus facilius perfici queat, aequatio haec ita poteſt transformari, vt loco ſinuum anomaliae verae r, ſinus anomaliae mediae p introducantur; id quod ſequenti modo praeſtabitur.

§. 269 Ponatur breuitatis gratia:

$$\tfrac{3}{4} + \tfrac{1}{8}kk + \tfrac{3}{64}k^4 = \alpha;\quad \tfrac{1}{3} + \tfrac{1}{8}kk = \beta;\quad \tfrac{5}{32} + \tfrac{3}{32}kk = \gamma$$
$$\tfrac{3}{40} = \delta \quad \text{et} \quad \tfrac{7}{192} = \epsilon$$

vt ſit:

$$r = p - 2k\,\mathrm{ſin}\,r - \alpha kk\,\mathrm{ſin}\,2r - \beta k^3\,\mathrm{ſin}\,3r - \gamma k^4\,\mathrm{ſin}\,4r - \delta k^5\,\mathrm{ſin}\,5r - \epsilon k^6\,\mathrm{ſin}\,6r$$

ac ponatur denuo

$$2k\,\mathrm{ſin}\,r + \alpha kk\,\mathrm{ſin}\,2r + \beta k^3\,\mathrm{ſin}\,3r + \gamma k^4\,\mathrm{ſin}\,4r + \delta k^5\,\mathrm{ſin}\,5r + \epsilon k^6\,\mathrm{ſin}\,6r = R$$

et

et cum ſit $r = p - R$ habebitur:

$$\sin r = (1 - \tfrac{1}{2}RR + \tfrac{1}{24}R^4)\sin p - (R - \tfrac{1}{6}R^3 + \tfrac{1}{120}R^5)\cos p$$

$$\sin 2r = (1 - 2RR + \tfrac{2}{3}R^4)\sin 2p - (2R - \tfrac{4}{3}R^3)\cos 2p$$

$$\sin 3r = (1 - \tfrac{9}{2}RR)\sin 3p - (3R - \tfrac{9}{2}R^3)\cos 3p$$

$$\sin 4r = (1 - 8RR)\sin 4p - 4R\cos 4p$$

$$\sin 5r = \sin 5p - 5R\cos 5p$$

$$\sin 6r = \sin 6p$$

negligendo ſcilicet terminos, qui ipſius k poteſtates ſexta altiores continent.

§. 270. Euolutio autem huius calculi fit maxime prolixa, ſi quidem ad ſextam poteſtatem ipſius k aſcendere velimus. Facile autem expedietur, ſi ad quartam ſubſiſtamus, tum autem reperietur:

$$r = p - (2k - \tfrac{1}{4}k^3)\sin p + (\tfrac{5}{4}kk - \tfrac{11}{24}k^4)\sin 2p - \tfrac{13}{12}k^3\sin 3p - \tfrac{103}{96}k^4\sin 4p$$

quae expreſſio ſatis accurate pro quauis anomalia media p conuenientem anomaliam veram indicabit. Calculus autem, ſi excentricitas k conſtet, facili negotio abſoluetur. Formula haec quoque ita repraeſentari poterit, vt ſit

$$r = p - 2k(1 - \tfrac{1}{8}kk)\sin p + \tfrac{5}{4}kk(1 - \tfrac{11}{30})kk\sin 2p - \tfrac{13}{12}k^3\sin 3p - \tfrac{103}{96}k^4\sin 4p$$

Hinc igitur tabulam conſtrui conueniet, vnde pro quauis anomalia media propoſita ipſi reſpondens anomalia vera excerpi queat.

§. 271. Cum autem inuenta fuerit anomalia vera r, longitudo Lunae regula Kepleriana inuenienda, quam

 ſupra

ſupra (206) per ζ indicauimus, ita exprimetur, vt ſit

$$\zeta = C + 1{,}0085272\,p$$

$$-1{,}0085272\left(\begin{matrix} 2k\sin r + \frac{3}{4}kk(1+\frac{5}{6}kk+\frac{7}{16}k^4)\sin 2r + \frac{5}{3}k^3(1+\frac{3}{8}kk)\sin 3r \\ + \frac{5}{32}k^4(1+\frac{3}{5}kk)\sin 4r + \frac{3}{40}k^5\sin 5r + \frac{7}{12}k\sin 6r \end{matrix}\right)$$

vbi $C + 1{,}0085272\,p$ exhibet longitudinem Lunae mediam; quae ſi vocetur $= \xi$, atque in coefficientium partibus minimis pro k ſcribatur valor proximus 0,0545, erit

	log. coeff.	val. in min. ſec.
$\zeta = \xi - 2{,}0170544\,k \sin r$	0,304718	$22675'' = 6°, 17', 55''$
$- 0{,}756770\,kk \sin 2r$	9,878964	$464 = 7, 44$
$- 0{,}336551\,k^3 \sin 3r$	9,527051	$11\frac{1}{2}$
$- 0{,}15786\,k^4 \sin 4r$	9,198282	$\frac{1}{3}$

vnde patet ſuperfluum futurum fuiſſe, ſi ſuperiores expreſſiones vltra quartam poteſtatem ipſius k extendere voluiſſemus.

CAPUT

CAPUT XVII.

INUESTIGATIO ELEMENTORUM MOTUS LUNAE.

§. 272.

Inuentis iam per Theoriam hiſce inaequalitatibus, quibus motus Lunae perturbatur, antequam eas ad computum aſtronomicum accommodare liceat, elementa, quae in eas ingrediuntur, per obſeruationes determinari oportet. Primo ſcilicet ad datam epocham cum longitudo Lunae media, tum eius anomalia media, ac locus nodi medius conſtitui debebit, vt eaedem res inde ad quoduis aliud tempus aſſignari queant. Deinde quoque ex obſeruationibus verus valor excentricitatis lunaris colligi debet, a quo potiſſimum quantitas praecipuarum inaequalitatum pendet. Excentricitas autem orbitae ſolaris pro ſatis certa haberi poterit, cum ſit $\epsilon = 0{,}0168$. Lunae vero excentricitas tam prope iam conſtat, vt inde ſine errore ad quamlibet anomaliam mediam vera ſatis exacte aſſignari poſſit. Etſi enim in anomalia vera error aliquot minutorum primorum committitur, inaequalitates Lunae inde non vltra aliquot minuta ſecunda afficiuntur.

§. 273. Quodſi autem ſtatim quasuis Lunae obſeruationes ad hunc finem adhibere velimus, ob tam ingentem inaequalitatum numerum, inueſtigatio elementorum maxime moleſta redderetur. Quocirca ex obſer-

obſeruationibus eas eligi conueniet, pro quibus numerus inaequalitatum multo fiat minor; dum ſcilicet diſtantia Lunae a ſole ſeu angulus η datum obtinet valorem. Commodiſſimae ergo erunt eae obſeruationes, quae in ipſis momentis coniunctionis vel oppoſitionis ſunt inſtitutae. Accuratas itaque obſeruationes eclipſium lunarium ad hoc negotium adhibebo, quoniam praeter haec tempora, vera vel coniunctionis vel oppoſitionis momenta non ſatis certo ex obſeruationibus colligi licet.

§. 274. Momento autem oppoſitionis verae Lunae et Solis, longitudo Lunae ſex ſignis diſtat a longitudine ſolis, ita vt ſit $\theta = \phi \pm 180°$, ideoque angulus $\eta = 180°$. Poſito autem pro η hoc valore longitudo Lunae vera ϕ ex media ξ per ſequentes formulas definietur, in quas formulae hactenus inuentae abeunt:

$$
\begin{array}{llll}
\phi = \xi & -2{,}0170544\,k\,\mathrm{ſin}\,r & -0{,}756770\,kk\,\mathrm{ſin}\,2r & -0{,}33655\,k^3\,\mathrm{ſin}\,3r \\
 & +0{,}0101460 & +0{,}004200 & \\
 & +0{,}4202260 & -0{,}573280 & \\
 & +0{,}0049920 & +0{,}003180 & \\
 & -0{,}0052860 & +0{,}150830 & \\
 & -0{,}000860 & -0{,}000002 & \\
 & +1{,}1959\,v & & \\
 & -0{,}0757\,v & & \\
 & +0{,}00932\,f & &
\end{array}
$$

$$
\begin{array}{l}
+(0{,}201385+0{,}021889-0{,}016368-0{,}3959\,v+4{,}1738\,v)\,e\,\mathrm{ſin}\,s \\
+(0{,}06645 - 0{,}02332 + 0{,}00840)\,ee\,\mathrm{ſin}\,2s \\
+(0{,}74760 - 0{,}81430 - 0{,}01420)\,ek\,\mathrm{ſin}\,(r-s) \\
+(-0{,}61850 - 0{,}23960 - 0{,}00610)\,ek\,\mathrm{ſin}\,(r+s)
\end{array}
$$

—

$$- (0{,}002823 + 0{,}000910) f \sin (2\varphi - 2\pi)$$
$$- 0{,}000028 f \sin (4\varphi - 4\pi)$$
$$+ (0{,}01521 - 0{,}00121) fk \sin (2\varphi - 2\pi - r)$$
$$+ 0{,}79079 fkk \sin (2\varphi - 2\pi - 2r)$$

§. 275. Cum igitur ſit $f = 1{,}09375$, et $\bar{\nu} = \frac{1}{11}$, erit has formulas colligendo:

$$\varphi = \xi - 1{,}572993 k \sin r - 1{,}17186 kk \sin 2r - 0{,}3365 k^3 \sin 3r$$
$$+ 0{,}21998 e \sin s + 0{,}05123 ee \sin 2s - 0{,}0809 ek \sin (r - s)$$
$$- 0{,}8642 ek \sin (r + s)$$
$$- 0{,}002989 \sin (2\varphi - 2\pi) + 0{,}01531 k \sin (2\varphi - 2\pi - r)$$
$$- 0{,}000031 \sin (4\varphi - 4\pi)$$
$$+ 0{,}86493 kk \sin (2\varphi - 2\pi - 2r)$$

et cum ſit $e = 0{,}0168$, erit hoc valore ſubſtituto:

$$\varphi = \xi - 1{,}572993 k \sin r - 1{,}17186 kk \sin 2r - 0{,}3365 k^3 \sin 3r$$
$$+ 0{,}003697 \sin s + 0{,}000014 \sin 2s - 0{,}001359 k \sin (r - s)$$
$$- 0{,}014523 k \sin (r + s)$$
$$- 0{,}002989 \sin (2\varphi - 2\pi) + 0{,}01531 k \sin (2\varphi - 2\pi - r)$$
$$- 0{,}000031 \sin (2\varphi - 4\pi)$$
$$+ 0{,}86493 kk \sin (2\varphi - 2\pi - 2r)$$

§. 276. Aſſumta iam hypotheſi quapiam non nimis a vero aberrante, vnde ad datum quoduis tem-

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pus definiri poſſit tam longitudo lunae, quam eius anomalia media, ex qua praeterea ope excentricitatis proxime cognitae anomalia vera aſſignari queat: haec elementa correctione indigebunt, quam ex obſeruationibus elici oporteat. Ponamus ergo longitudinem mediam ex tabulis deſumtam augeri debere m minutis ſecundis. Tum vero excentricitas ſuppoſita, quae ſit $= 0,0545$, augeri debeat $\frac{n}{10000}$, vt ſit $k = 0,0545 + \frac{n}{10000}$: ipſa vero anomalia vera tabularis, quae ſit $= v$, augmentum requirat μ minutorum ſecundorum, vt ſit $r = v + \mu''$: eritque ſin $r =$ ſin $v + \mu''$ coſ v; ſin $2r =$ ſin $2v + 2\mu''$ coſ $2v$; et ſin $3r =$ ſin $3v$ in terminis enim minimis haec correctio praetermitti poterit.

§. 277. Quod ſi haec omnia in minuta ſecunda conuertantur, prodibit longitudo lunae vera

$$\Phi = \text{Long. med.} + m''$$
$$- 17682'' \text{ ſin } v - 32,445\, n'' \text{ ſin } v - 0,085728\, \mu'' \text{ coſ } v$$
$$- 718'' \text{ ſin } 2v - 2,635\, n'' \text{ ſin } 2v - 0,006962\, \mu'' \text{ coſ } 2v$$
$$- 11'' \text{ ſin } 3v + 762'' \text{ ſin } s + 3'' \text{ ſin } 2s$$
$$- 15'' \text{ ſin } (r-s) - 163'' \text{ ſin } (r+s)$$
$$- 616'' \text{ ſin } (2\Phi-2\pi) + 172'' \text{ ſin } (2\Phi-2\pi-r)$$
$$- 6'' \text{ ſin } (4\Phi-4\pi) + 530 \text{ ſin } (2\Phi-2\pi-2r)$$

Cum

Cum autem poſtremus terminus ſit ſuſpectus, loco eius coefficientis 530 malumus ponere coefficientem indefinitum $100y$, atque ex obſeruationibus valorem ipſius y indagare. Deinde ſit error anomaliae verae i minutorum primorum, vt calculus commodior reddatur, atque ob $\mu = 60i$, neglectis terminis minimis erit:

$$\begin{aligned} \varphi = \text{Long. med.} + m'' \\ -17682'' \sin v - 32,445\, n'' \sin v - 5,143\, i'' \cos v \\ - 718'' \sin 2v - 2,635\, n \sin 2v - 0,417\, i \cos 2v \\ + 762 \sin s - 15 \sin (r-s) - 163 \sin (r+s) \\ - 616 \sin (2\varphi - 2\pi) + 172'' \sin (2\varphi - 2\pi - r) \\ + 100y \sin (2\varphi - 2\pi - 2r) \end{aligned}$$

§. 278. Oblata autem obſeruatione eclipſis lunae, quaeratur primum momentum medium huius eclipſis, pro quo colligatur longitudo ſolis, itemque longitudo nodi aſcendentis. Punctum autem ſoli oppoſitum nondum erit longitudo lunae vera in ecliptica; verumtamen longitudo lunae pro hoc momento eclipſis medio inueniri poterit ope ſequentis tabellae.

Subtrahatur longitudo nodi a longitudine ſolis, et aequatio tabulae ſecundum titulos adſcriptos applicetur puncto ſoli oppoſito in ecliptica.

gr.	{ O Sign. VI. Sign. } ſubtrahe	
0	0′, 0″	30
1	0, 32	29
2	1, 6	28
3	1, 39	27
4	2, 12	26
5	2, 45	25
6	3, 17	24
7	3, 49	23
8	4, 21	22
9	4, 53	21
10	5, 24	20
11	5, 56	19
12	6, 26	18
	adde { V Sign. XI. Sign. }	gr.

§. 279. Quanquam autem hoc momento, ad quod lunae longitudinem hinc colligimus, non vera lunae oppoſitio exiſtit, ſed luna ſecundum longitudinem a puncto ſoli oppoſito diſtat particula, quam haec tabula monſtrat; tamen tuto pro hoc momento ex formula noſtra longitudinem lunae inueſtigare poterimus, viſuri, quam

quam exacte ea conueniat cum longitudine eius ad hoc tempus ex obſeruatione concluſa. Cum enim luna hoc tempore nunquam vltra 5′ a vero oppoſitionis loco diſtet, ſi formula noſtra generali vti vellemus, foret angulus η minor 5 minutis primis; vnde facile perſpicitur, diſcrimen in loco lunae inde oriundum vix vnquam 12″ eſſe ſuperaturum. Quoniam itaque medium cuiusque eclipſis momentum ipſum tam accurate definiri nequit, vt non error dimidii minuti primi ſit pertimeſcendus, ſuperfluum ſane foret in calculo ad iſtiusmodi minutias attendere.

§. 280. Hanc ob cauſam quoque ex calculo, quem inibo, non ſummam praeciſionem expectari conueniet; quia ipſae obſeruationes, quibus vtar, non plenae accurationis ſunt capaces. Plus igitur me non effecturum confido, quam vt ſatis prope tam excentricitatem orbitae lunaris, quam longitudinem et anomaliam lunae mediam ad datam epocham definiam. Quod cum fuerit factum maiori confidentia theoriam ad quasuis alias obſeruationes transferre licebit; quae ſi nullis erroribus fuerint inquinatae, non admodum erit difficile reliquas elementorum correctiones, quibus formulae noſtrae ſunt innixae, inde concludere. Imprimis autem hic calculus veram excentricitatem orbitae lunaris ſatis exacte manifeſtabit, vt deinceps accuratius pro quauis anomalia media conuenientem anomalam veram definire valeamus. Hunc igitur in finem nonnullas eclipſes lunares Pariſiis inſtitutas calculo ſubiiciam.

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§. 281. Primae igitur eclipſis medium contigiſſe reperio Pariſiis A. 1712. Jan. $23^d, 7^h, 55', 16''$ temp. medio. Pro quo momento colligitur:

Longitudo ſolis θ . . .	$10^s, 3^\circ, 0', 54''$
Anomalia vera ſolis s . .	6, 24, 25, 13
Deinde ex tabulis meis-	
Longitudo lunae media .	4, 7, 18, 55
Anomalia lunae media .	2, 0, 18, 20
Anomalia lunae vera $v =$	1, 25, 6, 27
Longitudo nodi vera $\pi =$	9, 24, 34, 32
Diſt. nodi a ſole $\theta - \pi =$	0, 8, 26, 22
Hinc aequatio loci lunae .	— — 4, 33
Ergo longitudo lunae vera $\Phi =$	4, 2, 56, 21

§. 282. Hinc calculus ſequenti modo inſtituetur:

$v =$	1, 25, 6, 27	;	ſin $v = +$ ſin $55^\circ, 6', 27''$
			coſ $v = +$
$2v =$	3, 20, 12, 54	;	ſin $2v = +$ ſin 69, 47, 6
			coſ $2v = -$
$s =$	6, 24, 25, 13	;	ſin $s = -$ ſin 24, 25, 13
$v-s =$	7, 0, 41, 14	;	ſin $= -$ ſin 30, 41, 14
$v+s =$	8, 19, 31, 40	;	ſin $= -$ ſin 79, 31, 40
$\Phi-\pi =$	6, 8, 21, 49		
$2\Phi-2\pi =$	0, 16, 43, 38	;	ſin $= +$ ſin 16, 43, 38
$r =$	1, 25, 6, 27		
$2\Phi-2\pi-r =$	10, 21, 37, 11	;	ſin $= -$ ſin 38, 22, 49
$2\Phi-2\pi-2r =$	8, 26, 30, 44	;	ſin $= -$ ſin 86, 30, 44

+

+	9, 91393	+	9, 9139	+	9, 7575
—	4, 24753	−	1, 5111	−	0, 7104
—	4, 16146	−	1, 4250n	−	0, 4679i
+	9, 9724	+	9, 9724	−	9, 5385
—	2, 8561	−	0, 4208	−	9, 6201
—	2, 8285	−	0, 3932n	+	9, 1586i
—	9, 6163	−	9, 7078	−	9, 9927
+	2, 8819	−	1, 1761	−	2, 2122
—	2, 4982	+	0, 8839	+	2, 2049
+	9, 4588	−	9, 7930		
—	2, 7896	+	2, 2355	−	99, 8y
—	2, 2484	−	2, 0285		

aeq. +	aeq. —	
+ 8	− 14493	− 26, 6n
+ 160	− 674	− 2, 5n
+ 168	− 315	− 2, 9i
— 15766	− 177	+ 0, 1i
— 15598	− 107	− 99, 8y
— 259′, 58″	− 15766	

— 4°, 19, 58 . . aequatio.

Long. med. 4, 7, 18, 55 + m
aeq. − 4, 19, 58
4, 2, 58, 57 − 29, 1n − 2, 8i − 99, 8y = 4, 2, 56, 21
4, 2, 56, 21

Ergo o = 2, 36 − 29, 1n − 2, 8i − 99, 8y + m

§. 283.

§. 283. Secundae eclipſi medium contigit: Pariſiis A. 1713. Dec. 1^d, 15^h, $26'$, $34''$ temp. med. Pro quo momento colligitur

Longitudo Solis θ =	.	8, 9, 53, 40
Anomalia vera Solis s =	.	5, 1, 46, 43
Longitudo Lunae media	.	2, 5, 2, 26
Anomalia Lunae media	.	9, 12, 27, 42
Anomalia Lunae vera v =	.	9, 18, 24, 49
Longitudo nodi π =	.	8, 17, 46, 10
Diſtantia Solis a nodo	.	11, 22, 7, 30
Aequatio loci Lunae	.	+ 4, 17
Longitudo Lunae vera Φ =		2, 9, 57, 57

§. 284. Hinc calculus ſequens inſtituatur:

v	= 9, 18, 24, 49	; ſin v = − ſin 71, 35, 11 coſ v = +
$2v$	= 7, 6, 49	; ſin $2v$ = − ſin 36, 49 coſ $2v$ = −
s	= 5, 1, 46	; ſin s = + ſin 28, 14
$v - s$	= 4, 16, 39	; ſin = + ſin 43, 21
$v + s$	= 2, 20, 11	; ſin = + ſin 80, 11
$\Phi - \pi$	= 11, 22, 12	
$2\Phi - 2\pi$	= 11, 14, 24	; ſin = − ſin 15, 36
r	= 9, 18, 25	
$2\Phi - 2\pi - r$	= 1, 25, 59	; ſin = + ſin 55, 59
$2\Phi - 2\pi - 2r$	= 4, 7, 34	; ſin = + ſin 52, 26

— 9,97717	− 9,9772	+ 9,4996
— 4,24753	− 1,5111	− 0,7104
+ 4,22470	+ 1,4883*n*	− 0,2100 *i*

— 9,7776	− 9,7776	− 9,9034
— 2,8561	− 0,4208	− 9,6201
+ 2,6337	+ 0,1984*n*	+ 9,5235 *i*

+ 9,6749	+ 9,8366	+ 9,9936
+ 2,8819	− 1,1761	− 2,2122
+ 2,5568	− 1,0127	− 2,2058

— 9,4296	+ 9,9185	+ 79, 2*y*
— 2,7866	+ 2,2355	
+ 2,2192	+ 2,1530	

aeq. aff.	aequat.	
+ 16776	− 10	+ 30, 8*n*
+ 430	− 161	+ 1, 6*n*
+ 360	− 171	− 1, 6 *i*
+ 166		+ 0, 3 *i*
+ 143		
+ 17875		
−171		
+ 17704		
+ 2961′,4″	aequatio	

Long. media =	2, 5, 2, 26	+ *m*
aeq.	+ 4, 55, 4	
Long. vera	2, 9, 57, 30,	+ *m*
obſ.	2, 9, 57, 57	

Ergo *o* = − 0′,27″ + *m* + 32, 4*n* − 1,3*i* + 79,2*y*

§. 285. Tertiae eclipſis medium contigit Pariſiis A. 1717 Mart. $26^d, 15^h, 21', 20''$ temp. med. Pro quo tempore colligitur

Longitudo ſolis vera $\theta =$	$0^s, 6^\circ, 19', 56''$,
Anomalia ſolis vera $s =$	8, 28, 0, 17
Longitudo media lunae . .	6, 1, 37, 2
Anomalia media lunae . .	8, 24, 7, 21
Anomalia lunae vera $v =$	9, 0, 19, 10.
Longitudo nodi vera $\pi =$	6, 13, 30, 22
Diſtantia nodi a ſole . .	5, 22, 49, 29
aeq. pro loco lunae	+ 3, 57
Ergo longitudo lunae vera $\phi =$	6, 6, 23, 53

§. 286. Calculus igitur ita ſe habebit

$v =$	9, 0, 19, 10	; ſin $v = -$ ſin $89^\circ, 40', 50''$
		coſ. $= +$
$2v =$	6, 0, 38,	; ſin$2v = -$ ſin $0^\circ, 38'$
		coſ $= -$
$s =$	8, 28, 0	; ſin $s = -$ ſin $88^\circ, 0'$
$v - s =$	0, 2, 19	; ſin $= +$ ſin 2°, 19
$v + s =$	5, 28, 19	; ſin $= +$ ſin 1, 41
$\phi - \pi =$	5, 22, 53	
$2\phi - 2\pi =$	11, 15, 46	; ſin $= -$ ſin 14, 14
$r =$	9, 0, 19	
$2\phi - 2\pi - r =$	2, 15, 27	; ſin $= +$ ſin 75, 27
$2\phi - 2\pi - 2r =$	5, 15, 8	; ſin $= +$ ſin 14, 52

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— 9,99999	— 10,0000	+ 7,7425
— 4,24753	— 1,5111	— 0,7104
+ 4,24752	+ 1,5111*n*	— 8,4526*i*
— 8,0435	— 8,0435	— 9,9999
— 2,8561	— 0,4208	— 9,6201
+ 0,8996	+ 8,4643*n*	+ 9,6200*i*
— 9,9997	+ 8,6066	+ 8,4680
+ 2,8819	— 1,1761	— 2,2122
— 2,8816	— 9,7827	— 0,0802
— 9,3907	+ 9,9858	+ 25,65*y*
— 2,7893	+ 2,2355	
+ 2,1893	+ 2,2213	

aeq. aff.	aeq. neq.	
+ 17682	— 762	+ 32, 4*n*
+ 8	— 1	+ 0, 0*n*
+ 152	— 4	— 0, 0*i*
+ 166	— 767	+ 0, 2*i*
+ 18008	+ 18008	
	+ 17241	
	+ 287,21	
	+ 4,47,21 aequatio	

Long. media ☾ = 6, 1, 37, 2
aeq. + 4, 47, 21
Long. ☽ vera = 6, 6, 24, 23
obſ. = 6, 6, 23, 53
Ergo • = + 30 + *m* + 32,4*n* + 0,2*i* + 25,65*y*

§. 287. Quartae eclipſis medium erat Pariſiis A. 1718 Sept. $9^d, 8^h, 1', 1''$ temp. medio Pro quo tempore colligitur

Longitudo ſolis vera $\theta =$	5, 16, 40, 58
Anomalia ſolis vera $s =$	2, 8, 19, 59
Longitudo lunae media	11, 17, 25, 16
Anomalia lunae media	0, 10, 41, 28
Anomalia lunae vera $v =$	0, 9, 36, 52
Longitudo nodi vera $\pi =$	5, 15, 59, 35
Diſtantia nodi a ſole	0, 0, 41, 23
aeq. pro loco lunae	—— 22
Longitudo lunae obſ. $\Phi =$	11, 16, 40, 36

§. 288. Calculus ergo ſequens habebitur.

$v =$	0, 9, 36, 52	; ſin $v = +$ ſin 9, 36, 52
		coſ $= +$
$2v =$	0, 19, 14	; ſin $2v = +$ ſin 19, 14
		coſ $= +$
$s =$	2, 8, 20	; ſin $s = +$ ſin 68, 20
$v-s =$	10, 1, 17	; ſin $= -$ ſin 58, 43
$v+s =$	2, 17, 57	; ſin $= +$ ſin 77, 57
$\Phi-\pi =$	0, 0, 41	
$2\Phi-2\pi =$	0, 1, 22	; ſin $= +$ ſin 1, 22
$r =$	0, 9, 37	
$2\Phi-2\pi-r =$	11, 21, 45	; ſin $= -$ ſin 8, 15
$2\Phi-2\pi-2r =$	11, 12, 8	; ſin $= -$ ſin 17, 52

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+ 9,22274	+ 9,2227	+ 9,9938
— 4,24753	— 1,5111	— 0,7104
— 3,47027	— 0,7338 *n*	— 0,7042 *i*
+ 9,5177	+ 9,5177	+ 9,9750
— 2,8561	— 0,4208	— 9,6201
— 2,3738	— 9,9385 *n*	— 9,5951 *i*
+ 9,9682	— 9,9318	+ 9,9903
+ 2,8819	— 1,1761	— 2,2122
+ 2,8501	+ 1,1079	— 2,2025
+ 8,3775	— 9,1568	— 30,68 *y*
— 2,7896	+ 2,2355	
— 1,1671	— 1,3923	

	aeq. aff.	aeq. neg.	
	+ 708	— 2953	— 5, 4 *n*
	+ 13	— 237	— 0, 8 *n*
	+ 721	— 159	— 5, 1 *i*
	— 3389	— 15	— 0, 4 *i*
	— 2668	— 25	
aeq. =	— 44′, 28″	— 3389	

Long. ☾ med. 11, 17, 25, 16
aeq. — 44, 28

Long. vera 11, 16, 40, 48
obſ. 11, 16, 40, 36

Ergo $o = +12 + m - 6,2n - 5,5i - 30,68y$

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§. 289. Quintae eclipſis medium erat: Pariſiis A. 1719 Aug. $29^d, 8^h, 33', 19''$ temp. med. Pro quo tempore colligitur:

Longitudo ſolis vera θ =	5, 5, 47, 14
Anomalia ſolis vera s =	1, 27, 25, 24
Longitudo lunae media .	11, 2, 9, 40
Anomalia lunae media . .	10, 15, 59, 25
Anomalia lunae vera v =	10, 20, 5, 19
Longitudo nodi vera π =	4, 27, 44, 39
Diſtantia nodi a ſole =	0, 8, 2, 35
Aequ. pro loco lunae . .	— — 4, 22
Long. lunae obſeruata . .	11, 5, 42, 52

§. 290. Calculus ergo ita ſe habebit:

v = 10, 20, 5, 19	; ſin = − ſin 39, 54, 41 coſ = +
$2v$ = 9, 10, 11	; ſin $2v$ = − ſin 79, 49 coſ = +
v = 10, 20, 5	
s = 1, 27, 25	; ſin s = + ſin 57, 25
$v-s$ = 8, 22, 40	; ſin = − ſin 82, 40
$v+s$ = 0, 17, 30	; ſin = + ſin 17, 30
$\Phi-\pi$ = 0, 7, 58	
$2\Phi-2\pi$ = 0, 15, 16	; ſin = + ſin 15, 56
r = 10, 20, 5	
$2\Phi-2\pi-r$ = 1, 25, 15	; ſin = + ſin 55, 51
$2\Phi-2\pi-2r$ = 3, 5, 46	; ſin = + ſin 84, 14

−	9,80726	−	9,8073	+	9,8849
−	4,24753	−	1,5111	−	0,7104
+	4,05479	+	1,3184*n*	−	0,5953*i*
−	9,9931	−	9,9931	+	9,2475
−	2,8561	−	0,4208	−	9,6201
+	2,8492	+	0,4139*n*	−	8,8676*i*
+	9,9256	−	9,9964	+	9,4781
+	2,8819	−	1,1761	−	2,2122
+	2,8075	+	1,1725	−	1,6903
+	9,4386	+	9,9178	+	99,5*y*
−	2,7896	+	2,2355		
−	2,2282	+	2,1533		

aeq. aff.	aeq. neg.	
+ 11345	− 49	+ 20,8*n*
+ 707	− 170	+ 2,6*n*
+ 642	− 219	− 3,9*i*
+ 15	+ 12851	− 0,1*i*
+ 142	+ 12632	
+ 12851	210, 32	
	+ 3, 30, 32	aequatio

Long. lunae med.	11, 2, 9, 40
aeq.	+ 3, 30, 32
Long. lunae vera	11, 5, 40, 12
obſ.	11, 5, 42, 52

$= -2,40 + m + 23,4n - 4,0i + 99,5y$

§. 291. Sextae eclipſis medium erat Pariſiis A. 1722. Jun. $28^d, 13^h, 58', 41''$ temp. med. Pro quo tempore habetur:

Longitudo ſolis vera $\theta =$	3, 6, 51, 7
Anomalia ſolis vera $s =$	11, 28, 26, 56
Longitudo lunae media .	9, 9, 31, 50
Anomalia lunae media .	4, 28, 8, 18
Anomalia lunae vera $v =$	4, 24, 39, 53
Longitudo nodi vera $\pi =$	3, 2, 36, 2
Diſtantia nodi a ſole . .	0, 4, 15, 5
Aequatio loci lunae . .	— 2, 20
Longitudo lunae obſeruata	9, 6, 48, 47

§. 292. Calculus ergo ita ſe habebit

$v =$	4, 24, 39, 53	; ſin $v = +$ ſin 35°, 20′, 7″ coſ $v = -$
$2v =$	9, 19, 20	; ſin $2v = -$ ſin 70, 40 coſ $2v = +$
$r =$	4, 24, 40	
$s =$	11, 28, 27	; ſin $s = -$ ſin 1, 33
$r-s =$	4, 26, 13	; ſin $= +$ ſin 33, 47
$r+s =$	4, 23, 7	; ſin $= +$ ſin 36, 53
$\Phi-\pi =$	0, 4, 13	
$2\Phi-2\pi =$	0, 8, 26	; ſin $= +$ ſin 8, 26
$r =$	4, 24, 40	
$2\Phi-2\pi-r =$	7, 13, 46	; ſin $= -$ ſin 43, 46
$2\Phi-2\pi-2r =$	2, 19, 6	; ſin $= +$ ſin 79, 6

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+	9,76220	+	9,7622	−	9,9116
−	4,24753	−	1,5111	−	0,7104
−	4,00973	−	1,2733n	+	0,6220i
−	9,9748	−	9,9748	+	9,5199
−	2,8561	−	0,4208	−	9,6201
+	2,8309	+	0,3956n	−	9,1400i
−	8,4321	+	9,7451	+	9,7783
+	2,8819	−	1,1761	−	2,2122
−	1,3140	−	0,9212	−	1,9905
+	9,1663	−	9,8399	+	98,2y
−	2,7896	+	2,2355		
−	1,9559	−	2,0754		

	aeq. aff.	aeq. neg.		
	+ 678	− 10227	−	18,8n
	− 10563	− 21	+	2,5n
	− 9885	− 8		
	164,45	− 98	+	4,2i
aeq. =	− 2,44,45	− 90	−	0,1i
		− 119		
		−101563		

Long. lun. med.	9, 9, 31, 50
	− 2, 44, 45
Long. lun.	9, 6, 47, 5
obſ.	9, 6, 48, 47

Ergo $o = -1,42 + m - 16,3n + 4,1i + 98,9y$

§. 293. Septimae eclipſis medium obſeruatum eſt Pariſiis A. 1724 Oct. 31^d, 15^h, $34'$, $17''$ temp. med. Pro quo tempore colligitur

Longitudo ſolis vera θ =	7, 8, 56, 1
Anomalia ſolis vera s =	4, 0, 29, 44
Longitudo lunae media	1, 9, 23, 59
Anomalia lunae media	5, 22, 38, 2
Anomalia lunae vera v =	5, 21, 46, 51
Longitudo nodi vera	1, 16, 36, 22
Diſtantia nodi a ſole	5, 22, 19, 39
aequatio loci lunae	+ 4, 10
Long. lunae obſeruata	1, 9, 0, 11

§. 294. Calculus ergo ita ineatur:

v =	5, 21, 46, 51	; ſin v = + ſin 8°, 13′, 9″
		coſ = −
$2v$ =	11, 13, 34,	; ſin $2v$ = − ſin 16°, 26′
r =	5, 21, 47	coſ = +
s =	4, 0, 30	; ſin s = + ſin 59°, 30′
$r-s$ =	1, 21, 17	; ſin = + ſin 51, 17.
$r+s$ =	9, 22, 17	; ſin = − ſin 67, 43
$\Phi-\pi$ =	5, 22, 24	
$2\Phi-2\pi$ =	11, 14, 48	; ſin = − ſin 15, 12
r =	5, 21, 47	
$2\Phi-2\pi-r$ =	5, 23, 1	; ſin = + ſin 6, 59
$2\Phi-2\pi-2r$ =	0, 1, 14	; ſin = + ſin 1, 14

+

+	9, 15520	+	9, 1552	−	9, 9955
−	4, 24753	−	1, 5111	−	0, 7104
−	3, 40273	−	0, 6663*n*	+	0, 7059*i*
−	9, 4516	−	9, 4516	+	9, 9819
−	2, 8561	−	0, 4208	−	9, 6201
+	2, 3077	+	9, 8724*n*	−	9, 6020*i*
+	9, 9353	+	9, 8922	−	9, 9663
+	2, 8819	−	1, 1761	−	2, 2122
−	2, 8172	−	1, 0683	+	2, 1785
−	9, 4186	+	9, 0849	+	2, 15*y*
−	2, 7896	+	2, 2355		
+	2, 2082	+	1, 3024		

aeq. aff.	aeq. neg.	
+ 203	− 2528	− 4, 6*m*
+ 657	− 11	− 7, 5*n*
+ 151	− 2539	
+ 161	+ 1193	+ 5, 1*i*
+ 21	− 1346	− 0, 4*i*
+ 1193	− 22, 26	aequatio

Long. ☽ med. 1, 9, 23, 59
— 22, 26

Long. calc. 1, 9, 1, 33
Long. obſ. 1, 9, 0, 11

$o = + 1{,}22 + m - 3{,}9n + 4{,}7i + 2{,}15y$

§. 295. Octauae eclipſis medium obſeruatum eſt Pariſiis A. 1729. Febr. $13^d, 9^h, 6', 56''$ temp. med. Pro quo tempore colligitur:

Longitudo ſolis vera θ	=	10^s,	25°,	$13'$,	$23''$
Anomalia ſolis vera s	=	7,	16,	43,	34
Longitudo lunae media	.	5,	0,	5,	27
Anomalia lunae media	.	3,	18,	53,	24
Anomalia lunae vera v	=	3,	12,	54,	9
Longitudo nodi vera π	=	10,	24,	4,	30
Diſtantia nodi a ſole	=	0,	1,	8,	53
aequatio pro long. lunae	.	.	−	0,	37
Longitudo lunae obſeruata		4,	25,	12,	46

§. 296. Calculus ergo ita ſe habebit:

$v = 3, 12, 54, 9$	;	$\text{ſin } v = +\text{ſin } 77^\circ, 5', 51''$
		$\text{coſ } v = -$
$2v = 6, 25, 48$	;	$\text{ſin } 2v = -\text{ſin } 25, 48$
$r = 3, 12, 54$		$\text{coſ } 2v = -$
$s = 7, 16, 44$	;	$\text{ſin } s = -\text{ſin } 46, 44$
$r-s = 7, 26, 10$	;	$\text{ſin } = -\text{ſin } 56, 10$
$r+s = 10, 29, 38$	;	$\text{ſin } = -\text{ſin } 30, 22$
$\Phi-\pi = 0, 1, 8,$		
$2\Phi-2\pi = 0, 2, 16,$	;	$\text{ſin } = +\text{ſin } 2, 16$
$r = 3, 12, 54$		
$2\Phi-2\pi+r = 8, 19, 22$	;	$\text{ſin } = -\text{ſin } 79, 22$
$2\Phi-2\pi-2r = 5, 6, 28$	;	$\text{ſin } = +\text{ſin } 23, 32$

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+	9,98889	+	9,9889	−	9,3488
−	4,24757	−	1,5111	−	0,7104
−	4,23642	−	1,5000*n*	+	0,0502 *i*
−	9,6387	−	9,6387	−	9,9544
−	2,8561	−	0,4208	−	9,6201
+	2,4948	+	0,0595 *n*	+	9,5745 *i*
−	9,8622	−	9,9194	−	9,7037
+	2,8819	−	1,1761	−	2,2122
−	2,7441	+	1,0955	+	1,9159
+	8,5971	−	9,9925	+	39, 9*y*
−	2,7896	+	2,2355		
−	1,3867	−	2,2280		

aeq. aff.	aeq. neg.		
		−	31, 7*m*
+ 312	− 17235	+	1, 1*n*
+ 12	− 555		
+ 82	− 24	+	1, 1 *i*
+ 406	− 169	+	0, 3 *i*
	− 17983		
	+ 406.		
	− 17577		
	− 292, 57		
	− 4,52,57 aequatio		

Long. lunae media	=	5, 0, 5, 27
aeq.		− 4, 52, 57
Long. lunae calc.		4, 25, 12, 30
Long. lunae obſ.		4, 25, 12, 46

$o = -16 + m - 30,6n + 1,4i + 39,9y$

§. 297. Nonae eclipſis medium obſeruatum eſt Pariſiis A. 1729. Aug. 8^d, 13^h, $14'$, $14''$ temp. med. Pro quo tempore reperitur.

Longitudo ſolis vera	$\phi =$	4, 16, 17, 29
Anomalia ſolis vera	$s =$	1, 7, 47, 12
Longitudo lunae media		10, 11, 23, 57
Anomalia lunae media		8, 10, 36, 19
Anomalia lunae vera	$v =$	8, 16, 34, 40
Longitudo nodi vera	$\pi =$	10, 14, 58, 21
Diſtantia nodi a ſole		6, 1, 19, 8
Aequatio pro loco lunae		— 43
Long. lunae obſeruata		10, 16, 16, 46

§. 298. Calculus ergo ita inſtituetur:

v	= 8, 16, 34, 40	; ſin v = − ſin 76, 34, 40
		coſ v = −
$2v$	= 5, 3, 9	; ſin $2v$ = + ſin 26, 51
		coſ $2v$ = −
r	= 8, 16, 35	
s	= 1, 7, 47	; ſin s = + ſin 37, 47
$r - s$	= 7, 8, 48	; ſin = − ſin 38, 48
$r + s$	= 9, 24, 22	; ſin = − ſin 65, 38
$\phi - \pi$	= 6, 1, 19	
$2\phi - 2\pi$	= 0, 2, 38	; ſin = + ſin 2, 38
r	= 8, 16, 35	
$2\phi - 2\pi - r$	= 3, 16, 3	; ſin = + ſin 73, 57
$2\phi - 2\pi - 2r$	= 6, 29, 28	; ſin = − ſin 29, 28

— 9,93797	— 9,9880	— 9,3655
— 4,24755	— 1,5111	— 0,7104
+ 4,23550	+ 1,4991*n*	+ 0,0759*i*
+ 9,6548	+ 9,6548	— 9,9505
— 2,8561	— 0,4208	— 9,6201
— 2,5109	— 0,0756*n*	+ 9,5706*i*
+ 9,7872	— 9,7970	— 9,9595
+ 2,8819	— 1,1761	— 2,2122
+ 2,6691	+ 0,9731	+ 2,1717
+ 8,6622	+ 9,9827	— 49, 2*y*
— 2,7896	+ 2,2355	
— 1,4518	+ 2,2182	

aeq. aff.	aeq. neg.	
+ 17199	— 324	+ 31, 6*n*
+ 467	— 28	— 1, 2*n*
+ 9	— 352	+ 1, 2*i*
+ 148	+ 17988	+ 0, 4*i*
+ 165	+ 17636	
+ 17988	+ 293, 56	
	+ 4, 53, 56	aequatio

Long. ☽ med.	=	10, 11, 23, 57
aeq.		+ 4, 53, 56
Long. ☽ calc.	=	10, 16, 17, 53
Long. ☽ obſ.		10, 16, 16, 46

$$e = +1,7 + m + 30,4n + 1,6i - 49,2y$$

§. 299.

§. 299. Decimae eclipſis medium obſeruatum eſt Pariſiis A. 1731. Jun. $19^d, 13^h, 55', 13''$ temp. med. Pro quo tempore colligitur

Longitudo ſolis vera θ =	2^s,	$28°$,	$5'$,	$41''$
Anomalia ſolis vera s =	11,	19,	48,	47
Longitudo lunae media .	9,	1,	45,	1
Anomalia lunae media .	4,	15,	9,	43
Anomalia lunae vera v =	4,	10,	34,	21
Longitudo nodi vera π =	9,	8,	6,	38
Diſtantia nodi a ſole . .	5,	19,	59,	3
Aequatio pro loco lunae .		+	5,	24
Longitudo lunae obſeruata	8,	28,	11,	5

§. 300. Calculus ergo ita inſtituetur

v =	4, 10, 34, 21 ;	ſin v = + ſin 49, 25, 39
		coſ v = −
$2v$ =	8, 21, 9 ;	ſin $2v$ = − ſin 81, 9
		coſ $2v$ = −
r =	4, 10, 34	
s =	11, 19, 49 ;	ſin s = − ſin 10, 11
$r-s$ =	4, 20, 45 ;	ſin = + ſin 39, 15
$r+s$ =	4, 0, 23 ;	ſin = + ſin 59, 37
$\Phi-\pi$ =	5, 20, 4	
$2\Phi-2\pi$ =	11, 10, 8 ;	ſin = − ſin 29, 52
r =	4, 10, 34	
$2\Phi-2\pi-r$ =	6, 29, 34 ;	ſin = − ſin 29, 34
$2\Phi-2\pi-2r$ =	2, 19, 0 ;	ſin = + ſin 79, 0

+ 9,88057	+ 9,8806	− 9,8131
− 4,24753	− 1,5111	− 0,7104
− 4,12810	− 1,3917*n*	+ 0,2100 *i*
− 9,9948	− 9,9948	− 9,1871
− 2,8561	− 0,4208	− 9,6201
+ 2,8509	+ 0,4156*n*	+ 8,8072 *i*
− 9,2475	+ 9,8012	+ 9,9358
+ 2,8819	− 1,1761	− 2,2122
− 2,1294	− 0,9773	− 2,1480
− 9,5313	− 9,6932	+ 98, 1 *y*
− 2,7896	+ 2,2355	
+ 2,3209	− 1,9287	

	aeq. aff.	aeq. neg.	
	+ 709	− 13431	− 24, 8 *n*
	+ 209	− 135	+ 2, 6 *n*
	+ 918	− 9	+ 3, 3 *i*
	− 13801	− 141	+ 0, 1 *i*
	− 12883	− 85	
	− 214, 43	− 13801	
aeq.	− 3, 34, 43		

Long. lunae media	9, 1, 45, 1
aeq.	− 3, 34, 43
Long. lunae calc.	8, 28, 10, 18,
Long. lunae obſ.	8, 28, 11, 5

Ergo $0 = -47'' + m - 22,2n + 3,4i + 98,1y$

§. 301. Eclipſis undecimae medium obſeruatum eſt Pariſiis A. 1732 Dec. $1^d, 9^h, 48', 23''$ temp. med. Pro quo tempore colligitur:

Longitudo ſolis vera θ =	8, 10, 3, 6
Anomalia ſolis vera s =	5, 1, 29, 50
Longitudo lunae media .	2, 6, 8, 19
Anomalia lunae media . .	7, 19, 24, 12
Anomalia lunae vera v =	7, 24, 19, 39
Longitudo nodi vera π =	8, 10, 41, 14
Diſtantia nodi a ſole =	11, 29, 21, 52
Aequ. pro loco lunae . .	+ 21
Long. lunae obſeruata . .	2, 10, 3, 27

§. 302. Calculus ergo ita ſe habebit:

$v =$	7, 24, 19, 39	; ſin v = − ſin 54, 19, 39
		coſ v = −
$2v =$	3, 18, 39	; ſin $2v$ = + ſin 71, 21
		coſ = −
$r =$	7, 24, 20	
$s =$	5, 1, 30	; ſin s = + ſin 28, 30
$r-s =$	2, 22, 50	; ſin = + ſin 82, 50
$r+s =$	0, 25, 50	; ſin = + ſin 25, 50
$\Phi-\pi =$	11, 29, 22	
$2\Phi-2\pi =$	11, 28, 44	; ſin = + ſin 1, 16
$r =$	7, 24, 40	
$2\Phi-2\pi-r =$	4, 4, 4	; ſin = + ſin 55, 56
$2\Phi-2\pi-2r =$	8, 9, 24	; ſin = + ſin 69, 24

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— 9,90975	—	9,9097	—	9,7657
— 4,24753	—	1,5111	—	0,7104
+ 4,15728	+	1,4208 *n*	+	0,4761 *i*
+ 9,9766	+	9,9766	—	9,5048
— 2,8561	—	0,4208	—	9,6201
— 2,8327	—	0,3974 *n*	+	9,1249 *i*
+ 9,6787	+	9,9969	+	9,6444
+ 2,8819	—	1,1761	—	2,2122
+ 2,5606	—	1,1730	—	1,8566
— 8,3445	+	9,9182	—	93, 6 *y*
— 2,7896	+	2,2355		
+ 1,1341	+	2,1537		

aeq. aff.	aeq. neq.	
+ 14364	— 680	+ 26, 4 *m*
+ 364	— 15	— 2, 5 *n*
+ 14	— 72	+ 3, 0 *i*
+ 142	— 767	+ 0, 1 *i*
+ 14884	+ 14884	
	+ 14117	
	+ 235, 17	
	+ 3, 55, 17	aequatio

Long. lunae media		2, 6, 8, 19
aeq.	+	3, 55, 17
Long. lunae calc.		2, 10, 3, 36
obſ.		2, 10, 3, 27

o = + 9 + *m* + 23, 9 *n* + 3, 1 *i* − 93, 6 *y*

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§. 303. Eclipſis duodecimae medium obſeruatum eſt Pariſiis A. 1736 Mart. $26^d, 12^h, 14', 36''$ temp. med. Pro quo tempore colligitur

Longitudo ſolis vera $\theta =$	0ˢ,	6°,	35′,	42″
Anomalia ſolis vera $s =$	8,	27,	58,	24
Longitudo lunae media	6,	4,	5,	0
Anomalia lunae media	7,	3,	25,	43
Anomalia lunae vera $v =$	7,	7,	2,	56
Longitudo nodi vera $\pi =$	6,	6,	24,	31
Diſtantia nodi a ſole	6,	0,	11,	11
aeq. pro long. lunae	—			6
Longitudo lunae obſ.	6,	6,	35,	36

§. 304. Calculus ergo ita inſtituatur.

$v =$	7, 7, 2, 56	; ſin $v = -$ ſin 57, 2, 56
		coſ $= -$
$2v =$	2, 14, 6	; ſin $2v = +$ ſin 74, 6
		coſ $= +$
$r =$	7, 7, 3	
$s =$	8, 27, 58	; ſin $s = -$ ſin 87, 58
$r - s =$	10, 9, 5	; ſin $= -$ ſin 50, 55
$r + s =$	4, 5, 1	; ſin $= +$ ſin 54, 59
$\phi - \pi =$	6, 0, 11	
$2\phi - 2\pi =$	0, 0, 22	; ſin $= +$ ſin 0, 22
$r =$	7, 7, 3	
$2\phi - 2\pi - r =$	4, 23, 19	; ſin $= +$ ſin 36, 41
$2\phi - 2\pi - 2r =$	9, 16, 16	; ſin $= -$ ſin 73, 44

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— 9,77995	— 9,7799	— 9,9021
— 4,24753	— 1,5111	— 0,7104
+ 4,02748	+ 1,2910*n*	+ 0,6125*i*

+ 9,9831	+ 9,9831	+ 9,4377
— 2,8561	— 0,4208	— 9,6201
— 2,8392	— 0,4039*n*	— 9,0578*i*

— 9,9997	— 9,8900	+ 9,9133
+ 2,8819	— 1,1761	— 2,2122
— 2,8816	+ 1,0661	— 2,1255

+ 7,8061	+ 9,7763	— 96, 0*y*
— 2,7896	+ 2,2355	
— 0,5957	+ 2,0118	

	aeq. aff.	aeq. neg.	
			+ 19, 6*n*
	+ 10653	— 691	— 2, 5*n*
	+ 12	— 761	
	+ 103	— 133	+ 4, 1*i*
	+ 10768	— 4	— 0, 1*i*
	— 1589	— 1589	
	+ 9179		
	+ 152, 59		
aeq. =	+ 2,32,59		

Long. ☾ med.		6, 4, 5, 0
	aeq.	+ 2, 32, 59
Long. ☽ calc.		6, 6, 37, 59
	obſ.	6, 6, 35, 36

$\bullet = + 2', 23'' + m + 17, 1n + 4,0i - 96, 0y$

§. 305. Elipſis decima tertiae medium obſeruatum eſt Pariſiis A. 1736 Sept. $19^d, 14^h, 59', 36''$ temp. med. Pro quo tempore colligitur

Longitudo ſolis vera $\theta =$	$5^s, 27^\circ, 21', 39''$
Anomalia ſolis vera $s =$	2^s, 18, 43, 51
Longitudo lunae media . .	11, 27, 48, 53
Anomalia lunae media . .	0, 7, 25, 42
Anomalia lunae vera $v =$	0, 6, 40, 44
Longitudo nodi vera	5, 27, 15, 4
Diſtantia nodi a ſole . .	0, 0, 6, 35
aeq. pro long. lunae	— 4
Longitudo lunae obſeruata	11, 27, 21, 35

§. 306. Calculus ergo ita inſtituetur

$v =$ 0, 6, 40, 44	; ſin $v = +$ ſin 6, 40, 50
	coſ. $v = +$
$2v =$ 0, 13, 21	; ſin $2v = +$ ſin 13, 21
	coſ $2v = +$
$r =$ 0, 6, 41	
$s =$ 2, 18, 44	; ſin $s = +$ ſin 88, 44
$r - s =$ 9, 17, 57	; ſin $= -$ ſin 72, 3
$r + s =$ 2, 25, 25	; ſin $= +$ ſin 85, 25
$\phi - \pi =$ 0, 0, 6	
$2\phi - 2\pi =$ 0, 0, 12	; ſin $= +$ ſin 0, 12
$r =$ 0, 6, 41	
$2\phi - 2\pi - r =$ 11, 23, 31	; ſin $= -$ ſin 6, 29
$2\phi - 2\pi - 2r =$ 11, 16, 50	; ſin $= -$ ſin 13, 10

+

+	9, 06561	+	9,0656	+	9, 9970
—	4, 24753	—	1,5111	—	0, 7104
—	3, 31314	—	0, 5767n	—	0, 7074i
+	9, 3634	+	9,3634	+	9, 9881
—	2, 8561	—	0, 4208	—	9, 6201
—	2, 2195	—	9,7842n	—	9, 6082i
+	9, 9915	—	9,9783	+	9, 9986
+	2, 8819	—	1, 1751	—	2, 2122
+	2, 8734	+	1, 1534	—	2, 2108
+	7, 5429	—	9,0527	—	22, 6y
—	2, 7896	+	2,2355		
—	0, 3325	—	1,2882		

aeq. aff.		aeq. neg.		—	3, 8n
+	747	—	2057	—	0, 6n
+	14	—	166		
+	761	—	2	—	5, 1 i
—	2406	—	19	—	0, 4 i
—	1645	—	2406		
—	27′,25″	aequatio			

Long. ☽ med.	11, 27, 48, 53
aeq.	— 27, 25
Long. ☽ calc.	11, 27, 21, 28
Long. ☽ obſ.	11, 27, 21, 35

$0 = - 0,7'' + m - 4,4n - 5,5i - 22,6y$

§. 307.

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§. 307. Ex his ergo tredecim eclipſibus nacti ſumus aequationes, ex quibus cum tabularum, quibus ſum vſus, correctiones, tum verus valor aequationis ab angulo $2\Phi - 2\pi - 2r$ pendentis definiri debebit:

Aequationes autem inde ortae ſunt ſequentes.

I. $0 = +156'' + m - 29{,}1n - 2{,}8i - 99{,}8y$

II. $0 = -\ 27 + m + 32{,}4n - 1{,}3i + 79{,}2y$

III. $0 = +\ 30 + m + 32{,}4n + 0{,}2i + 25{,}6y$

IV. $0 = +\ 12 + m - 6{,}2n - 5{,}5i - 30{,}7y$

V. $0 = -160 + m + 23{,}4n - 4{,}0i + 99{,}5y$

VI. $0 = -102 + m - 16{,}3n + 4{,}1i + 98{,}2y$

VII. $0 = +\ 82 + m - 3{,}9n + 4{,}7i + 2{,}1y$

VIII. $0 = -\ 16 + m - 30{,}6n + 1{,}4i + 39{,}9y$

IX. $0 = +\ 67 + m + 30{,}4n + 1{,}6i - 49{,}2y$

X. $0 = -\ 47 + m - 22{,}2n + 3{,}4i - 98{,}1y$

XI. $0 = +\ 9 + m + 23{,}9n + 3{,}1i - 93{,}6y$

XII. $0 = +143 + m + 17{,}1n + 4{,}0i - 96{,}0y$

XIII. $0 = -\ 7 + m - 4{,}4n - 5{,}5i - 22{,}6y$

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§. 308. Hic ſtatim commode euenit, vt errores calculi ab obſeruationibus infra tria minuta prima ſubſiſtant, qui autem infra ſesquiminutum primum deprimuntur, ſimul ac litterae y valor tribuitur vnitati fere aequalis. Hincque ergo cognoſcimus valorem ipſius y, quem quinario maiorem inueneramus, merito nobis fuiſſe ſuſpectum, cum iam perſpiciamus, eum vnitatem ſuperare non poſſe. Quamobrem ponamus $y = 1$, ſeu in formula noſtra pro longitudine lunae ſcribamus terminum $100''$ ſin $(2\Phi - 2\pi - 2r)$. Quod autem ad litteras m, n et i attinet, tentanti mox patebit, quoscunque ipſis valores tribuamus, errores inde non admodum poſſe diminui; interim tamen decem circiter minutis ſecundis diminuentur, ſi ponatur $y = \frac{4}{5}$; $n = \frac{1}{2}$; $i = -3$ et $m = -4$; quo facto errores vix vnum minutum primum ſuperabunt.

CAPUT XVIII.

CONSTITUTIO ELEMENTORUM PRO TABULIS LUNARIBUS.

§. 309.

Tabulae autem, quibus in praecedenti calculo ſum vſus, praebent pro meridiano Pariſino ad epocham 1701 ſeu ad meridiem diei vltimi anni 1700 tempore medio

Longitudinem Lunae mediam 5ˢ, 20°, 19′, 47″
et Anomaliam Lunae mediam 6, 13, 26, 51

Hinc accuratius habebimus haec elementa pro eodem tempore eodemque loco ſcilicet

Longitudinem Lunae mediam 5ˢ, 20°, 19′, 43″
Anomaliam Lunae mediam 6, 13, 24, 0
unde Longitudo Apogei 11, 6, 55, 43

§. 310. Si haec elementa comparemus cum Tabulis aſtronomicis Cel. Caſſini et Monnierii, reperiemus pro eodem tempore et loco

	Caſſini	Monnier
Long. mediam Lunae	5, 20, 18, 19	5, 20, 19, 28
Anom. mediam Lunae	6, 13, 10, 48	6, 13, 13, 2
Long. Apogei	11, 7, 7, 27	11, 7, 6, 26

Hic quidem longitudo media ſatis conuenit cum ea, quam ex obſeruationibus concluſimus; verum anomalia media inuenta ſuperat Caſſinianam 13′, 12″, Monnierianam autem 11′, quod diſcrimen ſatis eſt notabile.

Verum

Verum ſi perpendamus motum lunae a tam multis variisque inaequalitatibus perturbari, mirum ſane non eſt, anomaliam mediam per ſolas obſeruationes accuratius definiri non potuiſſe; praeſertim cum error 15′ in anomalia media commiſſus in loco lunae ad ſummum errorem 1′, 45″ gignere valeat.

§. 311. Excentricitatem autem orbitae lunaris, quam ſtatueram $= 0,0545$ iam $\frac{1}{20000}$ vel $0,00005$ augeri oportet, ita vt nunc ſit excentricitatis valor $k = 0,05455$; qui a ſupra aſſumto tam parum diſcrepat, vt anomalia vera inde ex media collecta pro ſatis exacta haberi poſſit: aequationes autem ab excentricitate pendentes aliquod augmentum capient, quod nunc quidem diligentius definiri oportet. Primum ergo formulam pro longitudine lunae inuentam hinc corrigamus; deinde vero etiam formulas pro diſtantia lunae a terra, pro eius motu momentaneo, et pro loco nodi veraque inclinatione orbitae lunaris ad eclipticam hinc euoluamus.

§. 312. Ante omnia autem oportebit formulam exhibere, cuius ope ex data quauis anomalia lunae media p elicere liceat, conuenientem anomaliam veram r. Ac ſubſtituto quidem pro k vero eius valore nunc invento, coefficientibusque in minuta ſecunda conuerſis, formula ſupra (§. 306) exhibita ſequentem induet formam:

$$r = p - 22495'' \sin p + 766'' \sin 2p - 36'' \sin 3p$$
$$4,352086 \qquad 2,884229 \qquad 1,55630$$

Huius ergo formulae ope haud difficulter tabula computabitur, quae ad ſingulos anomaliae mediae gradus exhibeat valores anomaliae verae.

§. 313. Inuenta autem anomalia vera r, ſi habeatur quoque anomalia vera ſolis s, vna cum angulo η et longitudinibus Φ, θ, π ſaltem proxime, formula longitudinem veram Φ datae mediae ξ reſpondentem exhibens, ſequenti modo habebitur expreſſa:

	log. coeff.	
$\Phi = \xi - 22466''$ ſin r	4,351535	I
$-$ 462 ſin $2r$	2,66456	
$-$ 11 ſin $3r$	1,0518	
$+$ 701 ſin s	2,84572	II
$+$ 4 ſin $2s$	0,602	
$+$ 141 ſin $(r-s)$	2,1492	III
$-$ 118 ſin $(r+s)$	2,0719	IV
$-$ 175 ſin η	2,2430	V
$+$ 2115 ſin 2η	3,32531	
$+$ 4 ſin 3η	0,602	
$-$ 8 ſin 4η	0,903	
$+$ 59 ſin $(\eta-r)$	1,7708	VI
$+$ 352 ſin $(2\eta-2r)$	2,5465	
$-$ 2729 ſin $(2\eta-r)$	3,67477	VII
$-$ 93 ſin $(4\eta-2r)$	1,9685	
$+$ 56 ſin $(2\eta+r)$	1,7482	VIII
$+$ 59 ſin $(4\eta-r)$	1,7708	IX
$-$ 49 ſin $(\eta+s)$	1,6902	X
$-$ 76 ſin $(2\eta-s)$	1,8808	XI
$-$ 57 ſin $(2\eta+s)$	1,7559	XII
$+$ 154 ſin $(2\eta-r+s)$	2,1875	XIII

$+$

+	45 ſin $(2\eta - r - s)$	1,6532	XIV
—	411 ſin $(2\Phi - 2\pi)$	2,6138	XV
—	205 ſin $(2\theta - 2\pi)$	2,3117	XVI
—	6 ſin $(4\theta - 4\pi)$	0,778	
+	187 ſin $(2\Phi - 2\pi - r)$	2,2718	XVII
+	80 ſin $(2\Phi - 2\pi - 2r)$	1,9031	XVIII
—	15 ſin $(2\theta - 2\pi - r)$	1,176	XIX
—	10 ſin $(2\theta - 2\pi + r)$	1,000	XX

§. 314. Inaequalitates has ita diſpoſui, vt eas, quae vna tabula comprehendi poſſunt, coniunctim expoſuerim, quo facilius calculus expediri queat. Hinc igitur patet omiſſis iis inaequalitatibus, quae 10″ non ſuperant, locum lunae per viginti inaequalitates corrigi debere, antequam vera eius longitudo obtineatur.

§. 315. Haec autem expreſſio adhuc iſto defectu laborat, quod pleraeque inaequalitates ipſam lunae longitudinem veram Φ, quae tamen demum quaeritur, inuoluant, ideoque calculus, cum longitudo lunae etiamnunc eſt incognita, commòde expediri non poſſit. Quoniam tamen ſufficit longitudinem lunae proxime tantum noſſe, cum longitudo media per quatuor priores inaequalitates fuerit correcta, ea pro ſequentibus inaequalitatibus loco longitudinis verae vſurpari poterit, ſicque tandem longitudo lunae multo exactior reperietur. Quo facto ſi accuratior deſideretur, omnes inaequalitates poſt 4 priores denuo ad calculum reuocari conueniet, iisque euolutis longitudo lunae vera prodibit, quae nulla amplius correctione indigebit. Interim

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tamen ne calculum per ſe ſatis taedioſum bis repetere opus ſit, non difficulter hanc expreſſionem ita transformare licet, vt locus lunae per quatuor tantum priores inaequalitates correctus ſine errore in ſequentibus loco φ adhiberi poſſit.

§. 316. Cum autem longitudo lunae iam per obſeruationes fuerit cognita, haec expreſſio ſine vlla immutatione ad calculum accommodabitur, vt hoc modo conſenſus theoriae cum veritate exploretur. In inaequalitatibus enim determinandis pro littera φ vbique longitudo lunae obſeruata introducetur, calculoque peracto patebit, quantum locus lunae per calculum definitus etiamnunc diſcrepet ab eius loco vero obſeruato. Atque ſi hoc modo plurimae obſeruationes calculo ſubiiciantur, ex aberrationibus a veritate non ſolum elementa, quibus haec formula innititur, accuratius definire licebit, ſed etiam inaequalitates, quae nondum ſatis certae videntur, inde emendari poterunt. Quin etiam nouae inaequalitates, quas per Theoriam determinare non licuerat, hoc modo forte certius colligi poterunt.

§. 317. Antequam autem huiusmodi calculi ſpecimen exhiberi queat, neceſſe eſt vt aequationem pro loco nodi vero inueniendo ad calculum accommodemus. Formulae autem ſupra (219) exhibitae, ſi pro r ſubſtituamus valorem inuentum $r = p - 2k$ ſin r $- \frac{1}{4}kk$ ſin $2r$, pars: Conſt. $- 0,004053p$ indicabit longitudinem nodi mediam. Hincque longitudo nodi vera erit $\pi =$

		Log.coeff.
$\pi = \text{Long. med.}$	$-\ 107''\ \text{ſin}\ r$	2, 0294
	$-\ 6\ \text{ſin}\ 2r$	0, 778
	$+\ 551\ \text{ſin}\ s$	2, 7411
	$-\ 453\ \text{ſin}\ 2\eta$	2, 6561
	$-\ 129\ \text{ſin}\ (2\eta - r)$	2, 1106
	$-\ 33\ \text{ſin}\ (2\eta + r)$	1, 518
	$+\ 55\ \text{ſin}\ (2\eta - 2r)$	1, 740
	$+\ 420\ \text{ſin}\ (2\Phi - 2\pi)$	2, 6232
	$+\ 98\ \text{ſin}\ (2\Phi - 2\pi - r)$	1, 991
	$+\ 30\ \text{ſin}\ (2\Phi - 2\pi + r)$	1, 477
	$+\ 235\ \text{ſin}\ (2\Phi - 2\pi - 2r)$	2, 3711
	$+\ 5426\ \text{ſin}\ (2\theta - 2\pi)$	3, 73448
	$+\ 75\ \text{ſin}\ (4\theta - 4\pi)$	1, 875
	$-\ 53\ \text{ſin}\ (2\theta - 2\pi - r)$	1, 724
	$+\ 53\ \text{ſin}\ (2\theta - 2\pi + r)$	1, 724
	$-\ 90\ \text{ſin}\ (2\theta - 2\pi - s)$	1, 954
	$-\ 32\ \text{ſin}\ (2\theta - 2\pi + s)$	1, 505.

§. 318. In hoc calculo pleraſque inaequalitates omittere licet, ſiquidem tantum longitudinem lunae inveſtigare ſit propoſitum: manifeſtum enim eſt, etiamſi in loco nodi error plurium minutorum primorum committatur, inde vix errorem aliquot minutorum ſecundorum in longitudinem lunae redundare. Quodſi vero eclipſis cuiuſpiam omnia phaenomena diligenter definire velimus, tum locum nodi exactiſſime cognitum eſſe oportet. Praeterea vero pro latitudine aſſignanda vera inclinatio orbitae lunaris ad eclipticam ex media ι accuratiſſime erit definienda ope huius formulae:

$\iota = \iota$

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ADDITAMENTUM
CONTINENS ALIAS METHODOS
INUESTIGANDI MOTUS LUNAE INAEQUALITATES.

Qui methodum ante deſcriptam accuratius euoluerit, eam quidem in ſe ſpectatam ſatis bonam atque plerisque lunae inaequalitatibus definiendis aptam deprehendet; interim tamen fateri cogor, eam non ſolum maxime eſſe operoſam, ſed etiam ita comparatam, vt plures inaequalitates, quae tamen motum lunae imprimis afficere videntur, non ſatis exacte exhibeat, et quaſi in dubio relinquat. Cauſa huius incertitudinis manifeſto in hoc eſt ſita, quod omnes inaequalitates ita inter ſe ſunt connexae, vt nullius valor verus accurate definiri poſſit, quin ſimul reliquae inaequalitates omnes fuerint cognitae. Cum igitur eiusmodi methodo approximandi ſim vſus, vt primo quasdam inaequalitates tanquam cognitas aſſumſerim, ex quibus deinceps reliquas definiuerim, probe notandum eſt ab his inuentis iterum priores, quae erant aſſumtae, leuem quandam mutationem pati; quae ſi ſtatim ab initio nota fuiſſet, etiam reliquarum valores aliquantillum mutati prodiiſſent: at quaedam inaequalitates adeo ſunt lubricae, vt facta vel minima mutatione in iis, a quibus pendent, inde non exiguam alterationem trahant. Huc imprimis pertinet motus apogei, cuius inueſtigatio omnes omnino inaequa-

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litates implicat, ita vt ſine harum cognitione neutiquam accurate definiri queat.

Cum igitur haec methodus iſtis tantis incommodis ſit obnoxia, aliam maxime diuerſam tentaui viam, quae ab iis eſſet libera, etiamſi negare nequeam, etiam hanc ſuis non carere incommodis, quae tamen prorſus alius ſunt generis. Ex quo confido his duabus diuerſis methodis combinandis haud exiguum fructum in veram motuum lunarium cognitionem eſſe redundaturum. Praecipuum autem diſcrimen verſatur in electione anomaliae, quae in ſuperiore methodo non ita eſt aſſumta, vt diſtantia lunae a terra fieret vel maxima vel minima, ſi anomalia vel $= 0$ vel $= 180°$ ſtatuatur: neque enim differentiale diſtantiae dx euaneſcit, quando ſinus anomaliae in nihilum abit, ſed praeterea etiamnunc ab elongatione ſolis a luna ſeu angulo η pendet. Ita ſecundum hanc methodum neque apogaeum lunae neque perigaeum ibi ſtatuitur, ubi angulus, quem motus lunae directio cum radio vectore facit, eſt rectus; ſed plerumque in alia puncta incidunt, quae ab iis locis, vbi luna terrae vel eſt proxima, vel ab ea maxime remota, notabiliter ſint diuerſa. Etſi autem in hoc calculo non verae lineae abſidum poſitio conſideratur, hinc tamen methodus minime vitioſa eſt reputanda; propterea quod non eſt quaeſtio, quo nomine quaepiam orbitae lunaris puncta appellentur, dummodo cunctae inaequalitates recte exprimantur. Sed quoniam circa has ipſas inaequalitates nonnulla grauiora dubia ſunt orta, haud abs re fore arbitror, et alteram methodum hic proponere.

I. Sit

I.

Sit igitur vt ante: Maſſa ſolis $=\odot$; terrae $=♁$ et lunae $=☽$; atque vis attractiua terrae in diſtantia d vt $\frac{1}{dd}-\frac{1}{bb}$; manente vi ſolis quadrato diſtantiae exacte proportionali. Tum vero ſit

Longitudo lunae $=\varphi$; latitudo $=\psi$; et diſtantia curtata $=x$

Longitudo ſolis $=\theta$; eiusque a terra diſtantia $=y$

Longitudo nodi aſcendentis lunae $=\pi$ et inclinatio ad eclipticam $=\varrho$

ac ponatur breuitatis ergo elongatio lunae a ſole $\varphi-\theta=\eta$ et diſtantia $\sqrt{}(xx\ \text{ſec.}\ \psi^2-2xy\ \text{coſ}\ \eta+yy)=z$.

Quibus poſitis ſupra §. 20. vidimus motum lunae his quatuor aequationibus contineri:

I. $2dxd\varphi+xdd\varphi=-\frac{3}{4}dt^2.\odot\left(\frac{y}{z^3}-\frac{1}{yy}\right)\text{ſin}\ \eta$

II. $ddx-xd\varphi^2=-\frac{1}{2}dt^2(♁+☽)\text{coſ}\ \psi^3\left(\frac{1}{xx}-\frac{1}{bb}\right)$

$$-\frac{1}{2}dt^2.\odot\left(\frac{x-y\ \text{coſ}\ \eta}{z^3}+\frac{\text{coſ}\ \eta}{yy}\right)$$

III. $d\pi=-\frac{1}{2}dt^2.\odot\left(\frac{y}{z^3}-\frac{1}{yy}\right)\frac{\text{ſin}\ (\varphi-\pi)\ \text{ſin}\ (\theta-\pi)}{xd\varphi}$

IV. $dl\,\text{tang}\ \varrho=\frac{d\pi}{\text{tang}\ (\varphi-\pi)}$, et $\text{tang}\ \psi=\text{tg}\ \varrho\ \text{coſ}(\varphi-\pi)$

vbi elementum temporis dt ſumtum eſt pro conſtante.

II.

Quatenus hic motus ſolis ingreditur, is pro regulari atque regulis Kepleri conformi haberi poterit: habebimus ergo

$$2dyd\theta + ydd\theta = o \text{ et } ddy - yd\theta^2 = -\tfrac{1}{2}dt^2.\frac{\odot+\text{♁}}{yy}$$

vnde ſi ponamus orbitae ſolaris:

ſemiparametrum $= c$; excentricitatem $= e$ et anomaliam veram $= u$

erit $y = \frac{c}{1-e\cos u}$; $du = d\theta = \frac{dt}{yy}V\tfrac{1}{2}c(\odot+\text{♁})$

Sit a ſemiaxis tranſuerſus orbitae ſolis, ac tempore $= t$ ſol motu medio abſoluat angulum $= \omega$, quo pro menſura temporis t vtamur: erit ergo $d\omega = \frac{dt}{aa}V\tfrac{1}{2}a(\odot+\text{♁})$

ideoque $\tfrac{1}{2}dt^2 = \frac{a^3d\omega^2}{\odot+\text{♁}}$. At eſt $a = \frac{c}{1-ee}$. Hinc ergo fit

$$du = d\theta = \frac{ad\omega}{yy}Vac = \frac{aad\omega}{yy}V(1-ee) = \frac{d\omega(1-e\cos u)^2}{(1-ee)V(1-ee)}$$

ſicque tam du quam $d\theta$ per elementum $d\omega$ loco temporis introductum expreſſimus. Quia autem maſſa ſolis $\odot$ maſſam terrae ♁ tam enormiter excedit, ſine errore pro $\tfrac{1}{2}dt^2$ ſcribi poterit $\frac{a^3d\omega^2}{\odot}$, eruntque noſtrae aequationes pro luna:

$$\text{I.}\quad 2dxd\Phi + xdd\Phi = -a^3d\omega^2\left(\frac{y}{z^3} - \frac{1}{yy}\right)\sin\eta$$

$$\text{II.}\quad ddx - xd\Phi^2 = -\frac{a^3(\text{♁}+☽)d\omega}{\odot}\cos\psi^3\left(\frac{1}{xx} - \frac{1}{bb}\right)$$

$$-a^3d\omega^2\left(\frac{x-y\cos\eta}{z^3} + \frac{\cos\eta}{yy}\right)$$

III. dx

III. $d\pi = -\frac{a^3 d\omega^2}{x d\Phi}\left(\frac{y}{z^3}-\frac{1}{yy}\right)$ ſin $(\Phi-\pi)$ ſin $(\theta-\pi)$

IV. $d.l\,\text{tang}\,\rho = \frac{d\pi}{\text{tg}\,(\Phi-\pi)}$; atque ob $\text{tg}\,\psi = \text{tg}\,\rho\,\text{cſ}(\Phi-\pi)$, habebitur proxime $\text{coſ}\,\psi^3 = 1-\frac{3}{4}\text{tg}\,\rho^2-\frac{3}{4}\text{tg}\,\rho^2\text{cſ}\,2(\Phi-\pi)$.

III.

Incipiamus a duabus aequationibus prioribus, ac ponamus breuitatis gratia

$$a^3\left(\frac{y}{z^3}-\frac{1}{yy}\right)\text{ſin}\,\eta = \text{M} \quad \text{et}$$

$$\frac{a^3(☿+☽)}{\odot}\text{coſ}\,\psi^3\left(\frac{1}{xx}-\frac{1}{bb}\right)+a^3\left(\frac{x-y\,\text{coſ}\,\eta}{z^3}+\frac{\text{coſ}\,\eta}{yy}\right)=\frac{\text{A}}{xx}+\text{N}$$

quandoquidem haec poſterior expreſſio terminum inuoluit formae $\frac{\text{A}}{xx}$ prae ceteris incomparabiliter maiorem; atque habebimus has duas aequationes:

$$2dx\,d\Phi + x\,dd\Phi = -\text{M}d\omega^2 \quad \text{et} \quad ddx - x\,d\Phi^2 = -\frac{\text{A}d\omega^2}{xx}-\text{N}\,d\omega^2$$

quarum prior per $2x^3 d\Phi$ multiplicata ob $d\omega$ conſtans habebit integrale:

$$x^4\,d\Phi^2 = -2d\omega^2\int \text{M}x^3\,d\Phi$$

Tum prior multiplicata per $2x\,d\Phi$ addatur ad poſteriorem per $2dx$ multiplicatam, eritque, aggregatum:

$$2x\,dx\,d\Phi^2 + 2xx\,d\Phi\,dd\Phi + 2dx\,ddx = -2\text{M}x\,d\omega^2 d\Phi \frac{-2\text{A}\,d\omega^2 dx}{xx} - 2\text{N}\,d\omega^2 dx$$

Cuius integrale erit:

$$dx^2 + xx\,d\Phi^2 = +\frac{2\text{A}\,d\omega^2}{x} - 2d\omega^2\int(\text{M}x\,d\Phi+\text{N}\,dx)$$

IV.

Ponantur formulae integrales, quae in his expreſſionibus inſunt:

$$-\int M x^3 d\Phi = P \quad \text{et} \quad -\int (M x d\Phi + N dx) = Q$$

vt habeamus has duas aequationes:

$$x^4 d\Phi^2 = 2 P d\omega^2 \quad \text{et} \quad dx^2 + xx d\Phi^2 = \frac{2 A d\omega^2}{x} + 2 Q d\omega^2$$

vnde cum ſit $xx d\Phi^2 = \frac{2 P d\omega^2}{xx}$ erit

$$dx^2 = 2 d\omega^2 \left(Q + \frac{A}{x} - \frac{P}{xx}\right) \quad \text{et} \quad dx = + d\omega \sqrt{2\left(Q + \frac{A}{x} - \frac{P}{xx}\right)}$$

ſicque differentiale dx per $d\omega$ exprimitur. Deinde vero habetur

$$d\Phi = \frac{d\omega}{xx} \sqrt{2P}$$

eſtque per hypotheſin:

$$dP = -M x d\omega \sqrt{2P} \quad \text{et} \quad dQ = -\frac{M d\omega}{x} \sqrt{2P} + N d\omega \sqrt{2\left(Q + \frac{A}{x} - \frac{P}{xx}\right)}$$

vbi quidem ſignorum ambiguorum inferius locum habere ſtatuamus, quia motum ab apogeo numerare in animo eſt, ita vt hinc exeundo diſtantia x minuatur.

V.

Cum igitur differentiale dx in apogeo et perigeo euaneſcat, neceſſe eſt vt his locis formula irrationalis $\sqrt{\left(Q + \frac{A}{x} - \frac{P}{xx}\right)}$ in nihilum abeat, in reliquis autem locis valorem ſortiatur realem. Commodiſſime ergo haec formula per ſinum cuiuſpiam anguli v exhibebitur, qui cum in apogeo euaneſcat, in perigeo autem duobus re-

ctis aequalis fiat, anomaliam lunae referet: idque ſenſu vero, ita vt diſtantia x in apogeo prodeat maxima, in perigeo vero minima. Sit igitur vt formam motus regularis ſequamur:

ſemilatus rectum orbitae lunaris $= p$
excentricitas orbitae $= q$
et anomalia vera lunae $= v$

eritque hinc per eandem legem diſtantia $x = \frac{p}{1 - q\,\text{coſ}\,v}$. Verum hic quantitates p et q, quae in motu regulari eſſent conſtantes, nunc pro variabilibus ſunt habendae, earumque variabilitas per variabilitatem quantitatum P et Q, quae in motu regulari itidem ſunt conſtantes, determinari debebit.

VI.

Subſtituamus ergo valorem aſſumtum $x = \frac{p}{1 - q\,\text{coſ}\,v}$ in formula irrationali $\surd\left(Q + \frac{A}{x} - \frac{P}{xx}\right)$, quae abibit in

$$\frac{1}{p}\surd\,(Qpp + Ap\,(1 - q\,\text{coſ}\,v) - P\,(1 - q\,\text{coſ}\,v)^2)$$

et euoluta dabit

$$\frac{1}{p}\surd(Qpp + Ap - P - Apq\,\text{coſ}\,v + 2Pq\,\text{coſ}\,v - Pqq\,\text{coſ}\,v^2)$$

quae vt reducatur ad formam V ſin v, ſtatuatur

primo $2P - Ap = 0$
tum vero $Qpp + Ap - P = Pqq$

ac noſtra formula fiet $= \frac{1}{p}\surd Pqq\,\text{ſin}\,v^2 = \frac{q\,\text{ſin}\,v}{p}\surd P$, habebimusque

dx

$$dx = -\frac{q\,d\omega\,\text{ſin}\,v}{p} \sqrt{2\,P} \quad \text{et}$$

$$dQ = -\frac{M\,d\omega}{x} \sqrt{2\,P} + \frac{N\,q\,d\omega\,\text{ſin}\,v}{p} \sqrt{2\,P}$$

VII.

Cum iam ſit $2\,P - A\,p = o$; erit $P = \frac{1}{2}\,A\,p$: quo valore in altera formula ſubſtituto orietur:

$$Q\,p\,p + \tfrac{1}{2}\,A\,p = \tfrac{1}{2}\,A\,p\,q\,q \quad \text{ſeu} \quad Q = -\frac{A}{2\,p}(1-q\,q)$$

Sumantur nunc differentialia; eritque

$dP = -M\,x\,d\omega\sqrt{2\,P} = \frac{1}{2}\,A\,dp$, quae ob $2\,P = A\,p$ abit in hanc

$$-M\,x\,d\omega\sqrt{A\,p} = \tfrac{1}{2}\,A\,dp, \text{ ſiue } dp = -\frac{2\,M\,x\,d\omega}{A}\sqrt{A\,p}$$

vel etiam $\sqrt{A\,p} = -\int M\,x\,d\omega$

Simili modo erit

$$dQ = +\frac{A\,dp\,(1-q\,q)}{2\,p\,p} + \frac{A\,q\,dq}{p} = -\frac{M\,x\,d\omega\,(1-q\,q)}{p\,p}\sqrt{A\,p} + \frac{A\,q\,dq}{p}$$

ideoque

$$\frac{A\,q\,dq}{p} = M\,d\omega\left(\frac{x(1-q\,q)}{p\,p} - \frac{1}{x}\right)\sqrt{A\,p} + \frac{N\,q\,d\omega\,\text{ſin}\,v}{p}\sqrt{A\,p}$$

At eſt $\frac{x(1-q\,q)}{p\,p} - \frac{1}{x} = \frac{x}{p\,p}\left(1 - q\,q - \frac{p\,p}{x\,x}\right) = \frac{x}{p\,p}(1 - q\,q - 1 + 2\,q\,\text{cſ}v - q\,q\,\text{cſ}v^2)$

ſiue $\quad \frac{x(1-q\,q)}{p\,p} - \frac{1}{x} = \frac{q\,x}{p\,p}(2\,\text{coſ}\,v - q - q\,\text{coſ}\,v^2)$

Hinc ergo colligitur:

$$dq = \frac{M\,x\,d\omega}{A\,p}(2\,\text{coſ}\,v - q - q\,\text{coſ}\,v^2)\sqrt{A\,p} + \frac{N\,d\omega\,\text{ſin}\,v}{A}\sqrt{A\,p} \text{ ſiue}$$

$$dq = d\omega\left(\frac{M}{A}\left(2\,\text{coſ}\,v - \frac{q\,\text{ſin}\,v^2}{1-q\,\text{coſ}\,v}\right) + \frac{N}{A}\,\text{ſin}\,v\right)\sqrt{A\,p}$$

VIII.

VIII.

Inuenta iam relatione differentialium dx, dp et dq ad differentiale temporis $d\omega$ ſcilicet:

$$dx = -\frac{q\,d\omega \text{ſin}\,v}{q} \sqrt{Ap}; \quad dp = -\frac{2Mx\,d\omega}{A} \sqrt{Ap}$$

$$\text{et } dq = d\omega\left(\frac{M}{A}\left(2\,\text{coſ}\,v - \frac{q\,\text{ſin}\,v^2}{1-q\,\text{coſ}\,v}\right) + \frac{N}{A}\,\text{ſin}\,v\right)\sqrt{Ap}$$

ſupereſt, vt quoque relationem elementi anomaliae dv definiamus. Cum igitur ſit

$$x = \frac{p}{1-q\,\text{coſ}\,v}, \text{ erit } 1 - q\,\text{coſ}\,v = \frac{p}{x}; \text{ hincque differentiando}$$

$$q\,dv\,\text{ſin}\,v = dq\,\text{coſ}\,v + \frac{dp}{x} - \frac{p\,dx}{xx};$$

ſubſtituantur valores pro dq, dp et dx inuenti; ac diuiſione facta per $q\,\text{ſin}\,v$ prodibit

$$dv = \frac{d\omega}{xx}\sqrt{Ap} - \frac{d\omega}{q}\left(\frac{M}{A}\left(2\,\text{ſin}\,v + \frac{q\,\text{ſin}\,v\,\text{coſ}\,v}{1-q\,\text{coſ}\,v}\right) - \frac{N}{A}\,\text{coſ}\,v\right)\sqrt{Ap}$$

Pro elemento autem longitudinis $d\Phi$ ob $2P = Ap$, ex antecedentibus habemus:

$$d\Phi = \frac{d\omega}{xx}\sqrt{Ap} = \frac{d\omega\,(1-q\,\text{coſ}\,v)^2}{pp}\sqrt{Ap}$$

IX.

Ex his formulis ſtatim ſe offert motus apogei; cum enim longitudo apogei ſit $= \Phi - v$, erit eius differentiale pro tempuſculo $d\omega$:

$$d\Phi - dv = \frac{d\omega}{q}\left(\frac{M}{A}\left(2\,\text{ſin}\,v + \frac{q\,\text{ſin}\,v\,\text{coſ}\,v}{1-q\,\text{coſ}\,v}\right) - \frac{N}{A}\,\text{coſ}\,v\right)\sqrt{Ap}$$

cuius ergo integrale praebebit verum motum apogei cum omnibus inaequalitatibus, quibus perturbatur. Vnde

quidem perſpicitur, quod per ſe eſt manifeſtum, ſi quantitates M et N euaneſcerent, motum apogei fore nullum, ſeu apogeum perpetuo in loco fixo eſſe permanſurum. Deinde etiam iuuabit notaſſe has formulas:

$$d.\ q\cos v = -q\,d\Phi \sin v + \frac{2M}{A}\, d\omega \sqrt{Ap}$$

$$d.\ q\sin v = +q\,d\Phi \cos v + d\omega\left(\frac{N}{A} - \frac{M}{A}\cdot\frac{q\sin v}{1-q\cos v}\right)\sqrt{Ap}$$

Tandem quoque habemus ex motu ſolis $du = d\theta = \frac{d\omega(1-e\cos u)^2}{(1-ee)\sqrt{(1-ee)}}$ ideoque

$$d\eta = d\Phi - d\theta = d\omega\left(\frac{(1-q\cos v}{pp}\sqrt{Ap} - \frac{(1-e\cos u)^2}{(1-ee)\sqrt{(1-ee)}}\right)$$

X.

Inuentis nunc omnium differentialium relationibus ad elementum temporis $d\omega$, euoluamus valores litterarum M et N, ac primo quidem cum ſit $z = \sqrt{(yy - 2xy\cos\eta + xx\sec.\psi^2)}$; quoniam quantitas z nonniſi in terminis minimis occurrit, pro ſec. ψ tuto unitas ſcribi poterit, et quia y tantopere excedit x, erit proxime

$$\frac{1}{z^3} = \frac{1}{y^3} + \frac{3x}{y^4}\cos\eta + \frac{3xx}{2y^6}(5\cos\eta^2 - 1) \quad \text{ſiue}$$

$$\frac{1}{z^3} = \frac{1}{y^3} + \frac{3x}{y^4}\cos\eta + \frac{3xx}{4y^5}(3 + 5\cos 2\eta)$$

Ideoque hinc habebitur:

$$\frac{y}{z^3} - \frac{1}{yy} = \frac{3x}{y^3}\cos\eta + \frac{3xx}{4y^4}(3 + 5\cos 2\eta)$$

Vnde

Vnde obtinemus:

$$M = a^3\left(\frac{3x}{2y^3}\text{ſin }2\eta + \frac{3xx}{8y^4}(\text{ſin }\eta + 5\text{ ſin }3\eta)\right)$$

$$N = \frac{a^3(\delta + \mathfrak{D})}{\odot}\text{coſ }\psi^3\left(\frac{1}{xx} - \frac{1}{bb}\right) - \frac{A}{xx}$$

$$- a^3\left(\frac{x}{2y^3}(1+3\text{coſ}2\eta) + \frac{3xx}{8y^4}(3\text{coſ}\eta + 5\text{ coſ}3\eta)\right)$$

XI.

Cum ſit proxime $\text{coſ}\psi^3 = 1 - \frac{3}{4}\text{tang}\rho^2 - \frac{3}{4}\text{tang}\rho^2\text{coſ}2(\Phi - \pi)$, eius valor vnitate erit minor, atque ex parte conſtante, et parte variabili conſtabit, quae illa multo erit minor. Ponatur ergo

$\text{coſ}\psi^3 = \lambda + \Pi$; vt ſit $\Pi = 1 - \lambda - \frac{3}{4}\text{tang}\rho^2 - \frac{3}{4}\text{tang}\rho^2\text{coſ}2(\Phi - \pi)$ vbi λ denotat partem conſtantem vnitate proxime aequalem, Π vero partem variabilem.

Erit ergo:

$$N = \frac{\lambda a^3(\delta + \mathfrak{D})}{\odot}\left(\frac{1}{xx} - \frac{1}{bb}\right) - \frac{A}{xx} + \frac{a^3(\delta + \mathfrak{D})}{\odot}\Pi\left(\frac{1}{xx} - \frac{1}{bb}\right)$$

$$- a^3\left(\frac{x}{2y^3}(1+3\text{ coſ}2\eta) + \frac{3xx}{8y^4}(3\text{coſ}\eta + 5\text{ coſ}3\eta)\right)$$

Statuatur nunc $A = \frac{\lambda a^3(\delta + \mathfrak{D})}{\odot}$; vt fiat

$$N = -\frac{A}{bb} + A\Pi\left(\frac{1}{xx} - \frac{1}{bb}\right)$$

$$- a^3\left(\frac{x}{2y^3}(1+3\text{ coſ}2\eta) + \frac{3xx}{8y^4}(3\text{ coſ}\eta + 5\text{ coſ }3\eta)\right)$$

 ac

ac ponatur breuitatis gratia: $p = b(1+\xi)$

erit $\dfrac{\sqrt{Ap}}{pp} = \sqrt{\dfrac{A}{b^3(1+\xi)^3}} = (1 - \tfrac{3}{2}\xi + \tfrac{15}{8}\xi^2)\sqrt{\dfrac{A}{b^3}}$

ob ξ prae 1 vehementer paruum, ſitque porro:

$$\sqrt{\frac{A}{b^3}} = \sqrt{\frac{\lambda a^3(\text{♄}+\text{☽})}{\odot}} = m,$$

atque habebitur $d\Phi = m\,d\omega\,(1 - \tfrac{3}{2}\xi + \tfrac{15}{8}\xi^2)(1 - q\cos v)^2$

XII.

Subſtituantur nunc pro x et y valores $\dfrac{p}{1-q\cos v}$ et $\dfrac{c}{1-e\cos u}$, eritque

$$M = a^3\left(\frac{3p(1-e\cos u)^3}{2c^3(1-q\cos v)}\sin 2\eta + \frac{3pp(1-e\cos u)^4}{8c^4(1-q\cos v)^2}(\sin\eta + 5\sin 3\eta)\right)$$

$$N = -\frac{A}{bb} + A\Pi\left(\frac{(1-q\cos v)^2}{pp} - \frac{1}{bb}\right)$$

$$-a^3\left(\frac{p(1-e\cos u)^3}{2c^3(1-q\cos v)}(1+3\cos 2\eta) + \frac{3pp(1-e\cos u)^4}{8c^4(1-q\cos v)^2}(3\cos\eta + 5\cos 3\eta)\right)$$

vbi quidem quoque terminus $\dfrac{A\Pi}{bb}$ prae termino $\dfrac{A}{bb}$ omitti poteſt. Nunc vt hinc valores $\dfrac{M}{A}\sqrt{Ap}$ et $\dfrac{N}{A}\sqrt{Ap}$ commode exprimantur, erit

$$\frac{a^3 b}{Ac^3}\sqrt{Ab} = \frac{a^3}{mc^3} = \frac{1}{m(1-ee)^3} = \frac{1+3ee}{m}$$

quoniam in his terminis minimis pro $1-ee$ ſcribere licet 1. Tum vero ſit $\dfrac{b}{c} = n$, eritque n fractio valde parua.

XIII.

XIII.

Factis ergo his ſubſtitutionibus, ob $p = b\,(1+\xi)$ habebimus:

$$\frac{\mathrm{M}}{\mathrm{A}} \sqrt{\mathrm{A}p} = \frac{3\,(1+3ee)}{2m} \frac{(1-e\,\mathrm{coſ}\,u)^3}{1-q\,\mathrm{coſ}\,v}\,(1+\tfrac{3}{2}\xi)\ \mathrm{ſin}\ 2\eta$$

$$+ \frac{3n}{8m} \frac{(1-e\,\mathrm{coſ}\,u)^4}{(1-q\,\mathrm{coſ}\,v)^2}\,(1+\tfrac{5}{2}\xi)\,(\mathrm{ſin}\,\eta + 5\,\mathrm{ſin}\,3\eta)$$

Pro altera valore $\frac{\mathrm{N}}{\mathrm{A}}\sqrt{\mathrm{A}p}$ ſtatuatur terminus minimus:

$$\frac{\sqrt{\mathrm{A}b}}{bb} = \sqrt{\frac{\lambda a^3 b\,(\text{♁}+\text{☽})}{\odot b^4}} = i \quad ; \quad \text{eritque}$$

$$\frac{\mathrm{N}}{\mathrm{A}} \sqrt{\mathrm{A}p} = -\frac{(1+3ee)}{2m} \frac{(1-e\,\mathrm{coſ}\,u)^3}{1-q\,\mathrm{coſ}\,v}\,(1+\tfrac{3}{2}\xi).\,(1+3\,\mathrm{coſ}\,2\eta)$$

$$-\frac{3n}{8m} \frac{(1-e\,\mathrm{coſ}\,u)^4}{(1-q\,\mathrm{coſ}\,v)^2}\,(1+\tfrac{5}{2}\xi)\,(3\,\mathrm{coſ}\,\eta + 5\,\mathrm{coſ}\,3\eta)$$

$$+ m\,(1-q\,\mathrm{coſ}\,v)^2\,(1-\tfrac{3}{2}\xi)\,\Pi - i$$

vbi notari oportet, terminos per n multiplicatos ratione praecedentium eſſe minimos; tum vero quantitates ξ et Π atque multo magis i eſſe fractiones prae vnitate fere euaneſcentes.

XIV.

Quoniam hi ipſi termini quantitates M et N inuoluentes ſunt valde parui, in iis ſine errore altiores poteſtates vtriusque excentricitatis q et e negligi poſſunt. In terminis ergo primis ſimpliciter per m diuiſis excentricitates tantum ad duas dimenſiones intro-

 ducan-

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ducantur, in terminis autem per $\frac{n}{m}$ multiplicatis penitus omittantur, quia fractio n iam fere quadrato excentricitatis q aequiualet. In termino autem littera minima Π affecto, quia is per numerum m ſatis magnum, vtpote 13 fere, eſt multiplicatus, excentricitas q vnius dimenſionis retineatur.

His obſeruatis habebimus:

$$\frac{M}{A} V A p = \begin{cases} + \frac{3}{2m}(1 + \frac{3}{2}ee + \frac{1}{2}qq) \sin 2\eta + \frac{3q}{4m} \sin(2\eta - v) \\ + \frac{3q}{4m} \sin(2\eta + v) - \frac{9e}{4m} \sin(2\eta - u) - \frac{9e}{4m} \sin(2\eta + u) \\ + \frac{3qq}{8m} \sin(2\eta - 2v) + \frac{3qq}{8m} \sin(2\eta + 2v) \\ + \frac{9ee}{8m} \sin(2\eta - 2u) + \frac{9ee}{8m} \sin(2\eta + 2u) \\ - \frac{9eq}{8m} \sin(2\eta - v + u) - \frac{9eq}{8m} \sin(2\eta + v - u) \\ - \frac{9eq}{8m} \sin(2\eta - v - u) - \frac{9eq}{8m} \sin(2\eta + v + u) \\ + \frac{9}{4m} \xi \sin 2\eta + \frac{9}{8m} q \xi \sin(2\eta - v) + \frac{9}{8m} q \xi \sin(2\eta + v) \\ - \frac{27}{8m} e \xi \sin(2\eta - u) - \frac{27}{8m} e \xi \sin(2\eta + u) \\ + \frac{3n}{8m} \sin \eta + \frac{15n}{8m} \sin 3\eta \end{cases}$$

$\frac{N}{A} V A p =$

$$\frac{N}{A}\sqrt{A}\,p = \left\{\begin{array}{l}
-\frac{1}{2m}\left(1+\frac{9}{2}ee+\frac{1}{2}qq\right)-\frac{3}{2m}\left(1+\frac{9}{2}ee+\frac{1}{2}qq\right)\text{coſ}\,2\eta \\
-\frac{q}{2m}\text{coſ}\,v+\frac{3e}{2m}\text{coſ}\,u \\
-\frac{3q}{4m}\text{coſ}(2\eta-v)-\frac{3q}{4m}\text{coſ}(2\eta+v) \\
+\frac{9e}{4m}\text{coſ}(2\eta-u)+\frac{9e}{4m}\text{coſ}(2\eta+u) \\
-\frac{qq}{4m}\text{coſ}\,2v+\frac{3eq}{4m}\text{coſ}(v-u)+\frac{3eq}{4m}\text{coſ}(v+u) \\
-\frac{3ee}{4m}\text{coſ}\,2u \\
-\frac{3qq}{8m}\text{coſ}(2\eta-2v)-\frac{3qq}{8m}\text{coſ}(2\eta+2v) \\
-\frac{9ee}{8m}\text{coſ}(2\eta-2u)-\frac{9ee}{8m}\text{coſ}(2\eta+2u) \\
+\frac{9eq}{8m}\text{coſ}(2\eta-v+u)+\frac{9eq}{8m}\text{coſ}(2\eta+v-u) \\
+\frac{9eq}{8m}\text{coſ}\,2\eta-v-u)+\frac{9eq}{8m}\text{coſ}(2\eta+v+u) \\
-\frac{3}{4m}\xi-\frac{9}{4m}\xi\,\text{coſ}\,2\eta-\frac{3q}{8m}\xi\,\text{coſ}\,v \\
-\frac{9q}{8m}\xi\,\text{coſ}(2\eta-v)-\frac{9q}{8m}\xi\,\text{coſ}(2\eta+v) \\
+\frac{9e}{4m}\xi\,\text{coſ}\,u+\frac{27e}{8m}\xi\,\text{coſ}(2\eta-u) \\
+\frac{27e}{8m}\xi\,\text{coſ}(2\eta+u)-\frac{9n}{8m}\text{coſ}\,\eta-\frac{15n}{8m}\text{coſ}\,3\eta \\
+m\Pi-2mq\Pi\,\text{coſ}\,v-\frac{3}{2}m\xi\Pi-i
\end{array}\right.$$

XV.

XV.

Quaeramus igitur valores euolutos noſtrorum differentialium ad elementum temporis applicatorum: ac primo quidem habebimus:

$$\frac{d\Phi}{d\omega} = m\left(1+\tfrac{1}{2}qq\right) - 2mq\cos v + \tfrac{1}{2}mqq\cos 2v - \tfrac{3}{2}m\left(1+\tfrac{1}{2}qq\right)\xi$$
$$+ 3mq\xi\cos v - \tfrac{3}{4}mqq\xi\cos 2v + \tfrac{15}{8}m\xi\xi$$

$$\frac{du}{d\omega} = \frac{d\theta}{d\omega} = 1 + 2ee - 2e\cos u + \tfrac{1}{2}ee\cos 2u;$$ vnde concludimus

$$\frac{d\eta}{d\omega} = m\left(1+\tfrac{1}{2}qq\right) - 1 - 2ee - 2mq\cos v + 2e\cos u + \tfrac{1}{2}mqq\cos 2v$$
$$- \tfrac{1}{2}ee\cos 2u - \tfrac{3}{2}m\left(1+\tfrac{1}{2}qq\right)\xi + 3mq\xi\cos v$$
$$- \tfrac{3}{4}mqq\xi\cos 2v + \tfrac{15}{8}m\xi\xi$$

Deinde cum ſit $\frac{dp}{d\omega} = -2x.\frac{M}{A}\sqrt{Ap} = -\frac{2b(1+\xi)}{1-q\cos v}.\frac{M}{A}\sqrt{Ap}$, ob $p = b\,(1+\xi)$ erit $\frac{d\xi}{d\omega} =$

$$\left(-2\left(1+\tfrac{1}{2}qq\right) - 2q\cos v - qq\cos 2v - 2\xi - 2q\xi\cos v\right)\frac{M}{A}\sqrt{Ap}$$

ac valorem pro $\frac{M}{A}\sqrt{Ap}$ inuentum ſubſtituendo obtinebimus ſequentes formulas:

$\frac{d\xi}{d\omega} =$

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$$\frac{d\xi}{d\omega} = \left\{\begin{array}{l}
-\frac{3}{m}(1+\frac{3}{2}ee+\frac{3}{2}qq)\sin 2\eta - \frac{3q}{m}\sin(2\eta-v) - \frac{3q}{m}\sin(2\eta+v) \\
+\frac{9e}{2m}\sin(2\eta-u) + \frac{9e}{2m}\sin(2\eta+u) \\
-\frac{9qq}{4m}\sin(2\eta-2v) - \frac{9qq}{4m}\sin(2\eta+2v) \\
-\frac{9ee}{4m}\sin(2\eta-2u) - \frac{9ee}{4m}\sin(2\eta+2u) \\
+\frac{9eq}{2m}\sin(2\eta-v+u) + \frac{9eq}{2m}\sin(2\eta+v-u) \\
+\frac{9eq}{2m}\sin(2\eta-v-u) + \frac{9eq}{2m}\sin(2\eta+v+u) \\
-\frac{15}{2m}\xi\sin 2\eta - \frac{15q}{2m}\xi\sin(2\eta-v) - \frac{15q}{2m}\xi\sin(2\eta+v) \\
+\frac{45e}{4m}\xi\sin(2\eta-u) + \frac{45e}{4m}\xi\sin(2\eta+u) \\
-\frac{3n}{4m}\sin\eta - \frac{15n}{4m}\sin 3\eta
\end{array}\right.$$

XVI.

Porro cum ſit $\frac{q\sin v^2}{1-q\cos v} = \frac{q-q\cos 2v}{2(1-q\cos v)} =$ $\frac{1}{2}q - \frac{1}{2}q\,\text{c}2v + \frac{1}{4}qq\cos v - \frac{1}{4}qq\cos 3v$; erit

$$\frac{dq}{d\omega} = (2\,\text{c}v - \tfrac{1}{2}q + \tfrac{1}{2}q\,\text{c}2v - \tfrac{1}{4}qq\,\text{c}v + \tfrac{1}{4}qq\,\text{c}3v)\frac{M}{A}\sqrt{Ap} + \sin v.\frac{N}{A}\sqrt{Ap}$$

Facta ergo ſubſtitutione valorum pro $\frac{M}{A}\sqrt{Ap}$ et $\frac{N}{A}\sqrt{Ap}$ inuentorum, habebitur:

$$
\frac{dq}{d\omega} = \left\{
\begin{array}{l}
+ \frac{9}{4m}\left(1 + \frac{9}{2}ee + \frac{5}{12}qq\right)\sin(2\eta - v) + \frac{3}{4m}\left(1 + \frac{9}{2}ee + \frac{3}{4}qq\right) \\
\quad \sin(2\eta + v) - \frac{1}{2m}\left(1 + \frac{9}{2}ee + \frac{1}{4}qq\right)\sin v \\
+ \frac{3q}{4m}\sin 2\eta + \frac{3q}{2m}\sin(2\eta - 2v) + \frac{3q}{4m}\sin(2\eta + 2v) - \frac{q}{4m}\sin 2v \\
- \frac{27e}{8m}\sin(2\eta - v - u) - \frac{27e}{8m}\sin(2\eta - v + u) - \frac{9e}{8m}\sin(2\eta + v - u) \\
- \frac{9e}{8m}\sin(2\eta + v + u) + \frac{3e}{4m}\sin(v - u) + \frac{3e}{4m}\sin(v + u) \\
+ \frac{15qq}{16m}\sin(2\eta - 3v) + \frac{9qq}{16m}\sin(2\eta + 3v) - \frac{qq}{8m}\sin 3v \\
- \frac{9eq}{8m}\sin(2\eta - u) - \frac{9eq}{8m}\sin(2\eta + u) - \frac{9eq}{4m}\sin(2\eta - 2v + u) \\
- \frac{9eq}{4m}\sin(2\eta - 2v - u) - \frac{9eq}{8m}\sin(2\eta + 2v - u) - \frac{9eq}{8m}\sin(2\eta + 2v + u) \\
+ \frac{27ee}{16m}\sin(2\eta - v - 2u) + \frac{27ee}{16m}\sin(2\eta - v + 2u) \\
+ \frac{9ee}{16m}\sin(2\eta + v - 2u) + \frac{9ee}{16m}\sin(2\eta + v + 2u) \\
+ \frac{3eq}{8m}\sin(2v - u) + \frac{3eq}{8m}\sin(2v + u) \\
- \frac{3ee}{8m}\sin(v - 2u) - \frac{3ee}{8m}\sin(v + 2u) \\
+ \frac{27}{8m}\xi\sin(2\eta - v) + \frac{9}{8m}\xi\sin(2\eta + v) - \frac{3}{4m}\xi\sin v
\end{array}
\right.
$$

$$+ \frac{9q}{8m}$$

$$
\left. \begin{array}{l}
+\frac{9q}{8m}\xi\,\text{ſin}\,2\eta+\frac{9q}{4m}\xi\,\text{ſin}(2\eta-2v)+\frac{9q}{8m}\xi\,\text{ſin}(2\eta+2v) \\
-\frac{3q}{8m}\xi\,\text{ſin}\,2v+\frac{9e}{8m}\xi\,\text{ſin}(v-u)+\frac{9e}{8m}\xi\,\text{ſin}(v+u) \\
-\frac{81e}{16m}\xi\,\text{ſin}(2\eta-v-u)-\frac{81e}{16m}\xi\,\text{ſin}(2\eta-v+u) \\
-\frac{27e}{16}\xi\,\text{ſin}(2\eta+v-u)-\frac{27e}{16m}\xi\,\text{ſin}(2\eta+v+u) \\
+\frac{15n}{16m}\,\text{ſin}\,(\eta-v)-\frac{3n}{16m}\,\text{ſin}\,(\eta+v) \\
+\frac{45n}{16m}\,\text{ſin}\,(3\eta-v)+\frac{15n}{16m}\,\text{ſin}\,(3\eta+v) \\
+m\Pi\,\text{ſin}\,v-mq\Pi\,\text{ſin}\,2v-\frac{3}{2}m\xi\Pi\,\text{ſin}\,v-i\,\text{ſin}\,v
\end{array} \right.
$$

XVII.

Deinde cum ſit $\frac{q\,\text{ſin}\,v\,\text{coſ}\,v}{1-q\,\text{coſ}\,v}=\frac{q\,\text{ſin}\,2v}{2(1-q\,\text{coſ}\,v)}=$ $=\frac{1}{2}q\,\text{ſin}\,2v+\frac{1}{4}qq\,\text{ſin}\,v+\frac{1}{4}qq\,\text{ſin}\,3v$; erit pro motu elementari apogei :

$$\frac{q(d\Phi-dv)}{d\omega}=\left(2\,\text{ſin}\,v+\frac{1}{2}q\,\text{ſin}\,2v+\frac{1}{4}qq\,\text{ſin}\,v+\frac{1}{4}qq\,\text{ſin}\,3v\right)\frac{M}{A}\sqrt{Ap}-\text{coſ}\,v.\,\frac{N}{A}\sqrt{Ap}$$

ac facta ſubſtitutione obtinebitur :

$\frac{q(d\Phi-dv)}{d\omega}=$

$$
\frac{q(d\Phi - dv)}{d\omega} = \left\{
\begin{array}{l}
+\frac{9}{4m}\left(1+\frac{9}{2}ee+\frac{7}{12}qq\right)\text{coſ}(2\eta-v)-\frac{3}{4m}\left(1+\frac{9}{2}ee+\frac{3}{4}qq\right) \\
\qquad \text{coſ}(2\eta+v)+\frac{1}{2m}\left(1+\frac{9}{2}ee+\frac{3}{4}qq\right)\text{coſ}\,v \\
+\frac{3q}{4m}\text{coſ}\,2\eta+\frac{3q}{2m}\text{coſ}(2\eta-2v)-\frac{3q}{4m}\text{coſ}(2\eta+2v) \\
+\frac{q}{4m}\text{coſ}\,2v+\frac{q}{4m}-\frac{3e}{4m}\text{coſ}(v-u)-\frac{3e}{4m}\text{coſ}(v+u) \\
-\frac{27e}{8m}\text{coſ}(2\eta-v-u)-\frac{27e}{8m}\text{coſ}(2\eta-v+u) \\
+\frac{9e}{8m}\text{coſ}(2\eta+v-u)+\frac{9e}{8m}\text{coſ}(2\eta+v+u) \\
+\frac{159q}{16m}\text{coſ}(2\eta-3v)-\frac{99q}{16m}\text{coſ}(2\eta+3v)+\frac{qq}{8m}\text{coſ}\,3v \\
-\frac{9eq}{8m}\text{coſ}(2\eta-u)-\frac{9eq}{8m}\text{coſ}(2\eta+u)-\frac{3eq}{4m}\text{coſ}\,u \\
-\frac{3eq}{8m}\text{coſ}(2v-u)-\frac{3eq}{8m}\text{coſ}(2v+u) \\
-\frac{9eq}{4m}\text{coſ}(2\eta-2v+u)-\frac{9eq}{4m}\text{coſ}(2\eta-2v-u) \\
+\frac{9eq}{8m}\text{coſ}(2\eta+2v-u)+\frac{9eq}{8m}\text{coſ}(2\eta+2v+u) \\
+\frac{27ee}{16m}\text{coſ}(2\eta-v-2u)+\frac{27ee}{16m}\text{coſ}(2\eta-v+2u) \\
-\frac{9ee}{16m}\text{coſ}(2\eta+v-2u)-\frac{9ee}{16m}\text{coſ}(2\eta+v+2u) \\
+\frac{3ee}{8m}\text{coſ}(v-2u)+\frac{3ee}{8m}\text{coſ}(v+2u)
\end{array}\right.
$$

$+\frac{27}{8m}$

$$\left|\begin{array}{l}
+\frac{27}{8m}\xi\,\mathrm{coſ}(2\eta-v)-\frac{9}{8m}\xi\,\mathrm{coſ}(2\eta+v)+\frac{3}{4m}\xi\,\mathrm{coſ}v \\
+\frac{9q}{8m}\xi\,\mathrm{coſ}2\eta+\frac{9q}{4m}\xi\,\mathrm{cſ}(2\eta-2v)-\frac{9q}{8m}\xi\,\mathrm{cſ}(2\eta+2v) \\
+\frac{3q}{8m}\xi+\frac{3q}{8m}\quad\xi\,\mathrm{coſ}\,2v \\
-\frac{81e}{16m}\xi\,\mathrm{coſ}(2\eta-v-u)+\frac{27e}{16m}\xi\,\mathrm{coſ}(2\eta+v-u) \\
-\frac{81e}{16m}\xi\,\mathrm{coſ}(2\eta-v+u)+\frac{27e}{16m}\xi\,\mathrm{coſ}(2\eta+v+u) \\
-\frac{9e}{8m}\xi\,\mathrm{coſ}(v-u)-\frac{9e}{8m}\xi\,\mathrm{coſ}(v+u) \\
+\frac{15n}{16m}\mathrm{coſ}(\eta-v)+\frac{3n}{16m}\mathrm{coſ}(\eta+v) \\
+\frac{45n}{16m}\mathrm{coſ}(3\eta-v)-\frac{15n}{16m}\mathrm{coſ}(3\eta+v) \\
-m\Pi\,\mathrm{coſ}v+mq\Pi+mq\Pi\,\mathrm{coſ}2v+\frac{3}{2}m\xi\Pi\,\mathrm{coſ}v+i\,\mathrm{cſ}v
\end{array}\right.$$

XVIII.

Euoluamus ſimili modo valorem differentialium $d\pi$ et $d\varphi$, et cum ſit $\frac{y}{z^3}-\frac{1}{yy}=\frac{3x}{y^3}\mathrm{cſ}\eta+\frac{3xx}{4y^4}(3+5\,\mathrm{cſ}2\eta)$ et $d\Phi=\frac{d\omega}{xx}\sqrt{Ap}$; erit

$$d\pi=-\frac{a^3xd\omega}{\sqrt{Ap}}\left(\frac{3x}{y^3}\mathrm{coſ}\eta+\frac{3xx}{4y^4}(3+5\,\mathrm{coſ}2\eta)\right)\mathrm{ſin}(\theta-\pi)\,\mathrm{ſin}(\Phi-\pi)$$

Subſtitutis autem valoribus $x=\frac{p}{1-q\,\mathrm{cſ}v}$, $y=\frac{c}{1-e\,\mathrm{cſ}u}$; $p=b(1+\xi)$; $\sqrt{Ap}=m\sqrt{b^3p}=mbb(1+\frac{1}{2}\xi)$, $\frac{a^3}{c^3}=\frac{1}{(1-ee)^3}=1+3ee$ et $\frac{b}{c}=n$, erit

$$d\pi=-\frac{d\omega\,\mathrm{ſn}(\theta-\pi)\,\mathrm{ſn}(\Phi-\pi)}{m}\left[\frac{3(1+\frac{1}{2}\xi)(1+3ee)(1-e\,\mathrm{cſ}u)^3}{(1-q\,\mathrm{coſ}v)^2}\mathrm{cſ}\eta+\frac{3}{4}n(3+5\,\mathrm{cſ}2\eta)\right]$$

 Negle-

Neglectis igitur terminis, qui nullum valorem ſenſibilem continent, habebimus

$$\frac{d\pi}{d\omega} = \begin{cases} -\frac{3}{4m}(1+\frac{9}{2}ee+\frac{3}{2}qq)-\frac{3}{4m}(1+\frac{9}{2}ee+\frac{3}{2}qq)\,\text{coſ}\,2\eta \\ +\frac{3}{4m}(1+\frac{9}{2}ee+\frac{3}{2}qq)\,\text{cſ}\,2(\phi-\pi)+\frac{3}{4m}(1+\frac{9}{2}ee+\frac{3}{2}qq)\,\text{cſ}\,2(\theta-\pi) \\ -\frac{3q}{2m}\,\text{coſ}\,v-\frac{3q}{4m}\,\text{coſ}\,(2\eta-v)-\frac{3q}{4m}\,\text{coſ}\,(2\eta+v) \\ +\frac{9e}{4m}\,\text{coſ}\,u+\frac{9e}{8m}\,\text{coſ}\,(2\eta-u)+\frac{9e}{8m}\,\text{coſ}\,(2\eta+u) \\ +\frac{3q}{4m}\,\text{coſ}\,(2\phi-2\pi-v)+\frac{3q}{4m}\,\text{coſ}\,(2\phi-2\pi+v) \\ +\frac{3q}{4m}\,\text{coſ}\,(2\theta-2\pi-v)+\frac{3q}{4m}\,\text{coſ}\,(2\theta-2\pi+v) \\ -\frac{9e}{8m}\,\text{coſ}\,(2\phi-2\pi-u)-\frac{9e}{8m}\,\text{coſ}\,(2\phi-2\pi+u) \\ -\frac{9e}{8m}\,\text{coſ}\,(2\theta-2\pi-u)-\frac{9e}{8m}\,\text{coſ}\,(2\theta-2\pi+u) \\ -\frac{9}{8m}\xi-\frac{9}{8m}\xi\,\text{coſ}\,2\eta \\ +\frac{9}{8m}\xi\,\text{coſ}\,(2\phi-2\pi)+\frac{9}{8m}\xi\,\text{coſ}\,(2\theta-2\pi) \\ -\frac{11n}{16m}\,\text{coſ}\,\eta+\frac{3n}{8m}\,\text{coſ}\,(\phi+\theta-2\pi)-\frac{5n}{16m}\,\text{coſ}\,3\eta \\ +\frac{5n}{16m}\,\text{coſ}\,(3\phi-\theta-2\pi)+\frac{5n}{16m}\,\text{coſ}\,(3\theta-\phi-2\pi) \end{cases}$$

XIX.

XIX.

Simili autem modo praecedentem valorem per tang $(\Phi-\pi)$ diuidendo prodibit differentiale logarithmi tangentis inclinationis ϱ, erit enim

$$\frac{d.l\,\mathrm{tang}\,\varrho}{d\omega}=\begin{cases}+\frac{3}{4m}(1+\frac{9}{2}ee+\frac{3}{2}qq)\sin 2\eta-\frac{3}{4m}(1+\frac{9}{2}ee+\frac{3}{2}qq)\\ \qquad\sin 2(\Phi-\pi)-\frac{3}{4m}(1+\frac{9}{2}ee+\frac{3}{2}qq)\sin 2(\theta-\pi)\\ +\frac{3q}{4m}\sin(2\eta-v)+\frac{3q}{4m}\sin(2\eta+v)\\ -\frac{9e}{8m}\sin(2\eta-u)-\frac{9e}{8m}\sin(2\eta+u)\\ -\frac{3q}{4m}\sin(2\Phi-2\pi-v)-\frac{3q}{4m}\sin(2\Phi-2\pi+v)\\ -\frac{3q}{4m}\sin(2\theta-2\pi-u)-\frac{3q}{4m}\sin(2\theta-2\pi+u)\\ +\frac{9e}{8m}\sin(2\Phi-2\pi-u)+\frac{9e}{8m}\sin(2\Phi-2\pi+u)\\ +\frac{9e}{8m}\sin(2\theta-2\pi-u)+\frac{9e}{8m}\sin(2\theta-2\pi+u)\\ +\frac{9}{8m}\xi\sin 2\eta-\frac{9}{8m}\xi\sin(2\Phi-2\pi)-\frac{9}{8m}\xi\sin(2\theta-2\pi)\\ +\frac{n}{16m}\sin\eta-\frac{3n}{8m}\sin(\Phi+\theta-\pi)+\frac{5n}{16m}\sin 3\eta\\ -\frac{5n}{16m}\sin(3\Phi-\theta-2\pi)-\frac{5n}{16m}\sin(3\theta-\Phi-2\pi)\end{cases}$$

XX.

XX.

Quo iam facilius has formulas admodum complicatas euoluere queamus, quadruplicis generis terminos diſtingui conuenit. Primum ſcilicet genus eos complectitur terminos, qui tantum ab excentricitate orbitae lunaris pendent, neque excentricitatem ſolis, neque parallaxin ſolis ſeu literam n, neque inclinationem orbitae lunaris ſeu litteram Π inuoluunt. Ad ſecundum genus refero terminos, qui ad primum genus inſuper excentricitatem ſolis adiungunt. Ad tertium autem eos, qui praeterea parallaxin ſolis ſeu litteram n inducunt. In quarto autem eas inaequalitates, quae inſuper ab obliquitate orbitae lunaris proueniunt, complexurus ſum. Ab inaequalitatibus ergo primi generis exordiar, ideoque cum excentricitatem ſolis e, tum eius parallaxin, tum quoque obliquitatem orbitae lunaris reiiciam

INUESTIGATIO INAEQUALITATUM LUNAE PRIMI GENERIS.

XXI.

Neglectis ergo excentricitate ſolis cum eius parallaxi et obliquitate orbitae lunaris, has habebimus aequationes:

$\frac{d\xi}{d\omega} =$

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$$\frac{d\xi}{d\omega} = \begin{cases} -\frac{3}{m}(1+\tfrac{3}{4}qq)\sin 2\eta - \frac{3q}{m}\sin(2\eta-v) - \frac{3q}{m}\sin(2\eta+v) \\ -\frac{9qq}{4m}\sin(2\eta-2v) - \frac{9qq}{4m}\sin(2\eta+2v) \\ -\frac{15}{2m}\xi\sin 2\eta - \frac{15q}{2m}\xi\sin(2\eta-v) - \frac{15q}{2m}\xi\sin(2\eta+v) \end{cases}$$

$$\frac{dq}{d\omega} = \begin{cases} +\frac{9}{4m}(1+\tfrac{5}{12}qq)\sin(2\eta-v) + \frac{3}{4m}(1+\tfrac{3}{4}qq)\sin(2\eta+v) \\ -\frac{1}{2m}(1+\tfrac{3}{4}qq)\sin v - i\sin v \\ +\frac{3q}{4m}\sin 2\eta + \frac{3q}{2m}\sin(2\eta-2v) + \frac{3q}{4m}\sin(2\eta+2v) \\ -\frac{q}{4m}\sin 2v \\ +\frac{15qq}{16m}\sin(2\eta-3v) + \frac{9qq}{16m}\sin(2\eta+3v) - \frac{qq}{8m}\sin 3v \\ +\frac{27}{8m}\xi\sin(2\eta-v) + \frac{9}{8m}\xi\sin(2\eta+v) - \frac{3}{4m}\xi\sin v \\ +\frac{9q}{8m}\xi\sin 2\eta + \frac{9q}{4m}\xi\sin(2\eta-2v) + \frac{9q}{8m}\xi\sin(2\eta+2v) \\ -\frac{3q}{8m}\xi\sin 2v \end{cases}$$

 $\frac{q(d\Phi-dv)}{d\omega} =$

$$\frac{q(d\Phi - dv)}{d\omega} = \begin{cases} + \frac{9}{4m}(1 + \frac{7}{12}qq)\cos(2\eta - v) - \frac{3}{4m}(1 + \frac{1}{4}qq)\cos(2\eta + v) \\ + \frac{1}{2m}(1 + \frac{3}{4}qq)\cos v + i \cos v \\ + \frac{3q}{4m}\cos 2\eta + \frac{3q}{2m}\cos(2\eta - 2v) - \frac{3q}{4m}\cos(2\eta + 2v) \\ + \frac{q}{4m}\cos 2v + \frac{q}{4m} \\ + \frac{15qq}{16m}\cos(2\eta - 3v) - \frac{9qq}{16m}\cos(2\eta + 3v) + \frac{qq}{8m}\cos 3v \\ + \frac{27}{8m}\xi\cos(2\eta - v) - \frac{9}{8m}\xi\cos(2\eta + v) + \frac{3}{4m}\xi\cos v \\ + \frac{9q}{8m}\xi\cos 2\eta + \frac{9q}{4m}\xi\cos(2\eta - 2v) - \frac{9q}{8m}\xi\cos(2\eta + 2v) \\ + \frac{3q}{8m}\xi + \frac{3q}{8m}\xi\cos 2v \end{cases}$$

$$\frac{d\Phi}{d\omega} = m(1 + \tfrac{1}{2}qq) - 2mq\cos v + \tfrac{1}{2}mqq\cos 2v - \tfrac{3}{2}m(1 + \tfrac{1}{2}qq)\xi$$
$$+ 3mq\xi\cos v - \tfrac{3}{4}mqq\xi\cos 2v + \tfrac{25}{8}m\xi\xi$$

$$\frac{d\eta}{d\omega} = m(1 + \tfrac{1}{2}qq) - 1 - 2mq\cos v + \tfrac{1}{2}mqq\cos 2v - \tfrac{3}{2}m(1 + \tfrac{1}{2}qq)\xi$$
$$+ 3mq\xi\cos v - \tfrac{3}{4}mqq\xi\cos 2v + \tfrac{25}{8}m\xi\xi$$

XXII.

Hic autem primo patet valores litterarum ξ et q ſine cognitis rationibus $\frac{d\eta}{d\omega}$ et $\frac{dv}{d\omega}$ definiri non poſſe, has autem viciſſim ipſas quantitates ξ et q inuoluere. Cum autem ad valores ξ et q inueniendos non opus ſit ra-

tiones

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tiones $\frac{d\eta}{d\omega}$ et $\frac{dv}{d\omega}$ eo praeciſionis gradu noſſe, quo ipſi illi valores deſiderantur; patet ſi valores ξ et q prope tantum veri conſtent, iis in rationibus $\frac{d\eta}{d\omega}$ et $\frac{dv}{d\omega}$ adhibitis, eosdem multo exactiores repertum iri. Cum igitur, ſi motus eſſet regularis, foret $\xi = o$ et $q =$ conſtanti, hinc primam hypotheſin conſtituamus. Sit ergo

$$\xi = o \quad \text{et} \quad q = g$$

et neglectis terminis, qui ob harum litterarum errores affici poſſent, vtpote valde paruis prae reliquis, habebimus proxime

$$\frac{d\Phi}{d\omega} = m(1+\tfrac{1}{2}gg) - 2mg \operatorname{coſ} v; \quad \frac{d\eta}{d\omega} = (1+\tfrac{1}{2}gg) - 1 - 2mg \operatorname{coſ} v$$

$$\text{et } \frac{d\Phi - dv}{d\omega} = \frac{9}{4mg}(1+\tfrac{7}{12}gg)\operatorname{coſ}(2\eta - v) - \frac{3}{4mg}(1+\tfrac{1}{4}gg)\operatorname{coſ}(2\eta + v)$$
$$+ \frac{1}{2mg}(1+\tfrac{3}{4}gg)\operatorname{coſ} v + \frac{i}{g}\operatorname{coſ} v + \frac{1}{4m}$$

ideoque

$$\frac{dv}{d\omega} = m(1+\tfrac{1}{2}gg) - \frac{1}{4m} - (2mg + \frac{1}{2mg} + \frac{3g}{8m} + \frac{i}{g})\operatorname{coſ} v$$
$$- \frac{9}{4mg}(1+\tfrac{7}{12}gg)\operatorname{coſ}(2\eta - v) + \frac{3}{4mg}(1+\tfrac{1}{4}gg)\operatorname{coſ}(2\eta + v)$$

XXIII.

Ponamus ad has formulas abbreuiandas:

$$m(1+\tfrac{1}{2}gg) - 1 = \alpha; \qquad 2mg = \gamma$$

$$m(1+\tfrac{1}{2}gg) - \frac{1}{4m} = \beta; \quad 2mg + \frac{1}{2mg} + \frac{3g}{8m} + \frac{i}{g} = \delta$$

 et

et neglectis quadratis gg in reliquis terminis, habebimus has formulas ſimpliciores :

$$\frac{d\eta}{d\omega} = \alpha - \gamma \operatorname{coſ} v$$

$$\frac{dv}{d\omega} = \beta - \delta \operatorname{coſ} v - \frac{9}{4mg} \operatorname{coſ}(2\eta - v) + \frac{3}{4mg} \operatorname{coſ}(2\eta + v)$$

Tum vero pro valoribus ξ et q propius inueniendis has aequationes :

$$\frac{d\xi}{d\omega} = -\frac{3}{m}(1 + \tfrac{3}{2}gg) \operatorname{ſin} 2\eta - \frac{3g}{m} \operatorname{ſin}(2\eta - v) - \frac{3g}{m} \operatorname{ſin}(2\eta + v)$$

$$\frac{dq}{d\omega} = + \frac{9}{4m}(1 + \tfrac{5}{12}gg) \operatorname{ſin}(2\eta - v) + \frac{3}{4m}(1 + \tfrac{3}{4}gg) \operatorname{ſin}(2\eta + v)$$

$$- \frac{1}{2m}(1 + \tfrac{3}{4}gg) \operatorname{ſin} v - i \operatorname{ſin} v$$

$$+ \frac{3g}{4m} \operatorname{ſin} 2\eta + \frac{3g}{2m} \operatorname{ſin}(2\eta - 2v) + \frac{3g}{4m} \operatorname{ſin}(2\eta + 2v) - \frac{g}{4m} \operatorname{ſin} 2v$$

XXIV.

Fingamus ergo primo :

$$\xi = \mathfrak{A} \operatorname{coſ} 2\eta + \mathfrak{B} \operatorname{coſ}(2\eta - v) + \mathfrak{C} \operatorname{coſ}(2\eta + v)$$

vbi notandum eſt terminos binos poſteriores, vti in differentiali, multo eſſe minores primo. Quare cum etiam in differentialibus $d\eta$ et dv duplicis generis termini occurrant, quorum poſteriores prae primis ſint valde parui, in differentiatione ſolius primi termini totum differentialis $d\eta$ valorem pono, in duobus vero reliquis tantum valorem principalem ; ſic prodibit

$$\frac{d\xi}{d\omega} = -2\alpha \mathfrak{A} \operatorname{ſin} 2\eta + \gamma \mathfrak{A} \operatorname{ſin}(2\eta - v) + \gamma \mathfrak{A} \operatorname{ſin}(2\eta + v)$$

$$- (2\alpha - \beta) \mathfrak{B} \quad - (2\alpha + \beta) \mathfrak{C}$$

Collato ergo hoc differentiali cum forma propoſita obtinetur : $\mathfrak{A} =$

$$\mathfrak{A} = \frac{3}{2m\alpha}(1+\tfrac{3}{2}gg)$$

$$(2\alpha-\beta)\,\mathfrak{B} = \gamma\,\mathfrak{A} + \frac{3g}{m} \quad \text{ergo} \quad \mathfrak{B} = \frac{3(\gamma+2\alpha g)}{2m\alpha(2\alpha-\beta)}$$

$$(2\alpha+\beta)\,\mathfrak{C} = \gamma\,\mathfrak{A} + \frac{3g}{m} \quad \text{ergo} \quad \mathfrak{C} = \frac{3(\gamma+2\alpha g)}{2m\alpha(2\alpha+\beta)}$$

XXV.

Simili modo fingatur:

$$q = g + \mathrm{A}\operatorname{cof}(2\eta - v) + \mathrm{B}\operatorname{cof}(2\eta + v) + \mathrm{C}\operatorname{cof} v$$
$$+ \mathrm{D}\operatorname{cof} 2\eta + \mathrm{E}\operatorname{cof}(2\eta - 2v) + \mathrm{F}\operatorname{cof}(2\eta + 2v) + \mathrm{G}\operatorname{cof} 2v$$
$$+ \mathrm{H}\operatorname{cof} 4\eta + \mathrm{J}\operatorname{cof}(4\eta - 2v) + \mathrm{K}\operatorname{cof}(4\eta + 2v)$$

vbi linea prior continet terminos multo maiores, quam binae inferiores. Hinc ergo fit differentiando ſecundum regulam ſupra datam:

$$\frac{dq}{d\omega} = -(2\alpha-\beta)\mathrm{A}\operatorname{ſin}(2\eta-v) - (2\alpha+\beta)\mathrm{B}\operatorname{ſin}(2\eta+v) - \beta\mathrm{C}\operatorname{ſin} v$$
$$+\left(\tfrac{1}{2}(2\gamma-\delta)\mathrm{A} + \tfrac{1}{2}(2\gamma+\delta)\mathrm{B} + \frac{3}{2mg}\mathrm{C} - 2\alpha\mathrm{D}\right)\operatorname{ſin} 2\eta$$
$$+\left(\tfrac{1}{2}(2\gamma-\delta)\mathrm{A} - \frac{9}{8mg}\mathrm{C} - 2(\alpha-\beta)\mathrm{E}\right)\operatorname{ſin}(2\eta-2v)$$
$$+\left(\tfrac{1}{2}(2\gamma+\delta)\mathrm{B} - \frac{3}{8mg}\mathrm{C} - 2(\alpha+\beta)\mathrm{F}\right)\operatorname{ſin}(2\eta+2v)$$
$$+\left(\tfrac{1}{2}\delta\mathrm{C} - \frac{3\mathrm{A}+9\mathrm{B}}{8mg} - 2\beta\mathrm{G}\right)\operatorname{ſin} 2v$$
$$+\left(\frac{3\mathrm{A}+9\mathrm{B}}{8mg} - 4\alpha\mathrm{H}\right)\operatorname{ſin} 4\eta$$
$$+\left(-\frac{9\mathrm{A}}{8mg} - 2(2\alpha-\beta)\mathrm{J}\right)\operatorname{ſin}(4\eta-2v)$$
$$+\left(-\frac{3\mathrm{B}}{8mg} - 2(2\alpha+\beta)\mathrm{K}\right)\operatorname{ſin}(4\eta+2v)$$

 Hinc-

Hincque elicientur ſequentes coefficientium valores:

$$(2\alpha-\beta)\,A = -\frac{9}{4m}(1+\tfrac{5}{12}gg)$$

$$(2\alpha+\beta)\,B = -\frac{3}{4m}(1+\tfrac{3}{4}gg)$$

$$\beta C = \frac{1}{2m}(1+\tfrac{1}{4}gg) + t$$

$$2\alpha D = \tfrac{1}{2}(2\gamma-\delta)A + \tfrac{1}{2}(2\gamma+\delta)B + \frac{3}{2mg}C - \frac{3g}{4m}$$

$$2(\alpha-\beta)\,E = \tfrac{1}{2}(2\gamma-\delta)A - \frac{9}{8mg}C - \frac{3g}{2m}$$

$$2(\alpha+\beta)\,F = \tfrac{1}{2}(2\gamma+\delta)A - \frac{3}{8mg}C - \frac{3g}{4m}$$

$$2\beta G = -\frac{3A+9B}{8mg} + \tfrac{1}{2}\delta C + \frac{g}{4m}$$

$$4\alpha H = \frac{3A+9\beta}{8mg};\quad 2(2\alpha-\beta)J = -\frac{9}{8mg}A$$

$$2(2\alpha+\beta)K = -\frac{3}{8mg}B$$

XXVI.

Cum igitur his inuentis valoribus ſit multo verius:

$$z = \mathfrak{A}\,\mathrm{coſ}\,2\eta \text{ et } q = g + A\,\mathrm{coſ}(2\eta-v) + B\,\mathrm{coſ}(2\eta+v) + C\,\mathrm{cſ}\,v$$

vbi terminos minores data opera adhuc omitto, quia fortaſſe correctione egent, praecedentes operationes multo accuratius inſtituere atque ad ordinem terminorum vlteriorem progredi poterimus. Obtinebimus ergo:

$$\frac{d\Phi}{d\omega} = m\,(1+\tfrac{1}{2}gg - C) - 2mg\,\mathrm{coſ}\,v$$
$$-m(\tfrac{3}{2}\mathfrak{A}+A+B)\mathrm{cſ}\,2\eta - mA\,\mathrm{cſ}(2\eta-2v) - mB\,\mathrm{cſ}(2\eta+2v) + m(\tfrac{1}{2}gg-C)\mathrm{cſ}\,2v$$

hincque $\frac{d\eta}{d\omega} = \frac{d\Phi}{d\omega} - 1.$

Porro

Porro ob $\frac{1}{q} = \frac{1}{g} - \frac{A}{gg}\,\text{coſ}(2\eta - v) - \frac{B}{gg}\,\text{coſ}(2\eta + v) - \frac{C}{gg}\,\text{coſ}\,v$
erit

$$\frac{d\Phi - dv}{d\omega} = \frac{9}{4mg}(1 + \tfrac{7}{12}gg)\,\text{coſ}(2\eta - v) - \frac{3}{4mg}(1 + \tfrac{1}{4}gg)\,\text{coſ}(2\eta + v)$$

$$+ \frac{1}{2mg}(1 + \tfrac{3}{2}gg)\,\text{cſ}\,v + \frac{i}{g}\,\text{cſ}\,v + \frac{1}{4m}\left(1 - \frac{9A + 3B - 2C}{2gg}\right)$$

$$+ \frac{1}{4m}\left(3 - \frac{A - B - 3C}{gg}\right)\text{cſ}\,2\eta + \frac{1}{4m}\left(6 - \frac{2A - 9C}{2gg}\right)\text{cſ}(2\eta - 2v)$$

$$- \frac{1}{4m}\left(3 + \frac{2B - 3C}{2gg}\right)\text{cſ}(2\eta + 2v) + \frac{1}{4m}\left(1 + \frac{3A - 9B - 2C}{2gg}\right)\text{cſ}\,2v$$

$$+ \frac{3A - 9B}{8mgg}\,\text{coſ}\,4\eta - \frac{9A}{8mgg}\,\text{coſ}(4\eta - 2v) + \frac{3B}{8mgg}\,\text{coſ}(4\eta + 2v)$$

XXVII.

Ponatur ad abbreuiandum:

$$m\,(1 + \tfrac{1}{2}gg - C) - 1 = \alpha \quad ; \quad 2mg = \gamma$$

$$m\,(\tfrac{3}{2}\mathfrak{A} + A + B) = \varepsilon \quad ; \quad \text{vt ſit}$$

$$\frac{d\eta}{d\omega} = \alpha - \gamma\,\text{coſ}\,v - \varepsilon\,\text{coſ}\,2\eta$$

$$- mA\,\text{coſ}(2\eta - 2v) - mB\,\text{coſ}(2\eta + 2v) + m(\tfrac{1}{2}gg - C)\,\text{coſ}\,2v$$

Porro ſit

$$m\,(1 + \tfrac{1}{2}gg - C) - \frac{1}{4m}\left(1 - \frac{9A + 3B - 2C}{2gg}\right) = \beta$$

$$2mg + \frac{1}{2mg} + \frac{3g}{8m} + \frac{i}{g} = \delta$$

$$+ m(\tfrac{3}{2}\mathfrak{A} + A + B) + \frac{1}{4m}\left(3 - \frac{A - B - 3C}{gg}\right) = \zeta$$

mA

$$mA + \frac{1}{4m}\left(6 - \frac{2A-9C}{2gg}\right) = \eta$$

$$mB - \frac{1}{4m}\left(3 + \frac{2B-3C}{2gg}\right) = \theta$$

$$m\left(C - \tfrac{1}{2}gg\right) + \frac{1}{4m}\left(1 + \frac{3A-9B-2C}{2gg}\right) = \varkappa$$

vt habeatur

$$\frac{dv}{d\omega} = \beta - \delta\,\text{coſ}\,v - \frac{9}{4mg}\,\text{coſ}(2\eta - v) + \frac{3}{4mg}\,\text{coſ}(2\eta + v)$$
$$- \zeta\,\text{coſ}\,2\eta - \eta\,\text{coſ}(2\eta - 2v) - \theta\,\text{coſ}(2\eta + 2v) - \varkappa\,\text{coſ}\,2v$$
$$- \frac{3A+9B}{8mgg}\,\text{coſ}\,4\eta + \frac{9A}{8mgg}\,\text{coſ}(4\eta - 2v) - \frac{3B}{8mgg}\,\text{coſ}(4\eta + 2v)$$

vbi caueatur, ne coefficientes η, θ, cum angulis cognominibus confundantur.

XXVIII.

Opus plane non eſt, vt valores litterarum ξ et q accuratius determinemus, atque ad plures terminos, quam ante inuenimus, expediamus; verum hos ipſos terminos, quos ante inuenimus, accuratius obtinebimus, ſi litteris α et β eos valores tribuemus, quos nunc eis conuenire collegimus. Pluribus autem terminis non indigebimus tam ad longitudinem lunae Φ, quam ad eius anomaliam veram v ſatis exacte definiendam. Verum ad hoc ipſum negotium valores differentiales $\frac{d\Phi}{d\omega}$ et $\frac{d\Phi - dv}{d\omega}$, ac praecipue hunc poſteriorem, quo motus apogei continetur, accuratius euolui oportet, quoniam imprimis in motu medio apogei minimae particulae ingentis momenti eſſe poſſunt.

XXIX.

XXIX.

Cum igitur accuratius quam adhuc aſſumſimus ſit

$$\xi = \mathfrak{A} \cos 2\eta + \mathfrak{B} \cos(2\eta - v) + \mathfrak{C} \cos(2\eta + v) \quad \text{et}$$

$$q = g + A \cos(2\eta - v) + B \cos(2\eta + v) + C \cos v$$
$$+ D \cos 2\eta + E \cos(2\eta - 2v) + F \cos(2\eta + 2v) + G \cos 2v$$
$$+ H \cos 4\eta + J \cos(4\eta - 2v) + K \cos(4\eta + 2v)$$

erit terminus ad quartum vsque ordinem extenſis

I. II.

$$\frac{d\Phi}{d\omega} = m(1 + \tfrac{1}{2}gg - C) - (2mg - \tfrac{3}{2}mgC + mG)\cos v$$

III.

$$-m(\tfrac{3}{2}\mathfrak{A} + A + B)\cos 2\eta - mA\cos(2\eta - 2v) - mB\cos(2\eta + 2v)$$
$$- m(C - \tfrac{1}{2}gg)\cos 2v$$

IV.

$$+ m(\tfrac{3}{2}g\mathfrak{A} - \tfrac{3}{2}\mathfrak{B} + gA + \tfrac{1}{2}gB - D - E)\cos(2\eta - v)$$
$$+ m(\tfrac{3}{2}g\mathfrak{A} - \tfrac{3}{2}\mathfrak{C} + gB + \tfrac{1}{2}gA - D - F)\cos(2\eta + v)$$
$$+ m(\tfrac{1}{2}gA - E)\cos(2\eta - 3v) + m(\tfrac{1}{2}gB - F)\cos(2\eta + 3v)$$
$$+ m(\tfrac{1}{2}gC - G)\cos 3v$$
$$- m(H + J)\cos(4\eta - v) - m(H + K)\cos(4\eta + v) - mJ\cos(4\eta - 3v)$$
$$- mK\cos(4\eta + 3v)$$

vnde cum eſſet ante $\gamma = 2mg$, nunc accuratius erit

$$\gamma = 2mg - \tfrac{3}{2}mgC + mG$$

XXX.

Deinde cum nunc quoque ſit accuratius:

$$\frac{1}{q} = \left(\frac{1}{g} + \overset{\text{II.}}{\frac{AA+BB+CC}{2g^3}}\right)$$

$$\overset{\text{III.}}{-\frac{A}{gg}\operatorname{coſ}(2\eta-v) - \frac{B}{gg}\operatorname{coſ}(2\eta+v) - \frac{C}{gg}\operatorname{coſ} v}$$

$$\overset{\text{IV.}}{+\left(\frac{(A+B)C}{g^3}-\frac{D}{gg}\right)\operatorname{coſ} 2\eta + \left(\frac{AC}{g^3}-\frac{E}{gg}\right)\operatorname{coſ}(2\eta-2v)}$$

$$+\left(\frac{BC}{g^3}-\frac{F}{gg}\right)\operatorname{coſ}(2\eta+2v) + \left(\frac{2AB+CC}{2g^3}-\frac{G}{gg}\right)\operatorname{coſ} 2v$$

$$+\left(\frac{AB}{g^3}-\frac{H}{gg}\right)\operatorname{coſ} 4\eta + \left(\frac{AA}{2g^3}-\frac{J}{gg}\right)\operatorname{coſ}(4\eta-2v)$$

$$+\left(\frac{BB}{g2^3}-\frac{K}{gg}\right)\operatorname{coſ}(4\eta+2v)$$

Hinc quoque ad terminos quarti ordinis vsque valor formulae $\frac{d\Phi-dv}{d\omega}$ definiri poſſet, ſed expreſſio prodiret tantopere complicata, vt eius euolutio ſummam requireret patientiam; neque tamen hic labor vllius foret vſus, niſi forte in motu apogei exactius eruendo: ipſae enim inaequalitates nullius forent momenti; propterea quod error in anomalia commiſſus multo minorem errorem in longitudine producit.

XXXI.

XXXI.

Ponatur ergo longitudo apogei:

$$\Phi - v = \text{Conſt.}$$
$$+ A' \sin(2\eta - v) + B' \sin(2\eta + v) + C' \sin 2v$$
$$+ \Delta\omega + D' \sin 2\eta + E' \sin(2\eta - 2v) + F' \sin(2\eta + 2v) + G' \sin 2v$$
$$+ H' \sin 4\eta + J \sin(4\eta - 2v) + K \sin(4\eta + 2v)$$

et erit differentiando:

$$\frac{d\Phi - dv}{d\omega} =$$
$$(2\alpha - \beta) A' \cos(2\eta - v) + (2\alpha + \beta) B' \cos(2\eta + v) + \beta C' \cos v$$
$$+ \Delta - \tfrac{1}{2} \delta C' + \frac{9A'}{8mg} + \frac{3B'}{8mg} - \varepsilon D' - (mA - \eta) E'$$
$$- (mB + \theta) F' - \kappa G' - \frac{9AJ'}{8mgg} - \frac{3BK'}{3mgg}$$
$$\cos 2\eta \left(- \tfrac{1}{2} (2\gamma - \delta) A' - \tfrac{1}{2} (2\gamma + \delta) B' - \frac{3C'}{4mg} + 2\alpha D' \right.$$
$$\cos(2\eta - 2v) \left(- \tfrac{1}{2} (2\gamma - \delta) A' - \frac{9C'}{8mg} + 2(\alpha - \beta) E' \right.$$
$$\cos(2\eta + 2v) \left(- \tfrac{1}{2} (2\gamma + \delta) B' + \frac{3C'}{8mg} + 2(\alpha + \beta) F' \right.$$
$$\cos 2v \left(- \tfrac{1}{2} \delta C' - \frac{3A'}{8mg} - \frac{9B'}{8mg} + 2\beta G' \right.$$
$$\cos 4\eta \left(- \frac{3A'}{8mg} - \frac{9B'}{8mg} + 4\alpha H' \right.$$
$$\cos(4\eta - 2v) \left(\frac{9A'}{8mg} + 2(2\alpha - \beta) J' \right.$$
$$\cos(4\eta + 2v) \left(\frac{3B'}{8mg} + 2(2\alpha + \beta) K' \right.$$

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XXXIII.

Calculo autem praecipue in primis terminis accuratius expedito eſt:

$$\frac{d\Phi - dv}{d\omega} =$$

$$+ \frac{1}{4mg} \text{coſ}(2\eta - v) \left(9 + \tfrac{21}{4} gg + \frac{27\,AA + 18\,BB + 15\,CC}{4gg} \right.$$

$$\left. - \frac{3\,AB + 2\,AC + BC}{gg} - \frac{2D - 2E + 3G + 3H - 9J}{2g} \right)$$

$$+ \frac{1}{4mg} \text{coſ}(2\eta + v) \left(-3 - \tfrac{3}{4} gg - \frac{6\,AA - 9\,BB + 15\,CC}{4gg} \right.$$

$$\left. + \frac{9AB + AC + 2BC}{gg} - \frac{2D - 2F - 9G - 9H + 3K}{2g} \right)$$

$$+ \frac{1}{2mg} \text{coſ}\, v \left(1 + \tfrac{3}{4} gg + \frac{2\,AA + 2\,BB + 3\,CC}{4gg} \right.$$

$$\left. + \frac{AB + 6AC + 6BC}{2gg} - \frac{3D - 3E - 3F - G}{2g} + 2mi \right)$$

$$+ \frac{1}{4m} \left(1 - \frac{9\,A + 3\,B - 2\,C}{2gg} \right)$$

$$+ \frac{1}{4m} \left(3 - \frac{A - B - 3C}{gg} \right) \text{coſ}\, 2\eta + \frac{1}{4m} \left(1 + \frac{3A - 9B - 2C}{2gg} \right) \text{coſ}\, 2v$$

$$+ \frac{1}{4m} \left(6 - \frac{2A - 9C}{2gg} \right) \text{cſ}(2\eta - 2v) - \frac{1}{4m} \left(3 + \frac{2B - 3C}{2gg} \right) \text{cſ}(2\eta + 2v)$$

$$+ \frac{3A - 9B}{8mgg} \text{coſ}\, 4\eta - \frac{9A}{8mgg} \text{coſ}(4\eta - 2v) + \frac{3B}{8mgg} \text{coſ}(4\eta + 2v)$$

Simili

Simili autem modo ex valore ipſius $\frac{dq}{d\omega}$ accuratius erit

$$(2\alpha-\beta)\,A = -\frac{1}{4m}(9+\tfrac{15}{4}gg+\tfrac{9}{2}C)$$

$$(2\alpha+\beta)\,B = -\frac{1}{4m}(3+\tfrac{9}{4}gg+3C)$$

$$\beta C = \frac{1}{2m}(1+\tfrac{1}{4}gg+\tfrac{3}{4}A+2mi)$$

XXXIII.

Comparatione autem inſtituta reperitur:

$$(2\alpha-\beta)\,A' = \frac{1}{4mg}\Big(9+\tfrac{21}{9}gg+\frac{27AA+18BB+15CC}{4gg}$$
$$-\frac{3AB+2AC+BC}{gg}-\frac{2D-2E+3G+3H-9J}{2g}\Big)$$

ſeu

$$(2\alpha-\beta)A' = -\frac{1}{g}(2\alpha-\beta)A+\frac{1}{4mg}\Big(\tfrac{1}{2}gg-\tfrac{9}{2}C+\frac{27AA+18BB+15CC}{4gg}$$
$$-\frac{3AB+2AC+BC}{gg}-\frac{2D-2E+3G+3H-9J}{2g}\Big)$$

$$(2\alpha+\beta)B' = +\frac{1}{g}(2\alpha+\beta)B+\frac{1}{4mg}\Big(\tfrac{1}{4}gg+3C-\frac{6AA-9BB+15CC}{4gg}$$
$$+\frac{9AB+AC+2BC}{gg}-\frac{2D-2F-9G-9H+3K}{2g}\Big)$$

$$\beta C' = +\frac{1}{g}\beta C+\frac{1}{2mg}\Big(\tfrac{1}{2}gg-\tfrac{1}{4}A+\frac{2AA+2BB+3CC}{4gg}$$
$$+\frac{AB+6AC+6BC}{2gg}-\frac{3D-3E-3F-G}{2g}\Big)$$

 Quibus

Quibus valoribus ſubſtitutis obtinebitur pro apogei motu medio, qui in termino $\Delta\omega$ continetur:

$$\Delta = \frac{1}{4m} + \frac{2mg\delta - 1}{4mgg}C$$

$$+ \frac{\delta}{4\beta mg}\left(\tfrac{1}{2}gg - \tfrac{3}{4}A + \frac{2AA + 2BB + 3CC}{4gg} + \frac{AB + 6AC + 6BC}{2gg} - \frac{3D - 3E - 3F - G}{2g}\right)$$

$$- \frac{9}{32(2\alpha - \beta)mmgg}\left(\tfrac{3}{2}gg - \tfrac{9}{2}C + \frac{27AA + 18BB + 15CC}{4gg} - \frac{3AB + 2AC + BC}{gg} - \frac{2D - 2E + 3G + 3H - 9J}{2g}\right)$$

$$- \frac{3}{32(2\alpha + \beta)mmgg}\left(\tfrac{3}{2}gg + 3C - \frac{6AA - 9BB + 15CC}{4gg} + \frac{9AB + AC + 2BC}{gg} - \frac{2D - 2F - 9G - 9H + 3K}{2g}\right)$$

$$+ \varepsilon D' + (mA - \eta)E' + (mB + \theta)F' + \varkappa G' + \frac{9AJ'}{8mgg} + \frac{3BK'}{8mgg}$$

Quae expreſſio, cum omnino ſit ſimilis illi, quae methodo praecedente eſt inuenta, nullum etiam dubium relinquit, quin et hinc motus apogei prodituruſ ſit obſeruationibus conformis; ideoque littera illa ι omitti poterit.

XXXIV.

Hinc igitur patet ad motum apogei definiendum valores litterarum A, B, C et A′, B′, C′ ſumma accuratione inueſtigari debere, qui cum conſtent partibus duplicis ordinis, etiam ſi partes poſterioris ordinis prae

primo

primo admodum videantur paruae, eas tamen omni cura euolui oportet, propterea quod pro motu apogei partes primi ordinis ſe deſtruunt. Quod cum in determinatione reliquorum coefficientium vſu non eueniat, in his quoque non erit opus, vt partes iſtae minores in computum ducantur, ſed ſufficiet partibus principalibus vti. Scilicet etſi determinatio litterae Δ maxime eſt lubrica, atque a reliquorum coefficientium exactiſſimis valoribus pendet, reliqui tamen coefficientes tantam ſollertiam minime requirunt, ſed ſatis exacte ſine tanta opera definiri poſſunt.

XXXV.

Valores ergo reliquorum coefficientium ſequenti modo neglectis exiguis particulis ita ſe habebunt,

$$(2\alpha-\beta)\,A' = \frac{9}{4mg}\,; \quad (2\alpha+\beta)\,B' = -\frac{3}{4mg}\,; \quad \beta\,C' = \frac{1}{2mg}.$$

$$2\alpha D' = \tfrac{1}{2}(2\gamma-\delta)A' + \tfrac{1}{2}(2\gamma+\delta)B' + \frac{3C'}{4mg} + \frac{1}{4m}\left(3 - \frac{A-B-3C}{gg}\right)$$

$$2(\alpha-\beta)\,E' = \tfrac{1}{2}(2\gamma-\delta)\,A' + \frac{9C'}{8mg} + \frac{1}{4m}\left(6 - \frac{2A-9C}{2gg}\right)$$

$$2(\alpha+\beta)\,F' = \tfrac{1}{2}(2\gamma+\delta)\,B' - \frac{3C'}{8mg} - \frac{1}{4m}\left(3 + \frac{2B-3C}{2gg}\right)$$

$$2\beta\,G' = \tfrac{1}{2}\delta C' + \frac{3A'+9B'}{8mg} + \frac{1}{4m}\left(1 + \frac{3A-9B-2C}{2gg}\right)$$

$$4\alpha\,H' = \frac{3A'+9B'}{8mg} + \frac{8A-9B}{8mgg} = 0$$

$$2(2\alpha-\beta)J' = -\frac{9A'}{8mg} - \frac{9A}{8mgg} = 0 \quad \Big| \quad 2(2\alpha+\beta)K' = -\frac{3B'}{8mg} + \frac{3B}{8mgg} = 0$$

ob $A' = -\frac{A}{g}$; $B' = \frac{B}{g}$ et $C' = \frac{C}{g}$ proxime.

XXXVI.

XXXVI.

Cum autem ſit proxime: $\gamma = 2mg$; $\delta = 2mg + \frac{1}{2mg}$; his valoribus quoque ſubſtitutis fiet:

$$(2\alpha-\beta)\,A' = \frac{9}{4mg};\quad (2\alpha+\beta)\,B' = -\frac{3}{4mg};\quad \beta C' = \frac{1}{2mg};\ \text{ſiue:}$$

$$A' = -\frac{A}{g}\ ;\quad B' = \frac{B}{g}\ ;\quad C' = \frac{C}{g};$$

$$2\alpha D' = \frac{3}{4m} - mA + 3mB;$$

$$2(\alpha-\beta)E' = \frac{3}{2m} - mA$$

$$2(\alpha+\beta)F' = -\frac{3}{4m} + 3mB$$

$$2\beta G' = \frac{1}{4m} + mC$$

et reliqui coefficientes H', J', K' pro euaneſcentibus ſunt habendi. Valores autem litterarum A, B, C, etc. §. 25. ſunt exhibiti.

XXXVII.

Quaeramus nunc quoque longitudinem lunae ϕ, huncque in finem fingamus:

$$\phi = \text{Conſt.}$$

$$+\mathfrak{A}'\omega + \mathfrak{B}'\text{ſin}\,v + \mathfrak{C}'\text{ſn}\,2\eta + \mathfrak{D}'\text{ſn}(2\eta-2v) + \mathfrak{E}'\text{ſn}(2\eta+2v) + \mathfrak{F}'\text{ſn}\,2v$$

$$+\mathfrak{G}'\text{ſin}(2\eta-v) + \mathfrak{H}'\text{ſn}(2\eta+v) + \mathfrak{I}'\text{ſn}(2\eta-3v) + \mathfrak{K}'\text{ſn}(2\eta+3v) + \mathfrak{L}'\text{ſn}\,3v$$

$$+\mathfrak{M}'\text{ſin}(4\eta-v) + \mathfrak{N}'\text{ſin}(4\eta+v) + \mathfrak{P}'\text{ſin}(4\eta-3v) + \mathfrak{Q}'\text{ſin}(4\eta+3v)$$

ac

ac differentiatione inſtituta obtinebimus:

$$\frac{d\Phi}{d\omega} = + \mathfrak{A}' - \tfrac{1}{2}\,\delta\,\mathfrak{B}'$$

$$+ \cos v \left(ϐ\,\mathfrak{B}' - \tfrac{1}{2}\,\varkappa\,\mathfrak{B}' + \frac{9\mathfrak{D}'}{4mg} + \frac{3\mathfrak{E}'}{4mg} - \delta\,\mathfrak{F}' \right.$$

$$+ \cos 2\eta \left(- \frac{3\mathfrak{B}'}{4mg} + 2\alpha\,\mathfrak{C}' \right.$$

$$+ \cos(2\eta - 2v) \left(- \frac{9\mathfrak{B}'}{8mg} + 2\,(\alpha - ϐ)\,\mathfrak{D}' \right.$$

$$+ \cos(2\eta + 2v) \left(+ \frac{3\mathfrak{B}'}{8mg} + 2\,(\alpha + ϐ)\,\mathfrak{E}' \right.$$

$$+ \cos 2v \left(- \tfrac{1}{2}\,\delta\,\mathfrak{B}' + 2\,ϐ\,\mathfrak{F}' \right.$$

$$+ \cos(2\eta - v) \left(- \tfrac{1}{2}\zeta\mathfrak{B}' - \tfrac{1}{2}\eta\mathfrak{B}' - \gamma\mathfrak{C}' - (\gamma - \delta)\mathfrak{D}' + \frac{3\mathfrak{F}'}{4mg} + (2\alpha - ϐ)\mathfrak{G}' \right.$$

$$+ \cos(2\eta + v) \left(- \tfrac{1}{2}\zeta\mathfrak{B}' - \tfrac{1}{2}\theta\mathfrak{B}' - \gamma\mathfrak{C}' - (\gamma + \delta)\mathfrak{E}' - \frac{9\mathfrak{F}'}{4mg} + (2\alpha + ϐ)\mathfrak{H}' \right.$$

$$+ \cos(2\eta - 3v) \left(- \tfrac{1}{2}\eta\mathfrak{B}' - (\gamma - \delta)\,\mathfrak{D}' - \frac{9\mathfrak{F}'}{4mg} + (2\alpha - 3ϐ)\,\mathfrak{J}' \right.$$

$$+ \cos(2\eta + 3v) \left(- \tfrac{1}{2}\theta\,\mathfrak{B}' - (\gamma + \delta)\,\mathfrak{E}' + \frac{3\mathfrak{F}'}{4mg} + (2\alpha + 3ϐ)\,\mathfrak{K}' \right.$$

$$+ \cos 8v \left(- \tfrac{1}{2}\varkappa\mathfrak{B}' - \frac{3\mathfrak{D}'}{4mg} - \frac{9\mathfrak{E}'}{4mg} - \delta\mathfrak{F}' + 3\,ϐ\,\mathfrak{L}' \right.$$

$$+ \cos(4\eta - v) \left(- \frac{3A + 9B}{16mgg}\mathfrak{B}' + \frac{9A}{16mgg}\mathfrak{B}' - \frac{3\mathfrak{D}'}{4mg} + (4\alpha - ϐ)\mathfrak{M}' \right.$$

$$+ \cos(4\eta + v) \left(- \frac{3A + 9B}{16mgg}\mathfrak{B}' - \frac{3B}{16mgg}\mathfrak{B}' - \frac{9\mathfrak{E}'}{4mg} + (4\alpha + ϐ)\mathfrak{N}' \right.$$

$$+ \cos(4\eta - 3v) \left(+ \frac{9A}{16mgg}\mathfrak{B}' + \frac{9\mathfrak{D}'}{4mg} + (4\alpha - 3ϐ)\,\mathfrak{P}' \right.$$

$$+ \cos(4\eta + 3v) \left(- \frac{3B}{16mgg}\mathfrak{B}' + \frac{3\mathfrak{E}'}{4mg} + (4\alpha + 3ϐ)\,\mathfrak{Q}' \right.$$

XXXVIII.

Comparata iam hac forma cum valore ipſius $\frac{d\Phi}{d\omega}$ in §. 29. exhibito, obtinebitur

$$\mathfrak{A}' = \tfrac{1}{2}\,\delta\,\mathfrak{B}' + m\,(1 + \tfrac{1}{2}gg - \mathrm{C})$$

$$\beta\mathfrak{B}' = \tfrac{1}{2}\,\kappa\,\mathfrak{B}' - \frac{9\mathfrak{D}' - 3\mathfrak{E}'}{4mg} + \delta\,\mathfrak{F}' - 2mg + \tfrac{3}{2}\,mg\,\mathrm{C} - m\,\mathrm{G}$$

$$2\alpha\mathfrak{C}' = \frac{3\mathfrak{B}'}{4mg} - m\,(\tfrac{3}{2}\,\mathfrak{A} + \mathrm{A} + \mathrm{B})$$

$$2(\alpha - \beta)\,\mathfrak{D}' = \frac{9\mathfrak{B}'}{8mg} - m\,\mathrm{A}$$

$$2(\alpha + \beta)\,\mathfrak{E}' = -\frac{3\mathfrak{B}'}{8mg} - m\,\mathrm{B}$$

$$2\beta\,\mathfrak{F}' = \tfrac{1}{2}\,\delta\,\mathfrak{B}' - m\,(\mathrm{C} - \tfrac{1}{2}gg)$$

$$(2\alpha - \beta)\,\mathfrak{G}' = \tfrac{1}{2}\,(\zeta + \eta)\,\mathfrak{B}' + \gamma\mathfrak{C}' + (\gamma - \delta)\,\mathfrak{D}' - \frac{3\mathfrak{F}'}{4mg} + m\,(\tfrac{3}{2}\,g\,\mathfrak{A} - \tfrac{3}{2}\,\mathfrak{B} + g\,\mathrm{A} + \tfrac{1}{2}\,g\,\mathrm{B} - \mathrm{D} - \mathrm{E})$$

$$(2\alpha + \beta)\,\mathfrak{H}' = \tfrac{1}{2}\,(\zeta + \theta)\,\mathfrak{B}' + \gamma\mathfrak{C}' + (\gamma + \delta)\,\mathfrak{E}' + \frac{9\mathfrak{F}'}{4mg} + m\,(\tfrac{3}{2}\,g\,\mathfrak{A} - \tfrac{3}{2}\,\mathfrak{C} + g\,\mathrm{B} + \tfrac{1}{2}\,g\,\mathrm{A} - \mathrm{D} - \mathrm{F})$$

$$(2\alpha - 3\beta)\,\mathfrak{J}' = \tfrac{1}{2}\,\eta\,\mathfrak{B}' + (\gamma - \delta)\,\mathfrak{D}' + \frac{9\mathfrak{F}'}{4mg} + m\,(\tfrac{1}{2}g\,\mathrm{A} - \mathrm{E})$$

$$(2\alpha + 3\beta)\,\mathfrak{K}' = \tfrac{1}{2}\,\theta\,\mathfrak{B}' + (\gamma + \delta)\,\mathfrak{E}' - \frac{3\mathfrak{F}'}{4mg} + m\,(\tfrac{1}{2}g\,\mathrm{B} - \mathrm{F})$$

$$3\beta\,\mathfrak{L}' = \tfrac{1}{2}\,\kappa\,\mathfrak{B}' + \frac{3\mathfrak{D}' + 9\mathfrak{E}'}{4mg} + \delta\,\mathfrak{F}' + m\,(\tfrac{1}{2}g\,\mathrm{C} - \mathrm{G})$$

$$(4\alpha - \beta)\,\mathfrak{M}' = -\frac{6\mathrm{A} - 9\mathrm{B}}{16mgg}\,\mathfrak{B}' + \frac{3\mathfrak{D}'}{4mg} - m\,(\mathrm{H} + \mathrm{J})$$

$$(4\alpha + \beta)\,\mathfrak{N}' = +\frac{3\mathrm{A} - 6\mathrm{B}}{16mgg}\,\mathfrak{B}' + \frac{9\mathfrak{E}'}{4mg} - m\,(\mathrm{H} + \mathrm{K})$$

$$(4\alpha - 3\beta)\,\mathfrak{P}' = -\frac{9\mathrm{A}}{16mgg}\,\mathfrak{B}' - \frac{9\mathfrak{D}'}{4mg} - m\,\mathrm{J}$$

$$(4\alpha + 3\beta)\,\mathfrak{Q}' = +\frac{3\mathrm{B}}{16mgg}\,\mathfrak{B}' - \frac{3\mathfrak{E}'}{4mg} - m\,\mathrm{K}$$

XXXIX.

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XXXIX.

Inuentis iam valoribus litterarum $p = b(1+\xi)$ et q vna cum anomalia vera v, diſtantia curtata lunae a terra $x = \frac{p}{1-q\,\text{coſ}\,v}$ cognoſcetur: ac ſi deinceps latitudinis lunae ψ ratio habebitur, erit diſtantia vera $= \frac{p}{(1-q\,\text{cſ}\,v)\,\text{cſ}\,\psi}$. In Aſtronomia autem non tam diſtantia lunae, quam eius diameter apparens et parallaxis requiri ſolet; quarum vtraque cum ſit diſtantiae lunae a terra reciproce proportionalis, erit tam diameter apparens quam parallaxis vt $\frac{(1-q\,\text{coſ}\,v)\,\text{coſ}\,\psi}{p}$; vnde ſi vtriusque valor medius ex obſeruationibus fuerit definitus, ad quoduis tempus valor verus aſſignari poterit. Sit igitur ſiue diametri apparentis ſiue parallaxis horizontalis valor medius $= \sigma$, eritque is pro tempore dato $= \frac{b\sigma}{p}(1-q\,\text{coſ}\,v)\,\text{coſ}\,\psi$. Eſt autem proxime $\text{coſ}\,\psi = 1 - \frac{1}{4}\,\text{tang}\,\rho^2 - \frac{1}{4}\,\text{tg}\,\rho^2\,\text{cſ}\,2\,(\phi-\pi) = \frac{2}{3} + \frac{\lambda+\Pi}{3}$, et $\frac{b}{p} = 1 - \xi + \xi\xi$: vnde fit diameter ſeu parallaxis $= \frac{1}{3}\sigma\,(2+\lambda+\Pi)\,(1-\xi+\xi\xi)\,(1-q\,\text{coſ}\,v)$, quae euoluitur in hanc expreſſionem: ob $\frac{2+\lambda}{3} = 1$ proxime;

$$\tfrac{1}{3}(2+\lambda)\,\sigma\,[1-q\,\text{coſ}\,v-\xi+q\xi\,\text{coſ}\,v+\xi\xi+\tfrac{1}{2}\Pi-\tfrac{1}{3}q\,\Pi\,\text{coſ}\,v]$$

XL.

Pro praeſenti ergo caſu, quo parallaxin ſolis, eiusque excentricitatem vna cum inclinatione orbitae luna-

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ris negligimus, erit lunae diameter apparens vel parallaxis horizontalis

$$= \tfrac{1}{3}(2+\lambda)\sigma \text{ mult. per}$$

$$1 - \tfrac{1}{2}C - (g + \tfrac{1}{2}G)\,\text{coſ}\,v$$
$$- \tfrac{1}{2}(2\mathfrak{A}+A+B)\,\text{cſ}\,2\eta - \tfrac{1}{2}A\,\text{cſ}(2\eta-2v) - \tfrac{1}{2}B\,\text{cſ}(2\eta+2v) - \tfrac{1}{2}C\,\text{cſ}\,2v$$
$$- \tfrac{1}{2}(2\mathfrak{B}+D+E)\,\text{coſ}(2\eta-v) - \tfrac{1}{2}(2\mathfrak{C}+D+F)\,\text{coſ}(2\eta+v)$$
$$- \tfrac{1}{2}E\,\text{coſ}(2\eta-3v) - \tfrac{1}{2}F\,\text{coſ}(2\eta+3v) - \tfrac{1}{2}G\,\text{coſ}\,3v$$
$$- \tfrac{1}{2}(H+J)\,\text{cſ}(4\eta-v) - \tfrac{1}{2}(H+K)\,\text{cſ}(4\eta+v) - \tfrac{1}{2}J\,\text{cſ}(4\eta-3v) - \tfrac{1}{2}K\,\text{cſ}(4\eta+3v)$$

vbi quidem factor conſtans $\tfrac{1}{3}(2+\lambda)\sigma$ omitti poteſt, ſiquidem tantum proportio vel diametri apparentis vel parallaxis horizontalis deſideretur.

XLI.

Si nunc hos valores in numeris euoluere velimus, ex obſeruationibus primum colligimus has determinationes:

$\mathfrak{A}' = 13,3682$; $\Delta = 0,1123$ et proxime $g = 0,05445$

ac poſtremo quidem valore ipſius g tantum in terminis minimis vtar, in maioribus ipſam litteram g relicturus, vt deinceps ex collatione calculi cum obſeruationibus accuratius fortaſſe determinari poſſit. Habemus ergo

$$13,3682 = \tfrac{1}{2}\delta\mathfrak{B}' + m\,(1 + \tfrac{1}{2}gg - C)$$

vnde ob $\mathfrak{B}'$, gg et C numeros admodum paruos, ſtatim prope colligitur $m = 13,3682$. Tum vero eſt prope

$\mathfrak{B}' = -\frac{2mg}{\beta}$; $\beta = m$ et $\delta = 2mg + \frac{1}{2mg}$ ſeu $\beta = 13,3682$;

$\delta = 2,$

$\delta = 2,1419$; hinc $\frac{1}{2}\delta\mathfrak{B}' = -0,1165$, ergo accuratius

$$m(1+\tfrac{1}{2}gg - C) = 13,4847 = \alpha+1 \quad \text{et} \quad \alpha = 12,4847$$

Porro ob $C = \frac{1}{2mm}$ erit ſatis exacte . $m = 13,5039$

vnde ex valoribus A et B proxime collectis fit $\beta = 13,0644$

$$\gamma = 26,9524g.$$

At valor ipſius δ duabus conſtat partibus, altera per g multiplicata altera diuiſa, quibus ſeparatim expreſſis erit

$$\delta = 27,0355\,g + 0,0370.\,\frac{1}{g} = 2,1521 \text{ proxime.}$$

XLII.

Hinc iam computo inſtituto ſequentes ſupra aſſumtorum coefficientium eruuntur valores numerici:

$\mathfrak{A} = 0,008931$; $\mathfrak{B} = 0,03895g = 0,00209$; $\mathfrak{C} = 0,01219g = 0,00064$

ideoque

$$\xi = 0,008931\,cſ\,2\eta + 0,03895\,g\,cſ(2\eta - v) + 0,01219\,g\,cſ(2\eta + v)$$

Deinde reperitur:

$A = -0,013995$; $B = -0,001460$; $C = +0,002834$

$D = -0,001213.g + 0,00002198.\frac{1}{g} = -0,000280$ proxime

$E = +0,25834.g - 0,00001889.\frac{1}{g} = +0,014012$ proxime

$F = -0,00225.g - 0,00000207.\frac{1}{g} = -0,000161$ proxime

$G = +0,00218.g + 0,00001221.\frac{1}{g} = +0,000340$ proxime

$H = -0,00001022.\frac{1}{g} = -0,000184$

$J = +0,00004897.\frac{1}{g} = +0,000882$

$K = +0,00000053.\frac{1}{g} = +0,000009$

XLIII.

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XLIII.

Hinc porro pro motu apogei eiusque inaequalitatibus colligitur: $\Delta = 0,1123$; qui quidem valor ex obſeruationibus eſt deſumtus

$A' = -0,013995 \cdot \frac{1}{g} = -0,25703$ proxime

$B' = -0,001460 \cdot \frac{1}{g} = -0,02682$ proxime

$C' = +0,002834 \cdot \frac{1}{g} = +0,05205$ proxime

$D' = +0,007432$	$F' = -0,002249$
$E' = -0,259170$	$G' = -0,002176$

in minutis ſecundis

$A' = -53018'' = -14^\circ, 43', 38''$

$B' = -5532 = -1^\circ, 32, 12$

$C' = +10736 = +2, 58, 56$

$D' = +1533 = +0, 25, 33$

$E' = -53459 = -14, 50, 59$

$F' = -464 = -0, 7, 44$

$G' = +449 = +0, 7, 29$

Ergo longitudo apogei in minutis ſecundis

$$\phi - v = \text{Conſt.}$$

$$+0,1123\omega - 53018'' \sin(2\eta - v) + 1533'' \sin 2\eta$$
$$- 5532 \sin(2+v) - 53459 \sin(2\eta - 2v)$$
$$+ 10736 \sin v - 464 \sin(2\eta + 2v)$$
$$+ 449 \sin 2v$$

XLIV.

XLIV.

Iam pro longitudine ipſa inuenienda habentur primo ex §. 27. valores: $\gamma = 1,46756$

$\delta = +2,15210$; $\varepsilon = -0,027791$; $\zeta = +0,07138$

$\eta = -0,06974$; $\theta = -0,039809$; $\varkappa = -0,070920$

Deinde cum ſit proxime

$$6\,\mathfrak{B}' = -2mg \quad \text{ſeu} \quad \mathfrak{B}' = -0,11256$$

erit quoque proxime

$$\mathfrak{C}' = -0,003485 \;;\; \mathfrak{D}' = -0,014456$$

$$\mathfrak{E}' = +0,001509 \;;\; \mathfrak{F}' = -0,005334$$

Hinc ergo accuratius elicietur $\mathfrak{B}' = -0,11019$, ideoque hic et reliqui conficientes tam abſolute quam in numeris ſecundis erunt:

abſolute		in minutis ſecundis	
$\mathfrak{B}' = -0,11019$	. .	$\mathfrak{B}' = -22728''$	$= -6°,\ 18',\ 4''$
$\mathfrak{C}' = -0,00339$	. .	$\mathfrak{C}' = -\ 700$	$= -0,\ 11,\ 40$
$\mathfrak{D}' = -0,01742$	. .	$\mathfrak{D}' = -\ 3594$	$= -0,\ 59,\ 54$
$\mathfrak{E}' = +0,00149$	. .	$\mathfrak{E}' = +\ 306$	$= +0,\ 5,\ 6$
$\mathfrak{F}' = -0,00524$	. .	$\mathfrak{F}' = -\ 1081$	$= -0,\ 18,\ 1$
$\mathfrak{G}' = -0,01824$	. .	$\mathfrak{G}' = -\ 3762$	$= -1,\ 2,\ 42$
$\mathfrak{H}' = -0,00056$	. .	$\mathfrak{H}' = -\ 115$	$= -0,\ 1,\ 55$
$\mathfrak{J}' = +0,01368$	. .	$\mathfrak{J}' = +\ 2823$	$= +0,\ 47,\ 3$
$\mathfrak{K}' = +0,00023$	. .	$\mathfrak{K}' = +\ 47$	$= +0,\ 0,\ 47$
$\mathfrak{L}' = -0,00062$	. .	$\mathfrak{L}' = -\ 128$	$= -0,\ 2,\ 8$
$\mathfrak{M}' = -0,00119$	. .	$\mathfrak{M}' = -\ 246$	$= -0,\ 4,\ 6$
$\mathfrak{N}' = +0,00020$	. .	$\mathfrak{N}' = +\ 41$	$= +0,\ 0,\ 41$
$\mathfrak{P}' = +0,00184$	. .	$\mathfrak{P}' = +\ 379$	$= +0,\ 6,\ 19$
$\mathfrak{Q}' = -0,00001$	. .	$\mathfrak{Q}' = -\ 2$	$= -0,\ 0,\ 2$

XLV.

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XLV.

Hinc ergo ſi ad datum tempus iam cognita ſit anomalia lunae vera v cum angulo η, longitudo lunae per aequationes in minutis ſecundis expreſſas erit

$$\Phi = \text{Conſt.}$$

$$\begin{array}{lllll}
+13{,}3682\omega & -22728'' \sin v & & -\ 700'' & \sin 2\eta \\
 & -\ 1081 \ \sin 2v & & -\ 3594 & \sin(2\eta-2v) \\
 & -\ 128 \ \sin 3v & & +\ 306 & \sin(2\eta+2v) \\
 & -\ 3762'' \sin(2\eta-v) & & -\ 246'' & \sin(4\eta-v) \\
 & -\ 115 \ \sin(2\eta+v) & & +\ 41 & \sin(4\eta+v) \\
 & +\ 2823 \ \sin(2\eta-3v) & & +\ 379 & \sin(4\eta-3v) \\
 & +\ 47 \ \sin(2\eta+3v) & & -\ 2 & \sin(4\eta+3v)
\end{array}$$

vbi Conſt. $+ 13{,}3682\omega$ denotat longitudinem mediam; in reliquis autem terminis continentur inaequalitates periodicae pro hac hypotheſi.

XLVI.

Inde iam viciſſim anomalia vera lunae v colligitur, vt ſit

$$v =$$

$$\begin{array}{lllll}
13{,}2559\omega & -33464'' \sin v & & -\ 2233'' & \sin 2\eta \\
 & -\ 1530 \ \sin 2v & & +49864 & \sin(2\eta-2v) \\
 & -\ 128 \ \sin 3v & & +\ 770 & \sin(2\eta+2v) \\
 & +49256 \ \sin(2\eta-v) & & -\ 246 & \sin(4\eta-v) \\
 & +\ 5417 \ \sin(2\eta+v) & & +\ 41 & \sin(4\eta+v) \\
 & +\ 2823 \ \sin(2\eta-3v) & & +\ 379 & \sin(4\eta-3v) \\
 & +\ 47 \ \sin(2\eta+3v) & & -\ 2 & \sin(4\eta+3v)
\end{array}$$

vbi primus terminus $13{,}2559\omega$ deſignat anomaliam mediam lunae, quae ſit $= \zeta$: tum ex ea primum quaeratur ano-

anomalia Kepleriana, quae ſcilicet a ſola excentricitate pendet, ſitque ea $= \varkappa$; vt ſit

$\varkappa = \zeta - 33464'' \sin \varkappa - 1530'' \sin 2\varkappa - 128'' \sin 3\varkappa$

vnde quidem facile tabulae conſtruentur. Tum ſtatuatur $v = \varkappa + z$, et quia angulus z eſt modicus, inde is ſatis prope poterit definiri. Interim tamen expedire videtur aliquot operationibus iterandis iſtam anomaliam veram v determinari; dum ſcilicet primum valor non nimis a veritate abhorrens pro v aeſtimando aſſumitur, ex eoque deinceps exactior colligitur; qui ſi nimis ab aſſumto diſcrepare reperiatur, ex hoc denuo exactior quaeratur, donec nulla amplius correctione fuerit opus.

XLVII.

Formula denique, cui tam diameter lunae apparens geocentrica quam parallaxis horizontalis eſt proportionalis, ex §. 40. reperitur

$$\begin{aligned}1 &- 0{,}05470 \cos v &&- 0{,}00120 \cos 2\eta\\ &- 0{,}00142 \cos 2v &&+ 0{,}00700 \cos(2\eta-2v)\\ &- 0{,}00017 \cos 3v &&+ 0{,}00073 \cos(2\eta+2v)\\ &- 0{,}00898 \cos(2\eta-v) &&- 0{,}00035 \cos(4\eta-v)\\ &- 0{,}00042 \cos(2\eta+v) &&+ 0{,}00009 \cos(4\eta+v)\\ &- 0{,}00701 \cos(2\eta-3v) &&- 0{,}00044 \cos(4\eta-3v)\\ &+ 0{,}00008 \cos(2\eta+3v) &&- 0{,}00001 \cos(4\eta+3v)\end{aligned}$$

quorum quidem terminorum plures, qui pro parallaxi infra aliquot minuta ſecunda ſubſiſtunt, tuto omitti poterunt. His igitur tribus formulis pro anomalia vera v, longitudine Φ et parallaxi ſeu diametro apparente inuentis motus lunae contineretur, ſi quidem tam ſolis paral-

 laxis

laxis quam eius excentricitas et inclinatio orbitae lunaris ad eclipticam negligatur. Hae autem ſunt inaequalitates praecipuae, quae etiam ad reliquas eruendas adhiberi debent; vnde nunc ad inaequalitates ab excentricitate ſolis oriundas progrediamur.

INUESTIGATIO INAEQUALITATUM LUNAE SECUNDI GENERIS SEU AB EXCENTRICITATE SOLIS PENDENTIUM.

XLVIII.

Formulae noſtrae differentiales, quatenus ab excentricitate orbitae ſolaris pendent, omiſſis terminis, quos iam conſtat eſſe minimos, erunt

$$\frac{d\xi}{d\omega} = \text{Praec.} + \frac{9e}{2m}\,\text{ſin}\,(2\eta - u) + \frac{9e}{2m}\,\text{ſin}\,(2\eta + u)$$

$$\begin{aligned}\frac{dq}{d\omega} = \text{Praec.} &+ \frac{3e}{4m}\,\text{ſin}\,(v-u) + \frac{3e}{4m}\,\text{ſin}\,(v+u)\\ &- \frac{27e}{8m}\,\text{ſin}\,(2\eta - v - u) - \frac{27e}{8m}\,\text{ſin}\,(2\eta - v + u)\\ &- \frac{9e}{8m}\,\text{ſin}\,(2\eta + v - u) - \frac{9e}{8m}\,\text{ſin}\,(2\eta + v + u)\end{aligned}$$

$$\begin{aligned}\frac{q(d\Phi - dv)}{d\omega} = \text{Pr.} &- \frac{3e}{4m}\,\text{coſ}\,(v-u) - \frac{3e}{4m}\,\text{coſ}\,(v+u)\\ &- \frac{27e}{8m}\,\text{coſ}\,(2\eta - v - u) - \frac{27e}{8m}\,\text{coſ}\,(2\eta - v + u)\\ &+ \frac{9e}{8m}\,\text{coſ}\,(2\eta + v - u) + \frac{9e}{8m}\,\text{coſ}\,(2\eta + v + u)\end{aligned}$$

Quan-

Quanquam enim nunc tam ξ quam q etiam ab excentricitate e pendeant, tamen in his formulis, in quas hae quantitates ingrediuntur, haec mutatio earum ſine errore pro nihilo haberi poteſt; quoniam hi termini per ſe ſunt minimi, et quia iam terminos ab e et q ſimul pendentes omiſimus. Tum vero erit

$$\frac{d\Phi}{d\omega} = m(1+\tfrac{1}{2}qq) - 2mq\,\mathrm{coſ}\,v + \tfrac{1}{2}mqq\,\mathrm{coſ}\,2v - \tfrac{1}{2}m\xi + 3mq\xi\,\mathrm{coſ}\,v$$

$$\text{et } \frac{du}{d\omega} = \frac{d\theta}{d\omega} = 1 + 2ee - 2e\,\mathrm{coſ}\,u$$

XLIX.

Ad formulas has integrandas ſeu tantum ad eas integralium partes inueniendas, quae ab excentricitate ſolis e pendent, opus eſt vt formularum $\frac{d\eta}{d\omega}$, $\frac{dv}{d\omega}$ et $\frac{du}{d\omega}$ primum habeamus partes principales, tum vero etiam eas quae a ſimplici ſolis excentricitate e pendent: habebimus ergo primo

$$\frac{d\eta}{d\omega} = m(1+\tfrac{1}{2}gg) - 1 - 2ee - 2mg\,\mathrm{coſ}\,v + 2e\,\mathrm{coſ}\,u$$

$$\frac{dv}{d\omega} = m(1+\tfrac{1}{2}gg) - 2mg\,\mathrm{coſ}\,v + \frac{3e}{4mg}\mathrm{coſ}(v-u) + \frac{3e}{4mg}\mathrm{coſ}(v+u)$$
$$-\frac{1}{4m}\left(1-\frac{9A+3B-2C}{2gg}\right) + \frac{27e}{8mg}\mathrm{cſ}(2\eta-v-u) + \frac{27e}{8mg}\mathrm{cſ}(2\eta-v+u)$$
$$-\frac{9e}{8mg}\mathrm{cſ}(2\eta+v-u) - \frac{9e}{8mg}\mathrm{cſ}(2\eta+v+u)$$

ſeu introducendis, vt ſupra §. 27. breuitatis gratia, litteris

$$\alpha = m(1+\tfrac{1}{2}gg-C) - 2ee\;;\quad \gamma = 2mg$$
$$\beta = m(1+\tfrac{1}{2}gg-C) - \frac{1}{4m}\left(1-\frac{9A+3B-2C}{2gg}\right);\quad \delta = 2mg + \frac{1}{2mg} + \frac{3g}{8m}$$

 erit:

erit: $\frac{d\eta}{d\omega}\alpha - \gamma \operatorname{coſ} v + 2e \operatorname{coſ} u$

$$\frac{dv}{d\omega} = \mathfrak{E} - \delta \operatorname{coſ} v - \frac{9}{4mg}\operatorname{coſ}(2\eta - v) + \frac{3}{4mg}\operatorname{coſ}(2\eta + v)$$
$$+ \frac{3e}{4mg}\operatorname{coſ}(v-u) + \frac{3e}{4mg}\operatorname{coſ}(v+u) + \frac{27e}{8mg}\operatorname{coſ}(2\eta - v - u)$$
$$+ \frac{27e}{8mg}\operatorname{cſ}(2\eta - v + u) - \frac{9e}{8mg}\operatorname{cſ}(2\eta + v - u) - \frac{9e}{8mg}\operatorname{cſ}(2\eta + v + u)$$

et $\frac{du}{d\omega} = 1 - 2e \operatorname{coſ} u.$

L.

Fingamus nunc primo:

$$\xi = \mathfrak{A}\operatorname{cſ} 2\eta + \mathfrak{B}\operatorname{cſ}(2\eta - v) + \mathfrak{C}\operatorname{cſ}(2\eta + v) + \mathfrak{P}\operatorname{cſ}(2\eta - u) + \mathfrak{Q}\operatorname{cſ}(2\eta + u)$$

ac differentiando eos tantum ſumamus terminos, qui formulae differentiali reſpondent, quandoquidem reliquos iam inuenimus: eritque

$$\frac{d\xi}{d\omega} = -2e\,\mathfrak{A}\operatorname{ſin}(2\eta - u) - 2e\,\mathfrak{A}\operatorname{ſin}(2\eta + u)$$
$$-(2\alpha - 1)\,\mathfrak{P} \quad -(2\alpha + 1)\,\mathfrak{Q}$$

vnde colligitur:

$$(2\alpha - 1)\,\mathfrak{P} = -\frac{9e}{2m} - 2e\,\mathfrak{A}\,;\ (2\alpha + 1)\,\mathfrak{Q} = -\frac{9e}{2m} - 2e\,\mathfrak{A}$$

Cum igitur ſit $e = 0,0168$, erit in numeris:

$$\mathfrak{P} = -0,000247 \quad \text{et} \quad \mathfrak{Q} = -0,000227.$$

LI.

Fingatur porro: $q = g$

$$+ A\operatorname{coſ}(2\eta - v) + B\operatorname{coſ}(2\eta + v)\ C\operatorname{coſ} v + M\operatorname{coſ}(v - u) + N\operatorname{coſ}(v + u)$$
$$+ P\operatorname{coſ}(2\eta - v - u) + Q\operatorname{coſ}(2\eta - v + u) + R\operatorname{cſ}(2\eta + v - u) + S\operatorname{cſ}(2\eta + v + u)$$

ac

ac differentiando obtinebitur: $\frac{dq}{d\omega} =$

$$-2eA\,\text{ſn}(2\eta-v-u)-2eA\,\text{ſn}(2\eta-v+u)-2eB\,\text{ſn}(2\eta+v-u)-2eB\,\text{ſn}(2\eta+v+u)$$

$$-(2\alpha-\beta-1)P \quad -(2\alpha-\beta+1)Q \quad -(2\alpha+\beta-1)R \quad -(2\alpha+\beta+1)S$$

$$-(\beta-1)M\,\text{ſin}\,(v-u) - (\beta+1)H\,\text{ſin}\,(v+u)$$

Comparatione ergo inſtituta reperietur:

$$(\beta-1)M = -\frac{3e}{4m} \quad ; \quad (\beta+1)N = -\frac{3e}{4m}$$

$$(2\alpha-\beta-1)P = \frac{27e}{8m} - 2eA \quad ; \quad (2\alpha-\beta+1)Q = \frac{27e}{8m} - 2eA$$

$$(2\alpha+\beta-1)R = \frac{9e}{8m} - 2eB \quad ; \quad (2\alpha+\beta+1)S = \frac{9e}{8m} - 2eB$$

et in numeris

$$M = -0{,}00008 \quad ; \quad P = +0{,}00042 \quad ; \quad R = +0{,}00004$$

$$N = -0{,}00006 \quad ; \quad Q = +0{,}00036 \quad ; \quad S = +0{,}00004$$

LII.

Hic autem in differentiatione neglaximus partes ipſius $\frac{dv}{d\omega}$ ab excentricitate e pendentes, quarum tamen eodem iure ratio haberi debuiſſet, atque partis in differentiali $\frac{d\eta}{d\omega}$; inde autem multo plures termini accedent ad valorum ipſius q, ponatur ergo ob hos terminos:

$$q = g$$

$$+ A\,\text{coſ}(2\eta-v) + B\,\text{coſ}(2\eta+v) + C\,\text{coſ}\,v + M\,\text{coſ}(v-u)$$

$$+ N\,\text{coſ}(v+u) + D\,\text{coſ}(2\eta-u) + E\,\text{coſ}(2\eta+u)$$

$$+ P\,\text{coſ}(2\eta-v-u) + Q\,\text{coſ}(2\eta-v+u) + R\,\text{coſ}(2\eta+v-u)$$

$$+ S\,\text{coſ}(2\eta+v+u) + K\,\text{coſ}(2v-u) + L\,\text{coſ}(2v+u)$$

$$+ F\,\text{coſ}(2\eta - 2v - u) + G\,\text{coſ}(2\eta - 2v + u) + H\,\text{coſ}(2\eta + 2v - u)$$
$$+ J\,\text{coſ}(2\eta + 2v + u) + T\,\text{coſ}(4\eta - u) + V\,\text{coſ}(4\eta + u)$$
$$+ W\,\text{coſ}(4\eta - 2v - u) + X\,\text{coſ}(4\eta - 2v + u) + Y\,\text{coſ}(4\eta + 2v - u)$$
$$+ Z\,\text{coſ}(4\eta + 2v + u)$$

et ſumto differentiali pleno reperiotur:

$$\frac{dq}{d\omega} = + \text{ſin}\,(2\eta - v - u)\ [-2eA - (2\alpha - \beta - 1)\,P]$$
$$+ \text{ſin}\,(2\eta - v + u)\ [-2eA - (2\alpha - \beta + 1)\,Q] - (\beta - 1)\,M\,\text{ſin}\,(v - u)$$
$$+ \text{ſin}\,(2\eta \mp v - u)\ [-2eB - (2\alpha + \beta - 1)\,R]$$
$$+ \text{ſin}\,(2\eta + v + u)\ [-2eB - (2\alpha + \beta + 1)\,S] - (\beta + 1)\,N\,\text{ſin}\,(v + u)$$
$$+ \text{ſin}\,(2\eta - u)\left(+\frac{3eA}{8mg} - \frac{3eB}{8mg} - \frac{27eC}{16mg} - \frac{9eC}{16mg} - (2\alpha - 1)\,D\right)$$
$$+ \text{ſin}\,(2\eta + u)\left(+\frac{3eA}{8mg} - \frac{8eB}{8mg} - \frac{27eC}{16mg} - \frac{9eC}{16mg} - (2\alpha + 1)\,E\right)$$
$$+ \text{ſin}\,(2\eta - 2v + u)\left(+\frac{3eA}{8mg} + \frac{27eC}{16mg} - (2\alpha - 2\beta + 1)\,G\right)$$
$$+ \text{ſin}\,(2\eta - 2v - u)\left(+\frac{3eA}{8mg} + \frac{27eC}{16mg} - (2\alpha - 2\beta - 1)\,F\right)$$
$$+ \text{ſin}\,(2\eta + 2v - u)\left(-\frac{3eB}{8mg} + \frac{9eC}{16mg} - (2\alpha + 2\beta - 1)\,H\right)$$
$$+ \text{ſin}\,(2\eta + 2v + u)\left(-\frac{3eB}{8mg} + \frac{9eC}{16mg} - (2\alpha + 2\beta + 1)\,J\right)$$
$$+ \text{ſin}\,u\left(+\frac{27eA}{16mg} - \frac{27eA}{16mg} + \frac{9eB}{16mg} - \frac{9eB}{16mg} - \frac{3eC}{8mg} + \frac{3eC}{8mg}\right)$$
$$+ \text{ſin}\,(4\eta - 2v - u)\left(+\frac{27eA}{16mg} - (4\alpha - 2\beta - 1)\,W\right)$$
$$+ \text{ſin}\,(4\eta - 2v + u)\left(+\frac{27eA}{16mg} - (4\alpha - 2\beta + 1)\,X\right)$$

+

$$+\,\text{ſin}(4\eta+2v-u)\left(+\frac{9eB}{16mg}-(4\alpha+2\beta-1)\,Y\right)$$
$$+\,\text{ſin}(4\eta+2v+u)\left(+\frac{9eB}{16mg}-(4\alpha+2\beta+1)\,Z\right)$$
$$+\,\text{ſin}(2v-u)\left(+\frac{9eA}{16mg}-\frac{27eB}{16mg}-\frac{3eC}{8mg}-(2\beta-1)\,K\right)$$
$$+\,\text{ſin}(2v+u)\left(+\frac{9eA}{16mg}-\frac{27eB}{16mg}-\frac{3eC}{8mg}-(2\beta+1)\,L\right)$$
$$+\,\text{ſin}(4\eta-u)\left(-\frac{9eA}{16mg}-\frac{27eB}{16mg}-(4\alpha-1)\,T\right)$$
$$+\,\text{ſin}(4\eta+u)\left(-\frac{9eA}{16mg}-\frac{27eB}{16mg}-(4\alpha+1)\,V\right)$$

vnde reperitur:

$D = -0{,}000010$; $H = +0{,}000001$
$E = -0{,}000010$; $J = +0{,}000001$
$F = +0{,}000005$; $K = -0{,}000006$
$G = +0{,}000065$; $L = -0.000006$
$T = +0{,}000004$; $X = -0{,}000022$
$V = +0{,}000004$; $Y = -0{,}000000$
$W = -0{,}000023$; $Z = -0{,}000000$

L'II.

Ponamus nunc etiam pro motu apogei

$$\phi - v = \text{Conſt.} + \Delta\omega$$
$$+\,A'\,\text{ſin}(2\eta-v)+B'\,\text{ſin}(2\eta+v)+C'\,\text{ſin}\,v+M'\,\text{ſin}(v-u)+N'\,\text{ſin}(v+u)$$
$$+\,P'\,\text{ſin}(2\eta-v-u)+Q'\,\text{ſin}(2\eta-v+u)+R'\,\text{ſn}(2\eta+v-u)+S'\,\text{ſn}(2\eta+v+u)$$
$$+\,D'\,\text{ſin}(2\eta-u)+E'\,\text{ſin}(2\eta+u)+K'\,\text{ſin}(2v-u)+L'\,\text{ſn}(2v+u)+O'\,\text{ſn}\,u$$
$$+\,F'\,\text{ſn}(2\eta-2v-u)+G'\,\text{ſn}(2\eta-2v+u)+H'\,\text{ſn}(2\eta+2v-u)+J'\,\text{ſn}(2\eta+2v+u)$$
$$+\,W'\,\text{ſn}(4\eta-2v-u)+X'\,\text{ſn}(4\eta-2v+u)+Y'\,\text{ſn}(4\eta+2v-u)+Z'\,\text{ſn}(4\eta+2v+u)$$
$$+\,T'\,\text{ſin}(4\eta-u)+V'\,\text{ſin}(4\eta+u)$$

ec

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et ſumto differentiali pleno reperietur :

$$\frac{d\Phi - dv}{du} = \Delta + \cos(2\eta - v - u)\,[2eA' + (2\alpha - \beta - 1)P']$$

$$+ \cos(2\eta - v + u)\,[2eA' + (2\alpha - \beta + 1)Q'] + (\beta - 1)M'\cos(v - u)$$

$$+ \cos(2\eta + v - u)\,[2eB' + (2\alpha + \beta - 1)R']$$

$$+ \cos(2\eta + v + u)\,[2eB' + (2\alpha + \beta + 1)S'] + (\beta + 1)N'\cos(v + u)$$

$$+ \cos(2\eta - u)\left(-\frac{3e}{8mg}A' + \frac{3e}{8mg}B' + \frac{27e}{16mg}C' - \frac{9e}{16mg}C' + (2\alpha - 1)D'\right)$$

$$+ \cos(2\eta + u)\left(-\frac{3e}{8mg}A' + \frac{3e}{8mg}B' + \frac{27e}{16mg}C' - \frac{9e}{16mg}C' + (2\alpha + 1)E'\right)$$

$$+ \cos(2\eta - 2v - u)\left(-\frac{3e}{8mg}A' + \frac{27e}{16mg}C' + (2\alpha - 2\beta - 1)F'\right)$$

$$+ \cos(2\eta - 2v + u)\left(-\frac{3e}{8mg}A' + \frac{27e}{16mg}C' + (2\alpha - 2\beta + 1)G'\right)$$

$$+ \cos(2\eta + 2v - u)\left(+\frac{3e}{8mg}B' + \frac{9e}{16mg}C' + (2\alpha + 2\beta - 1)H'\right)$$

$$+ \cos(2\eta + 2v + u)\left(+\frac{3e}{8mg}B' + \frac{9e}{16mg}C' + (2\alpha + 2\beta + 1)J'\right)$$

$$+ \cos u\left(-\frac{27e}{16mg}A' - \frac{27e}{16mg}A' - \frac{9e}{16mg}B' - \frac{9e}{16mg}B' + \frac{3e}{8mg}C' + \frac{3e}{8mg}C' + O'\right)$$

$$+ \cos(4\eta - 2v - u)\left(-\frac{27e}{16mg}A' + (4\alpha - 2\beta - 1)W'\right)$$

$$+ \cos(4\eta - 2v + u)\left(-\frac{27e}{16mg}A' + (4\alpha - 2\beta + 1)X'\right)$$

$$+ \cos(4\eta + 2v - u)\left(-\frac{9e}{16mg}B' + (4\alpha + 2\beta - 1)Y'\right)$$

$$+ \cos(4\eta + 2v + u)\left(-\frac{9e}{16mg}B' + (4\alpha + 2\beta + 1)Z'\right)$$

+

$$+\operatorname{coſ}(2v-u)\left(+\frac{9e}{16mg}A'+\frac{27e}{16mg}B'\mp\frac{3e}{8mg}C'+(2\beta-1)K'\right)$$

$$+\operatorname{coſ}(2v+u)\left(+\frac{9e}{16mg}A'+\frac{27e}{16mg}B'+\frac{3e}{8mg}C'+(2\beta+1)L'\right)$$

$$+\operatorname{coſ}(4\eta-u)\left(+\frac{9e}{16mg}A'+\frac{27e}{16mg}B'+(2\alpha-1)T'\right)$$

$$+\operatorname{coſ}(4\eta+u)\left(+\frac{9e}{16mg}A'+\frac{27e}{16mg}B'+(2\alpha+1)V'\right)$$

LIV.

Singuli iam hi termini multiplicentur per q, cuius valor quidem erit $=g$, quoniam hi termini in ſuo genere iam ſunt minimi: ſed quoniam valor $\frac{d\Phi-dv}{du}$ adhuc hos terminos praecipuos continet:

$$(2\alpha-\beta)A'\operatorname{coſ}(2\eta-v)+(2\alpha+\beta)B'\operatorname{coſ}(2\eta+v)+\beta C'\operatorname{coſ}v$$

ſi et hi per q multiplicentur, inde naſcentur quoque termini angulum u inuoluentem, erit autem pro his, ſumtis partibus tantum praecipuis:

$$q=\text{Praec.}+P\operatorname{coſ}(2\eta-v-u)+Q\operatorname{coſ}(2\eta-v+u)$$

Ergo ad illos terminos per q multiplicatos inſuper accedent iſti:

$\operatorname{coſ}u\;[\tfrac{1}{2}(2\alpha-\beta)PA'+\tfrac{1}{2}(2\alpha-\beta)QA']\quad+\tfrac{1}{2}\beta PC'\operatorname{coſ}(2\eta-u)$

$\operatorname{coſ}(4\eta-2v-u)\;[\tfrac{1}{2}(2\alpha-\beta)PA'+\tfrac{1}{2}\beta PC']\quad+\tfrac{1}{2}\beta QC'\operatorname{coſ}(2\eta+u)$

$\operatorname{coſ}(4\eta-2v+u)\;[\tfrac{1}{2}(2\alpha-\beta)QA'+\tfrac{1}{2}\beta QC']$

$\operatorname{coſ}(2v+u)\;[\tfrac{1}{2}(2\alpha+\beta)PB'\quad+\operatorname{coſ}(4\eta-u)\;[\tfrac{1}{2}(2\alpha+\beta)PB'$

$\operatorname{coſ}(2v-u)\;[\tfrac{1}{2}(2\alpha+\beta)QB'\quad+\operatorname{coſ}(4\eta+u)\;[\tfrac{1}{2}(2\alpha+\beta)QB'$

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LV.

Hinc ergo obtinentur ſequentes determinationes:

$$2eg\,A' + (2\alpha-\beta-1)\,g\,P' = -\frac{27e}{8m}$$

$$2eg\,A' + (2\alpha-\beta+1)\,g\,Q' = -\frac{27e}{8m}$$

$$2eg\,B' + (2\alpha+\beta-1)\,g\,R' = +\frac{9e}{8m}$$

$$2eg\,B' + (2\alpha+\beta+1)\,g\,S' = +\frac{9e}{8m}$$

$$(\beta-1)\,g\,M' = -\frac{3e}{4m} \quad ; \quad (\beta+1)\,g\,N' = -\frac{3e}{4m}$$

$$-\frac{3e}{8m}(A'-B') + \frac{9e}{8m}C' + (2\alpha-1)\,g\,D' + \tfrac{1}{2}\beta\,PC' = 0$$

$$-\frac{3e}{8m}(A'-B') + \frac{9e}{8m}C' + (2\alpha+1)\,g\,E' + \tfrac{1}{2}\beta\,QC' = 0$$

$$-\frac{3e}{8m}A' + \frac{27e}{16m}C' + (2\alpha-2\beta-1)\,g\,F' = 0$$

$$-\frac{3e}{8m}A' + \frac{27e}{16m}C' + (2\alpha-2\beta+1)\,g\,G' = 0$$

$$+\frac{3e}{8m}B' + \frac{9e}{16m}C' + (2\alpha+2\beta-1)\,g\,H' = 0$$

$$+\frac{3e}{8m}B' + \frac{9e}{16m}C' + (2\alpha+2\beta+1)\,g\,J' = 0$$

$$-\frac{27e}{8m}A' - \frac{9e}{8m}B' + \frac{3e}{8m}C' + g\,O' + \tfrac{1}{2}(2\alpha-\beta)\,(P+Q)\,A' = 0$$

$$-\frac{27e}{8m}A' + (4\alpha-2\beta-1)\,g\,W' + \tfrac{1}{2}(2\alpha-\beta)\,PA' + \tfrac{1}{2}\beta\,PC' = 0$$

$$-\frac{27e}{8m}A' + (4\alpha-2\beta+1)\,g\,X' + \tfrac{1}{2}(2\alpha-\beta)\,QA' + \tfrac{1}{2}\beta\,QC' = 0$$

$$-\frac{9e}{16m}B' + (4\alpha+2\beta-1)\,g\,Y' = 0 \quad ; \quad -\frac{9e}{16m}B' + (4\alpha+2\beta+1)\,g\,Z' = 0$$

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$$+\frac{9e}{16m}A'+\frac{27e}{16m}B'+\frac{3e}{8m}C'+(2\beta-1)gK'+\tfrac{1}{2}(2\alpha+\beta)QB'=0$$

$$+\frac{9e}{16m}A'+\frac{27e}{16m}B'+\frac{3e}{8m}C'+(2\beta+1)gL'+\tfrac{1}{2}(2\alpha+\beta)PB'=0$$

$$+\frac{9e}{16m}A'+\frac{27e}{16m}B'+(4\alpha-1)gT'+\tfrac{1}{2}(2\alpha+\beta)PB'=0$$

$$+\frac{9e}{16m}A'+\frac{27e}{16m}B'+(4\alpha+1)gV'+\tfrac{1}{2}(2\alpha+\beta)QB'=0$$

LVI.

Valores ergo horum coefficientium iam ad minuta ſecunda reductorum erunt: $O'=+310''$

$P'=-1285''$	$M'=-293''$	$F'=+401''$
$Q'=-1087$	$N'=-251$	$G'=+5412$
$R'=+148$	$D'=-52$	$H'=-2$
$S'=+141$	$E'=-45$	$J'=-2$
	$W'=-91''$	$K'=+61''$
	$X'=-95$	$L'=+61$
	$Y'=-1$	$T'=+35$
	$Z'=-1$	$V'=+31$

Vnde ob excentricitatem orbitae ſolaris erit:

$$\xi=+0{,}008931\cos 2\eta-0{,}000247\cos(2\eta-u)$$
$$+0{,}002090\cos(2\eta-v)-0{,}000227\cos(2\eta+u)$$
$$+0{,}000640\cos(2\eta+v)$$

$$q=g-0{,}013595\cos(2\eta-v)-0{,}000280\cos 2\eta$$
$$-0{,}001460\cos(2\eta+v)+0{,}014012\cos(2\eta-2v)$$
$$+0{,}002834\cos v-0{,}000162\cos(2\eta+2v)$$
$$+0{,}000340\cos 2v$$
$$-0{,}000184\cos 4\eta+0{,}000420\cos(2\eta-v-u)$$
$$+0{,}000882\cos(4\eta-2v)+0{,}000360\cos(2\eta-v+u)$$
$$+0{,}000009\cos(4\eta+2v)$$

$$\Phi - v = \text{Conſt.} + 0{,}1123\,\omega$$

$$\begin{array}{lll}
-53018''\text{ſin}(2\eta - v) + & 1533''\text{ſin}\,2\eta & -1285''\text{ſin}(2\eta - v - u) \\
-\;5532\;\text{ſin}(2\eta + v) - & 53549\;\text{ſin}(2\eta - 2v) & -1087\;\text{ſin}(2\eta - v - u) \\
+10736\;\text{ſin}\,v\quad - & 464\;\text{ſin}(2\eta + 2v) + & 148\;\text{ſin}(2\eta + v - u) \\
\quad + & 449\;\text{ſin}\,2v & +\;141\;\text{ſin}(2\eta + v + u)
\end{array}$$

$$\begin{array}{lll}
-293''\text{ſin}(v - u) & +\;401''\text{ſin}(2\eta + 2v - u) & +\;61''\text{ſin}(2v - u) \\
-251\;\text{ſin}(v + u) & +\;5412\;\text{ſin}(2\eta - 2v + u) & +\;61\;\text{ſin}(2v + u) \\
-\;52\;\text{ſin}(2\eta - u) & -\;91\;\text{ſin}(4\eta - 2v - u) & +\;35\;\text{ſin}(4\eta - u) \\
-\;45\;\text{ſin}(2\eta + u) & -\;95\;\text{ſin}(4\eta - 2v + u) & +\;31\;\text{ſin}(4\eta + u) \\
+310\;\text{ſin}\,u & &
\end{array}$$

neglectis ſcilicet terminis minimis.

LVII.

Denique pro longitudine lunae vera Φ inuenienda, cum ſit

$$\frac{d\Phi}{d\omega} = \text{Praec.}$$

$$\begin{array}{ll}
+\,mg.\;0{,}00005\,\text{coſ}(2\eta - v - u) & -\,m.\;0{,}00005\,\text{coſ}(2\eta - u) \\
+\,mg.\;0{,}00002\,\text{coſ}(2\eta - v + u) & -\,m.\;0{,}00002\,\text{coſ}(2\eta + u) \\
+\,mg.\;0{,}00005\,\text{coſ}(2\eta + v - u) & -\,m.\;0{,}00042\,\text{coſ}(2\eta - 2v - u) \\
+\,mg.\;0{,}00002\,\text{coſ}(2\eta + v + u) & -\,m.\;0{,}00036\,\text{coſ}(2\eta - 2v + u) \\
+\,mg.\;0{,}00042\,\text{coſ}(2\eta - 3v - u) & \\
+\,mg.\;0{,}00036\,\text{coſ}(2\eta - 3v + u) &
\end{array}$$

ponatur $\Phi = \text{Conſt.}$

$$+\mathfrak{A}'\omega + \mathfrak{B}'\text{ſin}\,v + \mathfrak{D}'\text{ſin}(2\eta - 2v) + \mathfrak{G}'\text{ſin}(2\eta - v) + \mathfrak{J}'\text{ſin}(2\eta - 3v)$$

vna cum nouis terminis

$$\begin{array}{llll}
+\,\mathfrak{a}'\text{ſin}(2\eta - v - u) & +\,\mathfrak{e}'\text{ſin}(2\eta - u) & +\,\mathfrak{g}'\text{ſin}(2\eta - 2v - u) & +\,\mathfrak{l}'\text{ſin}\,u \\
+\,\mathfrak{b}'\text{ſin}(2\eta - v + u) & +\,\mathfrak{f}'\text{ſin}(2\eta + u) & +\,\mathfrak{h}'\text{ſin}(2\eta - 2v + u) & +\,\mathfrak{m}'\text{ſin}(2v - u) \\
+\,\mathfrak{c}'\text{ſin}(2\eta + v - u) & & +\,\mathfrak{i}'\text{ſin}(2\eta - 3v - u) & +\,\mathfrak{n}'\text{ſin}(2v + u) \\
+\,\mathfrak{d}'\text{ſin}(2\eta + v + u) & & +\,\mathfrak{k}'\text{ſin}(2\eta - 3v + u) & \\
& & & +\,\mathfrak{o}'
\end{array}$$

$$+\mathfrak{o}'\sin(2\eta+2v-u) + \mathfrak{q}'\sin(v-u) + \mathfrak{s}'\sin(4v-u)$$
$$+\mathfrak{p}'\sin(2\eta+2v+u) + \mathfrak{r}'\sin(v+u) + \mathfrak{t}'\sin(4v+u)$$

pro reliquorum terminorum, quos forma differentialis requirit, coefficientibus ponamus litteram I.

LVIII.

Differentiatione iam per regulas praecedentes inſtituta erit: $\frac{d\Phi}{d\omega} =$ Praec.

$$+\cos u\left(\frac{3e}{8mg}\mathfrak{B}'+\frac{3e}{8mg}\mathfrak{B}'-\frac{27e}{16mg}\mathfrak{G}'-\frac{27e}{16mg}\mathfrak{G}'+\mathfrak{l}'\right)$$
$$+(2\alpha+\beta-1)\mathfrak{c}'\cos(2\eta+v-u)$$
$$+\cos(2v-u)\left(\frac{3e}{8mg}\mathfrak{B}'+\frac{9e}{16mg}\mathfrak{G}'-\frac{81e}{16mg}\mathfrak{J}'+(2\beta-1)\mathfrak{m}'\right)$$
$$+(2\alpha+\beta+1)\mathfrak{d}'\cos(2\eta+v+u)$$
$$+\cos(2v+u)\left(\frac{3e}{8mg}\mathfrak{B}'+\frac{9e}{16mg}\mathfrak{G}'-\frac{18e}{16mg}\mathfrak{J}'+(2\beta+1)\mathfrak{n}'\right)$$
$$+\cos(2\eta-u)\left(+\frac{27e}{16mg}\mathfrak{B}'-\frac{9e}{16mg}\mathfrak{B}'-\frac{3e}{8mg}\mathfrak{G}'+(2\alpha-1)\mathfrak{e}'\right)$$
$$+\cos(2\eta+u)\left(+\frac{27e}{16mg}\mathfrak{B}'-\frac{9e}{16mg}\mathfrak{B}'-\frac{3e}{8mg}\mathfrak{G}'+(2\alpha+1)\mathfrak{f}'\right)$$
$$+\cos(2\eta-2v-u)\left(+\frac{27e}{16mg}\mathfrak{B}'+2e\mathfrak{D}'-\frac{9e}{8mg}\mathfrak{G}'-\frac{9e}{8mg}\mathfrak{J}'\right.$$
$$\left.+(2\alpha-2\beta-1)\mathfrak{g}'\right)$$
$$+\cos(2\eta-2v+u)\left(+\frac{27e}{16mg}\mathfrak{B}'+2e\mathfrak{D}'-\frac{3e}{8mg}\mathfrak{G}'-\frac{9e}{8mg}\mathfrak{J}'\right.$$
$$\left.+(2\alpha-2\beta+1)\mathfrak{h}'\right)$$

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$$+ \text{coſ}\,(4v-u)\left(+\frac{27e}{16mg}\mathfrak{J}' + (4\beta-1)\,\mathfrak{s}'\right)$$

$$+ \text{coſ}\,(4v+u)\left(+\frac{27e}{16mg}\mathfrak{J}' + (4\beta+1)\,\mathfrak{t}'\right)$$

$$+ \text{coſ}\,(2\eta+2v-u)\left(-\frac{9e}{16mg}\mathfrak{B}' + (2\alpha+2\beta-1)\,\mathfrak{v}'\right)$$

$$+ \text{coſ}\,(2\eta+2v+u)\left(-\frac{9e}{16mg}\mathfrak{B}' + (2\alpha+2\beta+1)\,\mathfrak{y}'\right)$$

$$+ \text{coſ}\,(2\eta-v-u)\left(-\frac{3e}{4mg}\mathfrak{D}' + 2e\mathfrak{G}' + (2\alpha-\beta-1)\,\mathfrak{a}'\right)$$

$$+ \text{coſ}\,(2\eta-3v+u)\left(-\frac{3e}{8mg}\mathfrak{D}' + 2e\mathfrak{J}' + (2\alpha-3\beta+1)\,\mathfrak{f}'\right)$$

$$+ \text{coſ}\,(2\eta-v+u)\left(-\frac{3e}{4mg}\mathfrak{D}' + 2e\mathfrak{G}' + (2\alpha-\beta+1)\,\mathfrak{b}'\right)$$

$$+ \text{coſ}\,(2\eta-3v-u)\left(-\frac{3e}{4mg}\mathfrak{D}' + 2e\mathfrak{J}' + (2\alpha-3\beta-1)\,\mathfrak{j}'\right)$$

$$+ \text{coſ}\,(v-u)\left(-\frac{27e}{8mg}\mathfrak{D}' + (\beta-1)\,\mathfrak{q}'\right)$$

$$+ \text{coſ}\,(v+u)\left(-\frac{27e}{8mg}\mathfrak{D}' + (\beta+1)\,\mathfrak{r}'\right)$$

$$+ \text{coſ}\,(4\eta-3v-u)\left(-\frac{27e}{8mg}\mathfrak{D}' + (4\alpha-3\beta-1)\,\mathfrak{l}\right)$$

$$+ \text{coſ}\,(4\eta-3v+u)\left(-\frac{27e}{8mg}\mathfrak{D}' + (4\alpha-3\beta+1)\,\mathfrak{l}\right)$$

$$+ \text{coſ}\,(3v-u)\left(+\frac{9e}{8mg}\mathfrak{D}' + (3\beta-1)\,\mathfrak{l}\right)$$

$$+ \text{coſ}\,(3v+u)\left(+\frac{9e}{8mg}\mathfrak{D}' + (3\beta+1)\,\mathfrak{l}\right)$$

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$$+\cos(4\eta-v-u)\left(+\frac{9e}{8mg}\mathfrak{D}'+(4\alpha-\beta-1)\,\mathrm{I}\right)$$

$$+\cos(4\eta-v+u)\left(+\frac{9e}{8mg}\mathfrak{D}'+(4\alpha-\beta+1)\,\mathrm{I}\right)$$

$$+\cos(4\eta-2v-u)\left(-\frac{27e}{16mg}\mathfrak{G}'+\frac{27e}{16mg}\mathfrak{I}'+(4\alpha-2\beta-1)\,\mathrm{I}\right)$$

$$+\cos(4\eta-2v+u)\left(-\frac{27e}{16mg}\mathfrak{G}'+\frac{27e}{16mg}\mathfrak{I}'+(4\alpha-2\beta+1)\,\mathrm{I}\right)$$

$$+\cos(4\eta-u)\left(+\frac{9e}{16mg}\mathfrak{G}'+(4\alpha-1)\,\mathrm{I}\right)$$

$$+\cos(4\eta+u)\left(+\frac{9e}{16mg}\mathfrak{G}'+(4\alpha+1)\,\mathrm{I}\right)$$

$$+\cos(2\eta-4v-u)\left(-\frac{9e}{8mg}\mathfrak{I}'+(2\alpha-4\beta-1)\,\mathrm{I}\right)$$

$$+\cos(2\eta-4v+u)\left(-\frac{9e}{8mg}\mathfrak{I}'+(2\alpha-4\beta+1)\,\mathrm{I}\right)$$

$$+\cos(4\eta-4v-u)\left(-\frac{81e}{3mg}\mathfrak{I}'+(4\alpha-4\beta-1)\,\mathrm{I}\right)$$

$$+\cos(4\eta-4v+u)\left(-\frac{81e}{8mg}\mathfrak{I}'+(4\alpha-4\beta+1)\,\mathrm{I}\right)$$

LIX.

Collectis hinc valoribus coefficientium aſſumtorum, obtinebitur longitudo lunae vt ſequitur:

$\Phi =$

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$$\phi = C + 13{,}3682\,\omega$$

$-22728'' \sin v$	$+2823'' \sin(2\eta-3v)$	$+100'' \sin u$
$-1081 \sin 2v$	$+47 \sin(2\eta+3v)$	$-23 \sin(v-u)$
$-128 \sin 3v$	$-246 \sin(4\eta-v)$	$-20 \sin(v+u)$
$-700 \sin 2\eta$	$+41 \sin(4\eta+v)$	$+22 \sin(2v-u)$
$-3594 \sin(2\eta-2v)$	$+379 \sin(4\eta-3v)$	$+21 \sin(2v+u)$
$+306 \sin(2\eta+2v)$	$-2 \sin(4\eta+3v)$	$+2 \sin(3v-u)$
$-3762 \sin(2\eta-v)$		$+2 \sin(3v+u)$
$-115 \sin(2\eta+v)$		$-2 \sin(4v-u)$
		$-2 \sin(4v+u)$

$+17'' \sin(2\eta-u)$	$-1' \sin(2\eta-4v-u)$
$+19 \sin(2\eta+u)$	$-1 \sin(2\eta-4v+u)$
$+1 \sin(4\eta-u)$	$-11 \sin(4\eta-2v-u)$
$+1 \sin(4\eta+u)$	$-10 \sin(4\eta-2v+u)$
$+6 \sin(2\eta-v-u)$	$-28 \sin(4\eta-3v-u)$
$+5 \sin(2\eta-v+v)$	$-23 \sin(4\eta-3v+u)$
$+60 \sin(2\eta-2v-u)$	$-98 \sin(4\eta-4v-u)$
$-194 \sin(2\eta-2v+u)$	$-245 \sin(4\eta-4v+u)$
$-6 \sin(2\eta+2v-u)$	$+2 \sin(4\eta-v-u)$
$-6 \sin(2\eta+2v+u)$	$+2 \sin(4\eta-v+u)$
$+6 \sin(2\eta-3v-u)$	
$+8 \sin(2\eta-3v+u)$	

LX.

Plurimae igitur prodeunt inaequalitates ab excentricitate ſolis pendentes, quarum nonnullae ita ſunt magnae,

gnae, vt ſine notabili errore omitti nequeant; cuiusmodi ſunt imprimis, quae ab angulis $2\eta-2v\mp u$ et $4\eta-4v\mp u$ pendent. Sed in his fere idem incommodum vſu venit, quo methodus praecedens premebatur, quod magnitudo harum inaequalitatum per Theoriam non ſatis accurate definiri queat. Cum enim pro his terminis inueniendis diuiſores $2\alpha-2\beta\mp 1$ et $4\alpha-4\beta\mp 1$ fiant perquam exigui, manifeſtum eſt in diuidendis terminos minimos neglectos non exigui fore momenti: praecipue cum pro litteris $\mathfrak{g}'$ et $\mathfrak{h}'$ termini maiores fere ſe mutuo deſtruxiſſent. Vnde cum ex valore Φ tantum termini maiores $\mathfrak{B}'$, $\mathfrak{D}'$, $\mathfrak{G}'$ et $\mathfrak{J}'$ eſſent adhibiti, perſpicuum eſt ſi etiam reliqui minores fuiſſent introducti, ex iis inſignem mutationem in valore coefficientium $\mathfrak{g}'$ et $\mathfrak{h}'$ orituram fuiſſe.

LXI.

De his autem inaequalitatibus tenendum eſt, eas per ſatis notabile temporis ſpatium vix immutari; nam inaequalitates ab angulo $2\eta-2v+u$, ob quantitatem $2\alpha-2\beta+1=-0,1594$ periodum habent annorum circiter $6\frac{1}{3}$ annorum, et interuallo 19 annorum ter tantum reuoluuntur: et inaequalitas ab angulo $4\eta-4v+u$ pendens ſpatio 29 annorum 19 periodos abſoluit. Ex quo cum iſtae inaequalitates per theoriam ſaltem propemodum fuerint definitae, eas deinceps per obſeruationes accuratius definiri conueniet: niſi forte quis laborem in ſe ſuſcipere voluerit, calculum hic adumbratum multo accuratius inſtituendi terminorumque hic omiſſorum rationem habendi; tum vero etiam valores ξ, q et $\Phi-v$

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multo maiori ſtudio, quam hic feci, euolui oporteret, quoniam in horum determinatione multa neglexi, quae in calculo tandem ad notabilem quantitatem excreſcere potuiſſent.

LXII.

Interim tamen hic notari conuenit, hac methodo eas tantum inaequalitates prodire incertas, quae ſatis longis periodis abſoluuntur; quae incertitudo minus officit, cum per obſeruationes facilius emendari poſſit: praecedente vero methodo etiam aliae inaequalitates minoribus periodis circumſcriptae aliquantum incertae prodierunt, quod ſane ingens erat incommodum. Vnde ex hac parte haec methodus poſterior priori anteferenda videtur: verum ſi ingentem inaequalitatum numerum ſpectemus, quibus non ſolum lunae longitudo afficitur, ſed etiam longitudo apogei, calculus tantopere fit operoſus, vt etiamſi has formulas accuratiſſime euoluerem, tamen in praxi difficillimi foret vſus. Quin etiam plurimae inaequalitates in motum apogei ingredi videntur, quarum effectum deinceps per alias longitudinis inaequalitates iterum deſtrui oportet, ita vt ſatius fuiſſet illas penitus omittere.

LXIII.

Multitudo autem harum inaequalitatum, quibus tam apogei, quam ipſius lunae longitudo turbatur, inde potiſſimum originem trahit, quod inaequalitates excentricitatis prae eius quantitate media admodum ſint notabiles, atque adeo quadrantem mediae quantitatis ſuperent; ita vt prae ea negligi minime queant. Multo plures autem

tem adhuc inaequalitates eſſent acceſſurae, ſi excentricitas lunae media adhuc eſſet minor, quo certe caſu calculi difficultates inſuperabiles euaſiſſent: hoc vero ipſo caſu methodus prior multo tractabilior redderetur, tum enim pleraeque inaequalitates ibi multo minores prodirent. Atque ob hanc cauſam minus expedire videtur, anomaliam lunae ita conſtituere, vt eius ſinus tam pro maximis quam pro minimis diſtantiis lunae a terra plane euaneſcat, etiamſi haec ratio naturae rei maxime conſentanea videatur.

LXIV.

Cum igitur numerus inaequalitatum iam tantopere increuerit, facile perſpicitur eum adhuc multo magis auctum iri, ſi eas inaequalitates, quae cum a parallaxi ſolis, tum ab eius inclinatione ad eclipticam eſſem euoluturus, quo labore propterea, cum eius vſus fere nullus futurus eſſet, ſuperſedebo. Interim tamen hinc tantum colligere licet, inaequalitates ab angulis $2\eta-2v\mp u$ et $4\eta-4v\pm u$ ortas, minime eſſe contemnendas; quae cum methodo praecedente ſint vel omiſſae vel non ſatis accurate determinatae, ſine dubio cauſam in ſe continent, quod etiam accuratiſſimae tabulae per obſeruationes emendatae adhuc vltra 4′ ſaepe a veritate aberrent.

LXV.

Sufficiat igitur methodum expoſuiſſe, cuius ope inaequalitates lunae tam ratione apogei, quam longitudinis ac latitudinis verae ex anomalia hic adhibita determinari queant; neque propterea laborem calculi reli-

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quarum inaequalitatum, quae vel ex ſolis parallaxi vel ex inclinatione orbitae lunaris ad eclipticam oriuntur, ſuſcipio; quippe quarum numerus, ſiquidem omnes, quae alicuius momenti eſſent futurae, perſequi vellem, in immenſum excreſceret. Non ſolum autem multitudo inaequalitatum hanc methodum omni vtilitate in praxi priuabit, ſed etiam ingentes aequationes, quas determinatio apogei, atque anomaliae inde pendentis requirit, ita ſunt comparatae, vt ipſae iam ſatis exactam tam longitudinis quam anomaliae cognitionem requirant; quae res etſi initio ſupponi poſſent, deinceps iterata eadem operatione accuratius definiendae, tamen quia correctio apogei vltra 30° gradus aſſurgere poteſt, calculus ob inaequalitatum multitudinem per ſe taediosus, nimis crebro repeti deberet, antequam de concluſione certi eſſe poſſemus.

APPLICATIO FORMULARUM INUENTARUM AD ALIOS CALCULOS LUNARES.

LXVI.

Cum igitur calculus inaequalitatum motus lunae hactenus duplici modo ſit inſtitutus, dum priori anomalia vera regulis Keplerianis conformis eſt aſſumta, poſteriori vero ita conſtituta, vt eius ſinus tam pro maximis lunae a terra diſtantiis quam pro minimis prorſus euaneſceret, quorum vterque vti vidimus incommodis non caret: ita etiam infinitis aliis modis lunae inaequalitates repraeſentari poterunt, quos breuiter expoſuiſſe haud abs

re

re fore arbitror. Nullum enim eſt dubium, quin inter hos infinitos modos quidam reperiantur, qui ipſi naturae rei magis ſint conſentanei, neque iis incommodis laborent, quibus vtrumque expoſitum non mediocriter impediri comperimus; etiamſi adhuc difficillimum videatur, inter hanc infinitam multitudinem modum conuenientiſſimum eligere.

LXVII.

Poſtquam autem inueſtigationem ab aequationibus differentio-differentialibus ad aequationes ſimpliciter differentiales produximus, etiamſi ad hoc anomalia vera v cuius ſinus in maximis ac minimis lunae a terra diſtantiis euaneſcat, ſimus vſi, tamen haec conditio iam iterum exui poteſt. Cum enim tam ſinus quam coſinus ipſius v vbique per quantitatem q ſit multiplicatus, loco harum duarum variabilium q et v iam alias duas variabiles in calculum introducere poterimus, quod commodiſſime fiet ponendo $q \operatorname{coſ} v = r$ et $q \operatorname{ſin} v = s$, vt ſit $qq = rr + ss$ et $\operatorname{tang} v = \frac{s}{r}$, tum enim vi formularum §. IX. exhibitarum habebimus iſtas aequationes:

$$dr = -s d\Phi + \frac{2M}{A} d\omega \sqrt{A p};$$

$$ds = r d\Phi + \left(\frac{N}{A} - \frac{Ms}{A(1-r)}\right) d\omega \sqrt{A p}.$$

 LXVIII.

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LXVIII.

Hinc autem porro erit: $x = \frac{p}{1-r}$;

$$dp = -\frac{2Mp\,d\omega}{A(1-r)}\,VAp \quad \text{et} \quad d\Phi = \frac{d\omega(1-r)^2}{pp}\,VAp,$$ tum vero vt ante $du = d\theta = \frac{d\omega(1-e\,\mathrm{coſ}\,u)^2}{(1-ee)\sqrt{(1-ee)}}$.

Deinde vero ſi ſtatuamus $p = b(1+\xi)$, reliquasque denominationes in §§. 11, 12, 13. factas adhibeamus, obtinebimus:

$$\frac{M}{A}VAp = \frac{3(1+3ee)}{2m}\,\frac{(1-e\,\mathrm{coſ}\,u)^3}{1-r}\,(1+\tfrac{3}{2}\xi)\,\mathrm{ſin}\,2\eta$$
$$+\frac{3n}{8m}\cdot\frac{(1-e\,\mathrm{coſ}\,u)^4}{(1-r)^2}(1+\tfrac{5}{2}\xi)\,(\mathrm{ſin}\,\eta+5\,\mathrm{ſin}\,3\eta)$$

$$\frac{N}{A}VAp = -\frac{(1+3ee)}{2m}\cdot\frac{(1-e\,\mathrm{coſ}\,u)^3}{1-r}\,(1+\tfrac{3}{2}\xi)\,(1+3\,\mathrm{coſ}\,2\eta)$$
$$-\frac{3n}{8m}\cdot\frac{(1-e\,\mathrm{coſ}\,u)^4}{(1-r)^2}\,(1+\tfrac{5}{2}\xi)\,(3\,\mathrm{coſ}\,\eta+5\,\mathrm{coſ}\,3\eta)$$
$$+m(1-r)^2(1-\tfrac{3}{2}\xi)\,\Pi-i$$

atque

$$x = \frac{b(1+\xi)}{1-r}\,; \quad d\xi = -\frac{2(1+\xi)\,d\omega}{1-r}\cdot\frac{M}{A}VAp;$$

ac tandem:

$$d\Phi = m\,d\omega\,(1-\tfrac{3}{2}\xi+\tfrac{15}{8}\xi\xi)\,(1-r)^2$$

LXIX.

LXIX.

Hinc iam omnium differentialium rationes ad $d\omega$ habentur, erit enim

$$\frac{d\xi}{d\omega} = -\frac{2(1+\xi)}{1-r}\cdot\frac{M}{A}\sqrt{Ap}:$$

$$\frac{d\Phi}{d\omega} = m\left(1-\tfrac{3}{2}\xi+\tfrac{15}{8}\xi\xi\right)(1-r)^2$$

$$\frac{du}{d\omega} = \frac{d\theta}{d\omega} = \frac{(1-e\,\mathrm{cof}\,u)^2}{(1-ee)\sqrt{(1-ee)}} \quad \text{et} \quad \frac{d\eta}{d\omega} = \frac{d\Phi-d\theta}{d\omega}.$$

$$\frac{dr}{d\omega} = -ms\left(1-\tfrac{3}{2}\xi+\tfrac{15}{8}\xi\xi\right)(1-r)^2 + 2.\frac{M}{A}\sqrt{Ap}$$

$$\frac{ds}{d\omega} = mr\left(1-\tfrac{3}{2}\xi+\tfrac{15}{8}\xi\xi\right)(1-r)^2 + \frac{N}{A}\sqrt{Ap} - \frac{s}{1-r}.\frac{M}{A}\sqrt{Ap}$$

$$\frac{d\pi}{d\omega} = -\frac{1}{m}\,\mathrm{ſin}\,(\theta-\pi)\,\mathrm{ſin}\,(\Phi-\pi)\left(\frac{3(1+3ee)(1+\tfrac{3}{2}\xi)}{(1-r)^2}\,\mathrm{cof}\,\eta + \tfrac{3}{4}n(3+5\,\mathrm{cof}\,2\eta)\right)$$

$$\frac{d.l\,\mathrm{tg}\,\rho}{d\omega} = -\frac{1}{m}\,\mathrm{ſin}\,(\theta-\pi)\,\mathrm{cof}(\Phi-\pi)\left(\frac{3(1+3ee)(1+\tfrac{3}{2}\xi)}{(1-r)^2}\,\mathrm{cof}\,\eta + \tfrac{3}{4}n(3+5\,\mathrm{cof}\,2\eta)\right)$$

at eſt $\Pi = 1 - \lambda - \tfrac{3}{4}\,\mathrm{tang}\,\rho^2 - \tfrac{3}{4}\,\mathrm{tang}\,\rho^2\;\mathrm{cof}\,2\,(\Phi-\pi)$, vbi pro λ aſſumi poteſt $1 - \tfrac{3}{4}\,\mathrm{tang}\,\iota^2$, denotante ι inclinationem mediam orbitae lunaris ad eclipticam, vt ſit $\Pi = \tfrac{3}{4}\,\mathrm{tang}\,\iota^2 - \tfrac{3}{4}\,\mathrm{tang}\,\rho^2\;[1+\mathrm{cof}\,2\,(\Phi-\pi)]$

LXX.

LXX.

Quodſi iam pro r ſtatueretur iſte valor $k \operatorname{coſ} v$, ita vt k eſſet quantitas conſtans, oriretur modus initio traditus inaequalitates lunae repraeſentandi, foret enim tum v anomalia vera Kepleri et k denotaret excentricitatem orbitae lunaris. Vnde patet etiam inaequalitates lunae per methodum primam erutas, ex his formulis inueniri poſſe, neque ad hoc aequationibus ſecundi gradus eſſe opus. Reduceretur autem hoc caſu indoles differentio-differentialium ad inuentionem quantitatis s, quem in finem pro s aſſumi deberet ſeries quaedam ſinuum angulorum η, v, u et $\Phi - \pi$ formatorum cum indefinitis coefficientibus, quos deinceps determinare liceret: hoc autem modo ſolutio primum tradita eſſet proditura.

LXXI.

Cum ſit $p = b(1+\xi)$ et $\sqrt{A} = m\sqrt{b^3}$, ob $dx = -\frac{sd\omega}{p}\sqrt{A}p$, fiat $\frac{dx}{d\omega} = -\frac{mbs}{\sqrt{(1+\xi)}} = -mbs(1 - \frac{1}{2}\xi + \frac{3}{8}\xi\xi)$, vnde patet ſi eiusmodi anomalia v introducatur, vt ſit $s = q \operatorname{ſin} v$, ſiue q ſit quantitas conſtans ſiue variabilis, tum hanc anomaliam tam in maximis quam minimis diſtantiis ſinum euaneſcentem eſſe habituram. Ac ſi pro q quantitas vel conſtans vel ex angulis cognitis compoſita aſſumatur, tum inde coefficientes aſſumti ac praeterea valor litterae r determinabitur. Sin autem pro q eiusmodi quantitas incognita aſſumatur, vt ſit praeterea $r = q \operatorname{coſ} v$, tum ſolutio ante expoſita reſultabit.

LXXII.

LXXII.

Semper autem vſus aſtronomicus exigit, vt anomalia vera quaedam angulo v contenta introducatur, id quod infinitis modis fieri poteſt. Quo autem quantitas r variabilitatem diſtantiarum lunae a terra accuratius exprimat, et valor ipſius ξ quam minimas mutationes ſubeat, neceſſe eſt, vt quantitas r huiusmodi contineat terminum k coſv, vbi k excentricitatem deſignet, qui ſit quaſi eius pars praecipua; hocque etiam locum habet, ſi pro r ſumatur q coſv, denotante q quantitatem variabilem, quippe cuius pars potior excentricitatem k praebere debet. Verum praeterea quantitas r alios terminos continere poteſt, qui ab angulo v vel pendeant vel non pendeant: ita poni poſſet: $r = k$ coſv + A coſ2η + B coſ4η + C coſ$(2\eta - v)$ etc. quo valore aſſumto litterae quoque s, ξ cum reliquis ſuos valores debitos obtinerent.

LXXIII.

Hoc modo illud incommodum euitari poteſt, quo methodum in hoc additamento traditam laborare vidimus, ſi excentricitas orbitae lunaris eſſet nimis parua, vel adeo euaneſcens; tum enim diſtantiae maximae et minimae non amplius ab anomalia penderent, ſed potius ab angulo η, atque imprimis quidem a coſinu dupli anguli 2η. Caſu ergo quo excentricitas plane euaneſcit, pro variabili r, cuius loco vtique noua variabilis introduci debet, non conueniet anomaliam v introducere, ſed praeſtabit aſſumi ſeriem coſinuum ex ſolis angulis 2η, μ

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et $\varphi-\pi$ conſtantem, quorum coefficientes etſi ſunt cónſtantes, tamen quia terminorum numerus in infinitum excurrit, vicem novae variabilis ſuſtinebunt. Tum autem valor ipſius s ex ſimili ſerie ſinuum eorundem angulorum conſtabit.

LXXIV.

Quodſi ergo rem generatim pro quacunque excentricitate expedire velimus, poterimus ad hos terminos, qui ex hypotheſi excentricitatis euaneſcentis prodeunt, adhuc adiungere terminos ex anomalia v formatos. Ita neglectis tam inaequalitatibus parallacticis, quam iis quae tum ab excentricitate orbitae ſolaris, tum ab inclinatione orbitae lunaris ad eclipticam pendent, poni conueniet:

$$r = k\,\text{coſ}\,v + A\,\text{coſ}\,2\eta + B\,\text{coſ}(2\eta-v) + C\,\text{coſ}(2\eta+v) + D\,\text{coſ}\,4\eta + E\,\text{coſ}(4\eta-v)\ \text{etc.}$$

$$s = \Delta k\,\text{ſin}\,v + \mathfrak{A}\,\text{ſin}\,2\eta + \mathfrak{B}\,\text{ſin}(2\eta-v) + \mathfrak{C}\,\text{ſin}(2\eta+v) + \mathfrak{D}\,\text{ſin}\,4\eta + \mathfrak{E}\,\text{ſin}(4\eta-v)\ \text{etc.}$$

$$\xi = ..\,\text{coſ}\,v + ..\,\text{coſ}\,2\eta + ..\,\text{coſ}(2\eta-v) + ..\,\text{coſ}(2\eta+v) + ..\,\text{coſ}\,4\eta + ..\,\text{coſ}(4\eta-v)\ \text{etc.}$$

Atque ſi hoc modo omnes angulorum 2η et v combinationes adhibeantur, hique valores in aequationibus ſupra datis ſubſtituantur, primo inde elicietur ratio $dv : d\omega$, ac deinceps coefficientes determinationes ſuas nanciſcentur.

LXXV.

Manebunt autem coefficientes vnius ſeriei veluti ipſius r indeterminati, propterea quod ipſa ſeries haec ab

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ab arbitrio noſtro pendet, dum pro *r* vel ſolum primum terminum *k* coſ *v*, vel quotquot lubuerit, aſſumere potuiſſemus. Hinc autem id commodi conſequemur, vt iſtos coefficientes ad ſcopum quam conuenientiſſime definire valeamus: ſcilicet eos ita definiri conueniet, vt primo nullius reliquorum coefficientium determinatio lubrica et incerta euadat, vti in vtraque methodo expoſita vſu venit: deinde vero vt nulli coefficientes fiant nimis magni praeter neceſſitatem, ita vt eorum effectus per alios terminos iterum deſtrui neceſſe ſit. Fateri quidem cogor calculum hoc modo inſtituendum admodum futurum eſſe prolixum, verum fortaſſe in ipſa operatione non contemnenda ſe offerent compendia; vnde confido hanc ſpeculationem, etiamſi mihi ipſi eam ſuſcipere non vacet, vſu non eſſe carituram.

BEROLINI, EX OFFICINA MICHAELIS.

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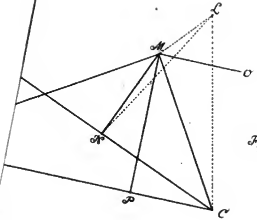
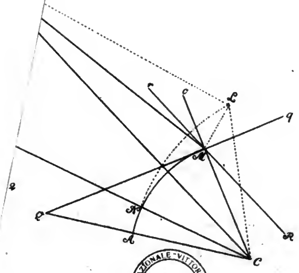


Fig. 1.



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